A STUDY ON MULTI OBJECTIVE SOLID AND MULTI OBJECTIVE INTERVAL SOLID TRANSPORTATION PROBLEMS AND THEIR SOLUTIONS WITH GREY SITUATION DECISION MAKING THEORY

[1]Feni Chevli, [2]Dr. Jayesh M. Dhodiya, [3]Dr. Mukesh Patel

[1]Assistant Professor,, [2]Assistant Professor,, [3]Assistant Professor
Mathematics Department,
Dr. S. & S. S. Ghandhy, GEC, Surat, India.

Abstract: **Nowadays, each business organization aims to provide efficient cost effective way to the customers for transport items in the specified time and minimum cost. For such kind of single and multi-objective problem they think of different ideas. Out of them one of the ideas can be using different type of transportation mode to save time and money. Taking into this factor a Transportation problem involving multiple objectives and constraints related with source, destination and conveyance formed is called as multi objective solid transportation problem (MOSTP). Now a day different approach is available for find solution of single solid transportation problem, MOSTP and MOISTP. In this paper we suggest grey situation decision making theory based method which transformed MOSTP and MOISTP into a single objective transportation problem for find efficient solution of MOSTP and MOISTP. With help of numerical example, the proposed method is illustrated.**

IndexTerms **- Solid Transportation Problem, multi objective solid transportation problem, multi objective interval solid transportation problem, Grey situation Decision making Theory, Efficient Solution, Conveyance.**

I. INTRODUCTION

The classical Transportation Problem (TP) is a special type of linear programming problem and it was originally developed by Hitchock [1]. The purpose of TP is to transport the goods from sources to destinations. TP is also used in inventory control, manpower planning, personnel allocation, allocation models etc. The Solid Transportation Problem (STP) is a generalization of the well-known classical TP. The necessity of considering this special type of TP arises when there are different types of products are to transported using heterogeneous transportation modes called conveyances. Thus, three item properties are taken into account in the constraints set of STP instead of two constraints (source and destination). The STP was stated by Shell [2] who discussed four different cases based on the given data on the item properties such as three planar sums, two planar sums, one planar and one axial sum, and three axial sums. The solid transportation problem (STP) may be considered as a special case of linear programming problem. In STP, bounds are given on three items namely, supply, demand and conveyance. In many industrial problems, a homogeneous product is delivered from an origin to a destination by means of different modes of transport called conveyances, such as trucks, cargo flights, goods trains, ships, etc.

2. MATHEMATICAL FORMULATIONS:

2.1 Mathematical formulation of Solid transportation problem:

An STP in typical form is defined as follows. Assume that a homogeneous product is to be transported from each of m sources to n destinations. The sources are production facilities, warehouses, or supply points that are characterized by the available capacities $a_i, i = 1,2,3,\ldots,m.$ The destinations are consumption points, warehouses, or demand points that are characterized by required levels of demands $b_j,\ j = 1,\ 2,\ 3,\ldots,n.$ Let e_k be the number of units transported by k^{th} type of the conveyance $e_k, k = 1,2,....,l$ from sources to destinations. The conveyances may be trucks, air freights, freight trains, and ships. A penalty or cost $c_{ijk} \geq 0$ and a cost function f_{ijk} are associated with transportation of a unit of the product from source i to the destination j by means of the conveyance k. The problem is to determine the unknown quantities x_{ijk} of the product to be transported from the each of the sources i to each of the destination j by each of the conveyances k so that the total transportation cost is minimized. The STP can be defined as a minimization problem with linear constraints in the following form.

$$\left.\begin{array}{l} \text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=i}^{n}\sum_{k=1}^{l} f_{ijk}(x_{ijk}), \text{p} = 1, 2, \ldots.., \text{P} \\ \text{Subject to} \\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = a_i, i = 1, 2, \ldots.., m \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = b_j, \text{j} = 1, 2, \ldots.., \text{n} \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k, \text{k} = 1, 2, \ldots.., l \end{array}\right\} \quad (1)$$

where (x_{ijk}) is a linear function, $x_{ijk} \geq 0$, $a_i \geq 0$, $b_j \geq 0$, $e_k \geq 0$, and $c_{ijk} \geq 0$ for all i, j, k. Also $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{l} e_k$ for balanced condition. Here (1) generalizes the classical TP, where there is only one conveyance. The most studied STP has been the linear STP, i.e. $f_{ijk}(x_{ijk}) = c_{ijk}x_{ijk}$ for all i, j, k.The existence of a feasible solution to this problem is guaranteed by Shell [2], and $m+n+l-2$ nonzero values of the decision variables becomes the nondegenerate basic feasible solution. The necessary definitions to reformulate the three axial sums problem (1) as a three planar sums have been showed by Haley [3] and also he described the procedure for the three planar sums, which is an extension of the modified distribution method for the classical TP.

2.2 Mathematical formulation of Multi Objective Solid transportation problem:

The Transportation Problem, originally developed by F.L. Hitchcock [1], is one of the most common combinatorial problems involving constraints that have been studied. In most of cases, it is required to solve the problem taking into account more than one decision criterion, thus giving place to the Multi objective Transportation Problem. A variety of approaches has been developed by many authors for the Linear Multi-objective Transportation Problem [5, 6, 7].

