Cordiality of Mixed Graphs’ One Point Union

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Abstract: Instead of taking one point union on a graph G alone we take it on two graphs G1 and G2. This structure is denoted by G = (G1,G2)k. Care is taken that G1 and G2 are repeated in G in a certain sequence. If both G1 and G2 appear alternately then the graph G is called balanced graph and for given k the number of copies of G1 and that of G2 differ at most by 1 in G. We discuss cordiality of G by taking G1 and G2 from C3, C4, C5 and house graph.

Keywords: mixed graphs, cordial labeling, cycle graph, union, structure.

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Introduction:

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [6]. A dynamic survey of graph labeling by J. Gallian [8] and Douglas West [9]. Cahit introduced the concept of cordial labeling [5]. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)−f(v)|. Further number of vertices labeled with 0 i.e. v(0) and the number of vertices labeled with 1 i.e.v(1) differ at most by one. Similarly number of edges labeled with 0 i.e. e(0) and number of edges labeled with 1 i.e.e(1) differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that if every tree is cordial; K3 is cordial if and only if n ≤ 3; Km,n is cordial for all m and n; the friendship graph C2n(i.e., the one-point union of t copies of C3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel Wn is cordial if and only if n is not congruent to 3 (mod 4). A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [8].

Let G1 and G2 be any two graphs. G(k) is one point union of k copies of G in which a fixed vertex x of G is chosen and K copies of G are fused together with x. By G = (G1,G2)k we have taken one point union of G1 and G2 and not G alone. For that a fixed vertex from G1 say p and a fixed vertex from G2 say q is chosen at which m copies of G1 and n copies of G2 are fused together to obtain G = (G1,G2)k. Note that m+n = k. If both G1 and G2 appear alternately then the graph G is balanced graph and for given k the number of copies of G1 and that of G2 differ at most by 1 in G, i.e. |m-n| ≤ 1. We have taken G1 and G2 from C3, C4, C5, house graph and C5.

4. Theorems proved:

4.1 Theorem. Mixed one point union of C3 and C5 given by G(k) = (C3, C5)k is cordial.

Proof: Define f:V(G)→{0,1} as follows. f gives different types of labeled copies of C3 and C5 as given below.

In construction of labeled copy of G(k) = (C3,C5)k we use ith copy as labeled copy of C3 as in fig.4.1 when i ≡ 1(mod2) and when i ≡ 0(mod2) we use labeled copy of C5 as in fig.4.2. Both copies are fused at vertex ‘a’ on it. The label numbers are as follows: On vertices we have v(i,0) = (2i+3x,3i+3x), and on edges e(i,1) = (4x+3,4x+2) for k is of type 2x+1 and i ≡ 0, 1, 2, ... if k = 2x, x = 1, 2, 3... v(i,0) = (3x+1,3x), and e(i,1) = (4x,4x). G(k) is balanced graph as alternate copies are C3 and C5 in labeled copy of G.

4.2 Theorem: Mixed one point union of C3 and house graph given by G(k) = (C3, house)k is cordial.

Proof: Define f:V(G)→{0,1} as follows. f gives different types of labeled copies of C3 and house as given below. In construction of G(k), when k = 1 Type A copy of house is used. For k = 2, labeled copy in fig.4.5 is used. For all rest of i when i ≥ 3, the ith copy in G(k) is Type A label if i ≡ 3(mod4), Type B label if i ≡ 0(mod4), labeled copy of C3 if i ≡ 1, 2 (mod 4). The label numbers in resultant graph are as follows: On vertices we have v(i,0) = (2i,3), and on edges e(i,1) = (3,3) for k = 1, for k = 2 we have v(i,0) = (3,4), and on edges e(i,1) = (4,5), for all k ≥ 3 we have:

i) if k is of type 4x+3, x = 0, 1, 2, ..., v(i,0) = (5i+6x, 6i+6x), and e(i,1) = (7+9x, 8+9x).

ii) if k = 4x, x = 1, 2, 3, ..., v(i,0) = (7i+6x, 8i+6x), and e(i,1) = (11+9x, 10+9x).

iii) if k is of type 4x+1, x = 0, 1, 2, ..., v(i,0) = (8i+9x, 9i+9x), and e(i,1) = (12+9x, 12+9x).

iv) if k is of type 4x+2, x = 0, 1, 2, ..., v(i,0) = (9i+6x, 10+6x), and e(i,1) = (13+9x, 124+9x).

Note that G(k) is not balanced graph.
Thus G is cordial graph.

4.3 Theorem: Mixed one point union of $C_3$ and house graph given by $G^{(k)} = (C_3, C_6)^{(k)}$ is cordial. Proof: Define $f: V(G) \to \{0,1\}$ as follows. $f$ gives different types of labeled copies of $C_3$ and $C_6$ as given below.