In most of the real-life situations, it is necessary to take into account more than one criterion or objective to reflect the problem more realistically to satisfy the given set of constraints. The objectives may be transportation cost, quantity of goods delivered, unfulfilled demand, average delivery time of the commodities, reliability of transportation, accessibility to the users, product deterioration.

A penalty or cost $c_{ijk}{}^{p} \geq 0$ is associated with transportation of a unit product from source 'i' to destination 'j' by means of the conveyance 'k' for the p^{th} decision criterion. The penalty cost could represent transportation cost, delivery time, quantity of goods delivered, duty paid, underused capacity, etc. One must determine the amount of product (unknown quantity) x_{ijk} to be transported from each source 'i' to each destination 'j' by means of each conveyance 'k' such that the total transportation cost is minimized.

Thus the Multi-Objective Solid Transportation Problem (MOSTP) is the problem of minimizing P objective functions which can be formulated as a linear programming problem as follows:

$$\left.\begin{array}{l} \textit{Minimize } Z^{p} = \sum_{i=1}^{m}\sum_{j=i}^{n}\sum_{k=1}^{l} f_{ijk}{}^{p}(x_{ijk}), \text{p} = 1, 2, \ldots.., \text{P} \\ \textit{Subject to} \\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = a_i, i = 1, 2, \ldots.., m \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = b_j, \text{j} = 1, 2, \ldots.., \text{n} \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k, \text{k} = 1, 2, \ldots.., l \end{array}\right\} \quad (2)$$

$x_{ijk} \geq 0$, $a_i \geq 0$, $b_j \geq 0$, $e_k \geq 0$, and $c_{ijk} \geq 0$ for all i, j *and* k. where superscripts on Z^{p} And on $f_{ijk}{}^{p}$ denote the p^{th} penalty criterion for $p = 1, 2, 3, \ldots, P$ and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{l} e_k$

2.3 Mathematical formulation of Multi objective Interval solid transportation problem:

A homogeneous product is to be transported from each of m sources to n destinations. The sources are production facilities, warehouses, or supply points, characterized by (uncertain) available capacities A_i, for $i = 1, 2, \ldots\ldots, m$.The destinations are consumption facilities, warehouses, or demand points, characterized by (uncertain) required levels of demand B_j, for $j = 1,2,......,n$ Let E_k $(k = 1,2,......,l)$ be the (uncertain) amount of this product which can be carried by l different modes of transport or conveyances. A penalty $c_{ijk}^{p} \geq 0$ is associated with transportation of a unit of the product from source i to destination j by means of the k^{th} conveyance for the p^{th} decision criterion. The decision criteria could be transportation cost, delivery time, quantity of goods delivered, unfulfilled demand, underused capacity, reliability of delivery, safety of delivery (spoilage), and many others [22].

One must determine the amounts x_{ijk} of the product to be transported from all sources i to all destinations j by means of each conveyance k so that all P decision criteria are taken into account in a way satisfactory to the decision maker.

The IMSTP can be stated as:

$$\left.\begin{array}{l} \min\ Z_p = \sum_{i=1}^{m}\sum_{j=i}^{n}\sum_{k=1}^{l} f_{ijk}{}^{p}(x_{ijk}),\ p = 1,2,.....,P \\ subject\ \text{to} \\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \in A_i, i = 1,2,.....,m \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \in B_j, \text{j} = 1,2,.....,\text{n} \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \in E_k, \text{k} = 1,2,.....,l \\ x_{ijk} \geq 0, \forall i,j,k \end{array}\right\} \quad (3)$$

where the subscript on Z_p and superscript on $f_{ijk}{}^{p}$ denote the p-th decision criterion, $A_i = [a_i^1, a_i^2], B_j = [b_j^1, b_j^2], E_k = [e_k^1, e_k^2], \forall i,j,k$ are intervals of possible (admissible) values for the supplies, demands and conveyance capacities respectively.

The STP is linear if the cost on a route is directly proportional to the amount transported ($f_{ijk}{}^{p}(x_{ijk}) = c_{ijk}{}^{p} x_{ijk}, \forall i,j,k,p$) ;otherwise, it is nonlinear ($f_{ijk}{}^{p}(x_{ijk}), \forall i,j,k,p$ are any type of practical or reasonable nonlinear function for the STP).

3. Literature review

The solid transportation problem (STP) may be considered as a special case of linear programming problem. In STP, bounds are given on three items namely, supply, demand and conveyance. In many industrial problems, a homogeneous product is delivered from an origin to a destination by means of different modes of transport called conveyances, such as trucks, cargo flights, goods trains, ships, etc.

In real world situation, multi-objective solid transportation problem need not to be deterministic because many parameters are not responsible for it. Many researchers focus on the solution of MOSTP. A.Nagarajan and K.Jeyaraman developed many models and methods for solving multi objective interval solid Transportation problems in stochastic environment [12, 13, 14, 21]. S.K.Das et al. [16], developed the theory and methodology for multi-objective transportation problem with interval cost, source and destination parameters. Expected value of fuzzy variable and fuzzy expected value models presented by Baoding Liu and P.Pandian and D.Anuradha [4, 17] proposed a new method called zero point method to find the optimal solution to STP.

The STP was proposed by Schell [2]. Haley [3] introduced the solution procedure of STP which is an extension of the modified distribution method. Bit et al. [16] developed the fuzzy programming model for a multi-objective STP. Vajda [11] proposed an algorithm for a multi-index transportation problem which is an extension of the modified distribution method. Basu et al. [8] provided an algorithm for finding the optimum solution of a solid fixed charge linear transportation problem. Gen et al.[24] gave a genetic algorithm for solving a bicriteria STP with fuzzy numbers. Jimenez and Verdegay [10] investigated interval multi-objective STP via genetic algorithms. Jimenez and Verdegay [9 obtained a solution procedure for uncertain STP. Many types of multi-objective STP models with uncertain variables are investigated by Dalman et al. [18]. Model parameters in most mathematical programming problems need to be addressed as interval parameters due to weak data for a certain evaluation, but with familiar extreme limits of the parameters. The interval uncertainty theory was presented by Mooer [19] Ishibuchi and Tanaka [15] used it for solving linear programming problems with interval objective functions by transforming those into the multi-objective programming problem. Chanas and Kuchta [20] suggested α -level-cut of the intervals. Thus, they obtained the deterministic non-linear functions in the objective and constraints. In interval uncertainty, Dalman et al. [18] suggested a solution procedure for solving non-linear multi-objective TP.

Bit et al [7] proposed a fuzzy programming approach to MSTP. In all of the cases above cited, the supplies and demands (and conveyance capacities for the solid case) are given as point values, but often there is uncertainty on this input data, and decision makers are more comfortable expressing them as intervals. Chanas et al. [21] proposed a solution approach to the interval TP, and here we propose a NGA based solution method to the Interval MSTP (IMSTP) taking into account too the case in which the objective functions are nonlinear.

4. GREY SITUATION DECISION MAKING THEORY AND FUZZY PROGRAMMING TECHNIQUE BASED APPROACH TO FIND SOLUTION OF MOSTP and MOISTP

4.1 GREY SITUATION DECISION MAKING THEORY AND FUZZY PROGRAMMING TECHNIQUE BASED APPROACH TO FIND SOLUTION OF MOSTP

Consider some notations to define the variables and the sets in multi-objective transportation problem.

Let $O = \{O_1, O_2, ..., O_m\}$ be the set of m-origins having $a_i \left(i = 1, 2, ..., m\right)$ units of supply respectively. Let $D = \{D_1, D_2, ..., D_n\}$ be the set of n-destinations with $b_j (j = 1, 2, ..., n)$ units of requirement respectively. Let $E = \{\mathrm{E}_1, \mathrm{E}_2, ..., \mathrm{E}_l\}$ be the set of l -conveyances having $e_k \left(k = 1, 2, ..., l\right)$ units of supply respectively.

There is a penalty p_{ijk} associated with transporting a unit of product from i^{th} source to destination with k conveyance. This penalty may be cost or delivery time or safety of delivery etc. A variable x_{ijk} represent the unknown quantity to be shipped from i^{th} source to j^{th} destination with k conveyance. The problem is to determine the transportation schedule when multiple objectives exist.

Grey situation decision making theory is used to minimize or maximize the total transportation penalty according to the problem which satisfying supply, demand and conveyance conditions. Consider the set of m-origins $O = \{O_1, O_2, ..., O_m\}$ as the set of events, the set of n destinations $D = \{D_1, D_2, ..., D_n\}$ as the set of countermeasure, $E = \{\mathrm{E}_1, \mathrm{E}_2, ..., \mathrm{E}_l\}$ as the set of conveyance the penalty p_{ijk} as the situation set denotes by $P = \{p_{ijk} = (O_i, D_j, \mathrm{E}_k) / O_i \in O, D_j \in D, \mathrm{E}_k \in E\}$

First of all confirm the decision making goals (objectives) and seek the corresponding effect measure matrix $U^{(r)}$

$$U^{(r)} = \left[u_{ijk}^{(\mathrm{r})}\right] = \begin{bmatrix} u_{111}^{(\mathrm{r})} & u_{112}^{(\mathrm{r})} & ...... & u_{1nl}^{(\mathrm{r})} \\ u_{211}^{(\mathrm{r})} & u_{212}^{(\mathrm{r})} & ...... & u_{2nl}^{(\mathrm{r})} \\ ...... & ...... & ...... & ...... \\ u_{m11}^{(\mathrm{r})} & u_{m12}^{(\mathrm{r})} & ...... & u_{mnl}^{(\mathrm{r})} \end{bmatrix}$$

Here, the data of decision making goals for transporting a product is the effect value $u_{ijk}^{(\mathrm{r})}$ of situation $p_{ijk} \in P$ with objective r = 1, 2... s.

Now, find the upper effect measure and lower effect measure by formula:

- Upper effect measure

  $$r_{ijk}^{(\mathrm{r})} = u_{ijk}^{(\mathrm{r})} \Big/ \max_i \max_j \max_k \left\{u_{ijk}^{(\mathrm{r})}\right\}$$

- Lower effect measure

  $$r_{ijk}^{(\mathrm{r})} = \min_i \min_j \min_k \left\{u_{ijk}^{(\mathrm{r})}\right\} \Big/ u_{ij}^{(\mathrm{r})}$$

and achieve the consistent matrix of effect measure $R^{(\mathrm{r})}$ by using upper effect measure and lower effect measure as

$$R^{(\mathrm{r})} = \left[r_{ijk}^{(\mathrm{r})}\right] = \begin{bmatrix} r_{111}^{(\mathrm{r})} & r_{112}^{(\mathrm{r})} & ...... & r_{1nl}^{(\mathrm{r})} \\ r_{211}^{(\mathrm{r})} & r_{212}^{(\mathrm{r})} & ...... & r_{2nl}^{(\mathrm{r})} \\ ...... & ...... & ...... & ...... \\ r_{m11}^{(\mathrm{r})} & r_{m12}^{(\mathrm{r})} & ...... & r_{mnl}^{(\mathrm{r})} \end{bmatrix}$$

Subtract each data of comprehensive matrix R of effect measure from 1 to convert combine maximization objective in minimization form and then find solutions of this grey theory based combined single objective problem from comprehensive matrix r_{ij} of effect measure and by using given supply, demand and conveyance as like LPP problem.

In fuzzy programming technique, we first find the lower bound as L_k and the upper bound as U_k for the K^{th} objective function $Z_k, k = 1, 2, ..., K$ where U_k is the highest acceptable level of achievement for objective k , L_k the aspired level of achievement for objective k and $d_k = U_k - L_k$ the degradation allowance for objective k . When the aspiration levels for each of the objective have been specified, a fuzzy model is formed and then the fuzzy model is converted into a crisp model. Here, in this developed approach we first utilized Grey situation decision making theory is utilized to Find the lower effect measure $r_{ij}^{(k)}$ and upper effect measure $r_{ij}^{(k)}$ and accomplish the consistent matrix of effect measure $R^{(k)} = [r_{ij}^{(k)}]$ for each objective k. These matrices of each objective are utilized as a cost matrix of each objective in fuzzy programming technique and solution are obtained. So here Grey situation decision making theory is utilized for normalization of data.

4.1.1 Algorithm for finding solution of MOSTP using grey situation decision making theory with linear membership function and hyperbolic membership function:

Input

Effect measure matrix $U^{(k)} = \left(U^{(1)}, U^{(2)}, ..., U^{(s)}; n \times m\right)$

Output
-| *Solution of MOSTP*
-| *Compute the efficient solution of MOSTP using the optimization model.*
-| *Solve MOSTP*
begin
***Read**: Example*
while example = MOSTP do
***for** k=1 to s do*
Enter effect measure matrix $U^{(k)}$
end
-| *Find the lower effect measure* $r_{ij}^{(k)}$ *and upper effect measure* $r_{ij}^{(k)}$ *and accomplish the*
-|*Consistent matrix of effect measure* $R^{(k)} = [r_{ij}^{(k)}]$.
***for** k=1 to s do*

$$r_{ij}^{(k)} = \min_i \min_j \left\{u_{ij}^{(k)}\right\} \Big/ u_{ij}^{(k)}$$

$$r_{ij}^{(k)} = u_{ij}^{(k)} \Big/ \max_i \max_j \left\{u_{ij}^{(k)}\right\}$$

$$R^{(k)} = \left[r_{ij}^{(k)}\right] = \begin{bmatrix} r_{11}^{(k)} & r_{12}^{(k)} & \ldots\ldots & r_{1m}^{(k)} \\ r_{21}^{(k)} & r_{22}^{(k)} & \ldots\ldots & r_{2m}^{(k)} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ r_{n1}^{(k)} & r_{n2}^{(k)} & \ldots\ldots & r_{nm}^{(k)} \end{bmatrix}$$

end
-| *Subtract each data of consistent matrix of effect measure from 1 to convert combine Maximization objective in minimization form for all objectives.*
-|*Find optimal solution to each objective by using simplex method.*
-|*Find pay off matrix by using each objective solution.*
-|*Define linear as well as hyperbolic membership function using payoff matrix.*
-|*Developed single objective optimization problem using fuzzy linear membership Function and hyperbolic function.*
-|*Solve model developed in step-6 and find compromise solution.*

4.2 **Algorithm for finding solution MOITP with linear and hyperbolic membership function using MGSD theory:**

Input: *Effect measure matrix* $U^{(k)} = \left(U^{(1)}, U^{(2)}, \ldots\ldots\ldots, U^{(s)}; n \times m\right)$

Output: *Solution of MOISTP*
-| *Compute the efficient solution of MOISTP using the optimization model of objective weight.*
-| *Solve MOISTP*
begin
Step-1 Read: Example
while example = MOISTP do
***for** k=1 to s do*
enter effect measure matrix $U^{(k)}$
end
-|*Find the lower effect measure* $r_{ij}^{(k)}$ *and upper effect measure* $r_{ij}^{(k)}$ *and accomplish the consistent matrix of effect measure* $R^{(k)} = [r_{ij}^{(k)}]$.
***for** k=1 to s do*

$$r_{ij}^{(k)} = \min_i \min_j \left\{u_{ij}^{(k)}\right\} \Big/ u_{ij}^{(k)}$$

$$r_{ij}^{(k)} = u_{ij}^{(k)} \Big/ \max_i \max_j \left\{u_{ij}^{(k)}\right\}$$

$$R^{(k)} = \left[r_{ij}^{(k)}\right] = \begin{bmatrix} r_{11}^{(k)} & r_{12}^{(k)} & \ldots\ldots & r_{1m}^{(k)} \\ r_{21}^{(k)} & r_{22}^{(k)} & \ldots\ldots & r_{2m}^{(k)} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ r_{n1}^{(k)} & r_{n2}^{(k)} & \ldots\ldots & r_{nm}^{(k)} \end{bmatrix}$$

end
-| *Get the comprehensive matrix of effect measure for situation* s_{ij} *is* $R = [r_{ij}]$.
-| *Apply same process for remaining objectives*
-| *Subtract each data of comprehensive matrix R of effect measure from 1 to convert combine maximization objective in minimization form for all objectives.*

-| *Find optimal solution to each objective by using the standard method.*
-| *Find upper & lower value for each objective.*
-| *Solve above problem using fuzzy linear membership function and hyperbolic function*.
-| *Find solutions for multi-objective transportation problem from comprehensive matrix* $R=[r_{ij}]$ *of effect measure using modified distribution method in LINGO package.*
end

5. **Numerical illustrations:**

This section discussed about multi objective solid transportation problem solution with Grey situation decision making theory and fuzzy programming technique based approach as well as its comparison with other approach.

5.1 Numerical illustration-1:

To illustrate the efficiency of the proposed method we consider the following numerical example [25]:
The problem has the following characteristics:
Supplies: $a_1=24, a_2=8, a_3=18, a_4=10$
Demands: $b_1=11, b_2=19, b_3=21, b_4=9$
Conveyances capacities: $e_1=17, e_2=31, e_3=12$
Where $O_i = origin$ i,$D_j = destination$ j,$C_k = conveyance$ k
Penalties $c_{ijk}{}^1$

	D_1			D_2			D_3			D_4		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
O_1	15	18	17	12	22	13	10	4	12	8	11	13
O_2	17	20	19	21	21	22	21	19	18	30	10	23
O_3	14	11	12	25	34	33	20	16	15	21	23	22
O_4	22	18	13	24	35	32	18	21	14	13	23	20

Penalties $c_{ijk}{}^2$

	D_1			D_2			D_3			D_4		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
O_1	6	7	8	10	6	5	11	3	7	10	9	6
O_2	13	8	11	12	2	9	20	15	13	17	15	13
O_3	5	6	7	11	9	7	10	5	2	15	14	18
O_4	13	6	6	17	11	18	12	16	12	18	14	7

Solution:
Step-1: Confirm the decision-making goals time, cost and effect measure matrix under three goals are given below.

$$U^{(1)}=\begin{bmatrix}15&18&17&12&22&13&10&4&12&8&11&13\\17&20&19&21&21&22&21&19&18&30&10&23\\14&11&12&25&34&33&20&16&15&21&23&22\\22&18&13&24&35&32&18&21&14&13&23&20\end{bmatrix} U^{(2)}=\begin{bmatrix}6&7&8&10&6&5&11&3&7&10&9&6\\13&8&11&12&2&9&20&15&13&17&15&13\\5&6&7&11&9&7&10&5&2&15&14&18\\13&6&6&17&11&18&12&16&12&18&14&7\end{bmatrix}$$

Step-2: Select the minimum number from c_{ijk} where c_{ijk} =element of i supply, j demand, k conveyance row wise and take the ratio of minimum number and c_{ijk} for all objective and Select the minimum number from c_{ijk} column wise and take the ratio of minimum number and c_{ijk} for all objectives

For transporting a product, time and cost are less than its the batter, so use lower effect measure. So the lower effect measure for first data $r_{111}^{(1)}=\dfrac{\min\limits_{1}\min\limits_{1}\{u_{111}\}}{u_{111}}=\dfrac{4}{15}$

Similarly obtain lower effect measure for each data. Therefore the consistent matrices of effect measure are given below and subtract each data from 1 to convert combine maximization objective in minimization form therefore

$$R^{(1)}=\begin{bmatrix}0.7333&0.7778&0.7647&0.6667&0.8182&0.6923&0.6&0&0.6667&0.5&0.6364&0.6923\\0.4118&0.5&0.4737&0.5238&0.5238&0.5455&0.5238&0.7895&0.4444&0.7333&0&0.5652\\0.2143&0&0.0833&0.56&0.6765&0.6667&0.5&0.75&0.2667&0.6190&0.5652&0.5\\0.4091&0.3889&0.4783&0.5&0.6286&0.5938&0.4444&0.8095&0.1429&0.3846&0.5652&0.35\end{bmatrix}$$

$$R^{(2)} = \begin{bmatrix} 0.5 & 0.5714 & 0.625 & 0.7 & 0.6667 & 0.4 & 0.7273 & 0 & 0.7143 & 0.7 & 0.6667 & 0.5 \\ 0.8462 & 0.75 & 0.8182 & 0.8333 & 0 & 0.7778 & 0.9 & 0.8667 & 0.8462 & 0.8824 & 0.8667 & 0.8462 \\ 0.6 & 0.6667 & 0.7143 & 0.8182 & 0.7778 & 0.7143 & 0.8 & 0.6 & 0 & 0.8667 & 0.8571 & 0.8889 \\ 0.6154 & 0 & 0 & 0.6471 & 0.8182 & 0.7222 & 0.5 & 0.8125 & 0.8333 & 0.6667 & 0.5714 & 0.1429 \end{bmatrix}$$

Step-3: Find solutions for multi-objective transportation problem from consistent matrix of effect measure using simplex method. And we have

$$\text{Pay-off matrix} = \begin{bmatrix} Z_1(X^1) & Z_2(X^1) \\ Z_1(X^2) & Z_2(X^2) \end{bmatrix} = \begin{bmatrix} 11.571 & 20.9563 \\ 21.8545 & 14.5608 \end{bmatrix}$$

Applying fuzzy linear membership function, we get
Solution of this model is
$\lambda = 0.6388595 \ \ \& \ Z_1 = 808.7965, Z_2 = 329.565148$
Applying fuzzy hyperbolic membership function the model is
Maximize λ
Constraints Subject to the

Solution of this model is (Where, $a_1 = a_2 = a_3 = 1$)
$\lambda = 0.841084 \ \ \& \ Z_1 = 808.7965, Z_2 = 329.565148$

5.1.1 Comparison of multi objective Grey situation decision making theory approach with other approaches

Table-1 indicates the grey situation decision making theory approach can provided an alternative approach to find the solution of MOSTP-1.

Table-1: Comparison of developed approach and other approaches of MOSTP-1

multi objective Grey situation decision making theory approach	**Other approach [25]**
By GSD $Z_1 = 808.7965, Z_2 = 329.565148$	A genetic algorithm for the multi objective solid transportation problem: a fuzzy approach $Z_1 = 715, Z_2 = 394$

5.2 Numerical illustration-2:

To illustrate the efficiency of the proposed method we consider another numerical example presented in [23]:
Suppose that there are two coal mines to supply the coal for three cities, and two kinds of conveyances are available to be determined i.e. train and cargo ship. Here, the decision maker should make a transportation plan for the next month such that the transportation cost minimized, simultaneously. To illustrate the recommended solution procedures for an MOSTP, let us consider the following data:
Here $\tilde{c}_{ijk} = [\underline{c}_{ijk}, \overline{c}_{ijk}]$
Interval transportation cost for 1st objective $\tilde{c}_{ijk}{}^1 = [\underline{c}_{ijk}{}^1, \overline{c}_{ijk}{}^1]$

$$\underline{c}_{ijk}{}^1 = \begin{bmatrix} \underline{c}_{111}{}^1 & \underline{c}_{112}{}^1 & \underline{c}_{121}{}^1 & \underline{c}_{122}{}^1 & \underline{c}_{131}{}^1 & \underline{c}_{132}{}^1 \\ \underline{c}_{211}{}^1 & \underline{c}_{212}{}^1 & \underline{c}_{221}{}^1 & \underline{c}_{222}{}^1 & \underline{c}_{231}{}^1 & \underline{c}_{232}{}^1 \end{bmatrix} = \begin{bmatrix} 6.5 & 10 & 5 & 7 & 11 & 8 \\ 9 & 10.5 & 6.5 & 7 & 12 & 15 \end{bmatrix}$$

$$\overline{c}_{ijk}{}^1 = \begin{bmatrix} \overline{c}_{111}{}^1 & \overline{c}_{112}{}^1 & \overline{c}_{121}{}^1 & \overline{c}_{122}{}^1 & \overline{c}_{131}{}^1 & \overline{c}_{132}{}^1 \\ \overline{c}_{211}{}^1 & \overline{c}_{212}{}^1 & \overline{c}_{221}{}^1 & \overline{c}_{222}{}^1 & \overline{c}_{231}{}^1 & \overline{c}_{232}{}^1 \end{bmatrix} = \begin{bmatrix} 10 & 14 & 10 & 11 & 15 & 13 \\ 14 & 14 & 8.5 & 11 & 16.5 & 17 \end{bmatrix}$$

Interval transportation cost for 2nd objective $\tilde{c}_{ijk}{}^2 = [\underline{c}_{ijk}{}^2, \overline{c}_{ijk}{}^2]$

$$\underline{c}_{ijk}{}^2 = \begin{bmatrix} \underline{c}_{111}{}^2 & \underline{c}_{112}{}^2 & \underline{c}_{121}{}^2 & \underline{c}_{122}{}^2 & \underline{c}_{131}{}^2 & \underline{c}_{132}{}^2 \\ \underline{c}_{211}{}^2 & \underline{c}_{212}{}^2 & \underline{c}_{221}{}^2 & \underline{c}_{222}{}^2 & \underline{c}_{231}{}^2 & \underline{c}_{232}{}^2 \end{bmatrix} = \begin{bmatrix} 9.5 & 12 & 6.5 & 6.5 & 10.5 & 13.5 \\ 12 & 15 & 8 & 10 & 13 & 13.5 \end{bmatrix}$$

$$\overline{c}_{ijk}{}^2 = \begin{bmatrix} \overline{c}_{111}{}^2 & \overline{c}_{112}{}^2 & \overline{c}_{121}{}^2 & \overline{c}_{122}{}^2 & \overline{c}_{131}{}^2 & \overline{c}_{132}{}^2 \\ \overline{c}_{211}{}^2 & \overline{c}_{212}{}^2 & \overline{c}_{221}{}^2 & \overline{c}_{222}{}^2 & \overline{c}_{231}{}^2 & \overline{c}_{232}{}^2 \end{bmatrix} = \begin{bmatrix} 12.5 & 14.5 & 11 & 10 & 12 & 14 \\ 13 & 19 & 13 & 13.5 & 17 & 15.5 \end{bmatrix}$$

The following notations $\tilde{a}_i, \tilde{b}_j$ and $\tilde{e}_k \, \forall$i=1,2,....,m,j=1,2,....,n,k=1,2,....,l are used to express the interval supply capacities, the interval demands, and the interval transportation capacities, respectively.

Interval parameters of supplies,
$\tilde{a}_1 = [22.5, 27], \tilde{a}_2 = [30, 36]$
Interval parameters of demands,
$\tilde{b}_1 = [15, 20.5], \tilde{b}_2 = [18.5, 23.5], \tilde{b}_3 = [13.5, 19.5]$
Interval parameters of transportation capacities.
$\tilde{e}_1 = [47.5, 52], \tilde{e}_2 = [52, 57.5]$

Solution:

Step-1:

$$U^1 = \begin{bmatrix} [6.5,10] & [10,14] & [5,10] & [7,11] & [11,15] & [8,13] \\ [9,14] & [10.5,14] & [6.5,8.5] & [7,11] & [12,16.5] & [15,17] \end{bmatrix},$$

$$U^2 = \begin{bmatrix} [9.5,12.5] & [12,14.5] & [6.5,11] & [6.5,10] & [10.5,12] & [13.5,14] \\ [12,13] & [15,19] & [8,13] & [10,13.5] & [13,17] & [13.5,15.5] \end{bmatrix}$$

Step-2: Convert each interval objective of transportation problem in fixed single valued transportation problem by using formula $[0.5(a+b)] + \in .(b-a)$ where interval is given by in the form of $[a,b]$ and $a < b. \in$ is parameter. Here $-0.5 \le \in \le 0.5$

For $\in = 0$ $U^1 = \begin{bmatrix} 8.25 & 12 & 7.5 & 9 & 13 & 10.5 \\ 11.5 & 12.25 & 7.5 & 9 & 14.25 & 16 \end{bmatrix}$, $U^2 = \begin{bmatrix} 11 & 13.25 & 8.75 & 8.25 & 11.25 & 13.75 \\ 12.5 & 17 & 10.5 & 11.75 & 15 & 14.5 \end{bmatrix}$

Step-3: Select the minimum number from c_{ijk} *where* $c_{ijk} = element\ of\ i\ supply, j\ demand, k\ conveyance$ row wise and take the ratio of minimum number and c_{ijk} for all objective and Select the minimum number from c_{ijk} *where* $c_{ijk} = element\ of\ i\ supply, j\ demand, k\ conveyance$ column wise and take the ratio of minimum number and c_{ijk} for all objective

For transporting a product, time and cost are less than its the batter, so use lower effect measure. So the lower effect measure for first data $r_{111}^{(1)} = \frac{\min_1 \min_1 \{u_{111}\}}{u_{111}} = \frac{7.5}{9}$

Similarly obtain lower effect measure for each data. Therefore the consistent matrices of effect measure are given below and subtract each data from 1 to convert combine maximization objective in minimization form therefore

$$R^{(1)} = \begin{bmatrix} 0.0909 & 0.375 & 0 & 0.1667 & 0.4231 & 0.2857 \\ 0.3478 & 0.3878 & 0 & 0.1667 & 0.4737 & 0.5312 \end{bmatrix}, R^{(2)} = \begin{bmatrix} 0.25 & 0.3774 & 0.0571 & 0 & 0.2667 & 0.4 \\ 0.16 & 0.3824 & 0.1667 & 0.2979 & 0.3 & 0.2759 \end{bmatrix}$$

Step-4: Find solutions for multi-objective transportation problem from consistent matrix of effect measure using simplex method. We have

$$\text{Pay-off matrix} = \begin{bmatrix} Z_1(X^1) & Z_2(X^1) \\ Z_1(X^2) & Z_2(X^2) \end{bmatrix} = \begin{bmatrix} 6.4879 & 12.54245 \\ 16.82735 & 6.36465 \end{bmatrix}$$

Applying fuzzy linear membership function, we get the following model
Solution of this model is

$$\lambda = 0.5965777 \quad \& \quad Z_1 = 532.6203, Z_2 = 604.7216$$

Applying fuzzy hyperbolic membership function the model is $\lambda = 0.761139$ & $Z_1 = 532.6203, Z_2 = 604.7216$
Same process applying for remaining values of $\in$.

Table: 2 for remaining values of $\in$

Value of $\in$	**1st objective** Z_1	**2nd objective** Z_2	**Linear** λ	**hyperbolic** λ
–0.5	420.9951	517.7306	0.9759645	0.996703
–0.4	446.1476	534.3097	0.9573252	0.995881
–0.3	471.7445	552.8467	0.9059093	0.992392
–0.2	491.4978	567.9615	0.7641623	0.959688
–0.1	510.8989	586.0501	0.6563514	0.867176
0	532.6203	604.7216	0.5965777	0.761139
0.1	545.6448	592.1629	0.5375526	0.610789
0.2	564.2672	605.0057	0.5381794	0.612576
0.3	581.5031	620.5507	0.5437651	0.628359
0.4	598.9166	636.1298	0.5479967	0.640138

0.5	612.6369	653.1974	0.5780226	0.718346

5.3.1 Comparison of multi objective Grey situation decision making theory approach with other approaches:

Table-3 indicates the grey situation decision making theory approach can provided an alternative approach to find the solution of MOISTP.

Table-3: Comparison of developed approach and other approaches of MOISTP

multi objective Grey situation decision making theory approach	**Other approach [23]**
By GSD for $\in = 0, Z_1 = 532.6203, Z_2 = 604.7216$	By A Fuzzy programming approach for interval multi objective solid transportation problem $Z_1 = [407.625, 608.5]$ $Z_2 = [583.125, 730.625]$

6. **Conclusion:**

In this paper linear membership function & hyperbolic membership function are used to solve MOSTP and MOISTP using Multi objective grey situation decision making theory. Using MGSD theory MOSTP and MOISTP converted into simple transportation problem. And it can be solved by different standard method to get optimal solution. It provides alternative optimal solution of MOSTP and MOISTP.

REFERENCES

[1] F.L.Hitchcock, The distribution of a product from several sources to numerous localities, J.Math.Phys.,20(1941),224-230.

[2] E.Shell, Distribution of a product by several properties, Directorate of Management Analysis, Proc. 2nd Symp.on Linear programming, Vol. 2, 1955, pp. 615-642,DCS/Comptroller H.Q. U.S.A.F., Washington, DC.

[3] K.B. Haley, The solid transportation problem, Oper. Res., 11 (1962), 446–448.

[4] P.Pandian and G. Natarajan, A new method for finding an optimal solution for transportation problems, International J. of Math. Sci. and Engg. Appls. ,4 (2010), 59-65.

[5] S.M.Lee and L.J.Moore ,optimizing transportation problems with multiple objectives ,AIEE Transactions,5(1973),333-338

[6] A.K. Bit, M.P. Biswal and S.S. Alam, Fuzzy programming approach to multi criteria decision making transportation problem, Fuzzy Sets and Systems, vol. 50, pp. 135-141, 1992.

[7] A.K. Bit, M.P. Biswal, S.S. Alam, Fuzzy programming approach to multi objective solid transportation problem. Fuzzy Sets and Systems, vol. 57, pp. 183-194, 1993.

[8] M. Basu, B.B.Pal and A. Kundu, An algorithm for finding the optimum solution of solid fixed charge transportation problem, 31 (1994), 283-291.

[9] F. Jimenez and J.L. Verdegay, Uncertain solid transportation problems, Fuzzy Sets and Systems, 100 (1998), 45–57.

[10] F. Jimencz and J.L. Verdegay, Interval multiobjective solid transportation problem via genetic algorithms, Management of Uncertainty in Knowledge-Based Systems, II (1996), 787–792.

[11] S.Vajda, Readings in Linear Programming, Pitman, London, 1988.

[12] A.Nagarajan and K.Jeyaraman, Mathematical modeling of solid fixed cost bi-criterion indefinite quadratic transportation problem under stochastic environment,*Emerging journal of engineering science and technology*, Volume-02, Issue No-03, March 2009 –May 2009, pp.106 – 127.

[13] A.Nagarajan and K.Jeyaraman, Solution of expected value model for multi-objective interval solid transportation problem under stochastic environment using fuzzy programming approach, *Emerging journal of engineering science and technology,* Volume-04, Issue No-07, September 2009 – November 2009, pp. 53 – 70.

[14] A.Nagarajan and K.Jeyaraman, Chance constrained goal programming models for multi-objective interval solid transportation problem under stochastic environment, *International journal of applied mathematics analysis and applications,* Volume-05, Issue No-02, July- December 2010, pp. 229 – 255.

[15] H.Ishibuchi, H.Tanaka, multiobjective programming in optimization of the interval objective function, *European Journal of Operations Research* **48** (1990)219-225.

[16] S.K.Das, A.Goswami and S.S.Alam, Multi-objective transportation problem with interval cost, source and destination parameters, *European Journal of Operations Research* **117**(1999)100-112.

[17] P. Pandian and D. Anuradha, A New Approach for Solving Solid Transportation Problems, ,Applied Mathematical Sciences, Vol. 4, 2010, no. 72, 3603 – 3610.

[18] Dalman H, Nuran G, Sivri M (2016) A fuzzy set-based approach to multi-objective multi-item solid transportation problem under uncertainty. Int J Fuzzy Syst 18(4):716–729

[19] Moore RE (1966) Interval analysis, vol 4. Prentice-Hall, Englewood Cliffs

[20] Chanas S, Kuchta D (1996) Multiobjective programming in optimization of interval objective functions—a generalized approach. Eur J Oper Res 94(3):594–598

[21] S. Chanas, M. Delgado, J.L. Verdegay, M.A. Vila (1993). Interval and fuzzy extensions of classical transportation problems. Transportation Planning and Technology 17(2):203-218. Special Issue: Application of Fuzzy Set Theory to Transportation 1

[22] Fernado jimenez, Jose verdegay, Interval Multiobjective Solid Transportation Problem via Genetic Algorithms Dpto. Informatica y Sistemas Facultad de Informatica Universidad de Murcia.

[23] Hasan Dalman,A Fuzzy programming approach for interval multiobjective solid transportation problem, NTMSCI 4, No. 4, 114-127 (2016)

[24] Gen, M., K. Ida, E. Kono and Y. Li:"Solving Bicriteria Solid Transportation Problem by Genetic Algorithm", *Proc. of16th Inter. Conf. Compt. ~ Indust. Engg.*Ashikaga, Japan, pp.572-575, 1994

[25] Cadenas, J. M., Jiménez, F., Manuel, J., & Fern, C. (1994). A genetic algorithm for the multiobjective solid transportation problem: a fuzzy approach.