To construct one point union two or more copies are fused at point ‘a’ on it.

When $k = 1$ we have Type A to be used.

ii) When $k = 2$ type B is to be used and label numbers are $v(0,1) = (5,5)$, $e(0,1) = (5,6)$.

iii) If $k \equiv 3 \pmod{6}$ Type A is used. Write $k = 6x+3$, $x = 0, 1, 2, \ldots$ we have label distribution given by $v(0,1) = (13x + 7, 13x + 7)$, $e(0,1) = (16x + 8, 16x + 8)$.

iv) If $k \equiv 4 \pmod{6}$ Type A is used. Write $k = 6x+4$, $x = 0, 1, 2, \ldots$ we have label distribution given by $v(0,1) = (13x + 9, 13x + 9)$, $e(0,1) = (16x + 11, 16x + 10)$.

v) If $k \equiv 5 \pmod{6}$ Type B is used. Write $k = 6x+5$, $x = 0, 1, 2, \ldots$ we have label distribution given by $v(0,1) = (13x + 12, 13x + 11)$, $e(0,1) = (16x + 13, 16x + 14)$.

vi) If $k \equiv 0 \pmod{6}$ Type A is used. Write $k = 6x$, $x = 1, 2, \ldots$ we have label distribution given by $v(0,1) = (13(x-1) + 14, 13(x-1) + 13)$, $e(0,1) = (16(x-1) + 16, 16(x-1) + 16)$.

vii) If $k \equiv 1 \pmod{6}$ Type A is used. Write $k = 6x+1$, $x = 1, 2, \ldots$ we have label distribution given by $v(0,1) = (13(x-1) + 16, 13(x-1) + 15)$, $e(0,1) = (16(x-1) + 19, 16(x-1) + 18)$.

viii) If $k \equiv 2 \pmod{6}$ Type C is used. Write $k = 6x+2$, $x = 1, 2, \ldots$ we have label distribution given by $v(0,1) = (13(x-1) + 18, 13(x-1) + 18)$, $e(0,1) = (16(x-1) + 21, 16(x-1) + 22)$. Thus the mixed graphs one point union is cordial.

4.4 Theorem: Mixed one point union of $C_3$ and house graph given by $G^{(k)} = (C_3, house)^{(k)}$ is cordial. Proof: Define $f: V(G) \to \{0,1\}$ as follows. $f$ gives different types of labeled copies of $C_3$ and house as given below. To construct one point union two or more two or more copies are fused at point ‘a’ on it. In construction of $G^{(k)}$, when $k \equiv 0 \pmod{4}$ then $k = 4x$, $x = 1, 2, \ldots$ Fuse Type B with Type C at point ‘a’ on it. We have,

label numbers given by $v(0,1) = (8x, 8x+1)$, $e(0,1) = (11x, 11x)$.

When $k \equiv 2 \pmod{4}$ then $k = 4x + 2$. First obtain labeled copy for $k = 4x$ and fuse a copy of type B at vertex ‘a’ on it with vertex ‘a’ on $G^{(4x)}$. We have label numbers given by $v(0,1) = (8x + 4, 8x + 5)$, $e(0,1) = (11x + 5, 11x + 6)$. When $k \equiv 1 \pmod{4}$ then $k = 4x + 1$, $x = 0, 1, 2, \ldots$ First obtain labeled copy of $G^{(4x)}$ and fuse it at vertex ‘a’ on it with Type A label at vertex ‘a’ on it. We have label
numbers given by \( v_i(0,1) = (8x+2,8x+3) \), \( e_i(0,1) = (11x+3,11x+3) \). When \( k \equiv 3 \pmod{4} \) then \( k = 4x + 3 \). First obtain labeled copy of \( G^{4x+3} \) and fuse it at vertex ‘a’ on it with Type A label at vertex ‘a’ on it. We have label numbers given by \( v_i(0,1) = (8x+6,8x+7) \), \( e_i(0,1) = (11x+8,11x+9) \). Thus \( G^{k} \) is balanced and cordial.

4.5 Theorem: Mixed one point union of \( C_6 \) and \( C_3 \) graph given by \( G^k = (C_6, C_3)^k \) is cordial.

Proof: Define \( f: V(G) \rightarrow \{0,1\} \) as follows. \( f \) gives different types of labeled copies of \( C_3 \) and \( C_6 \) as given below.

In design of \( G^k \) when we first obtain a block of \( G^{(6)} \). Fuse Type A, Type B and two copies of Type C at vertex ‘a’ on it to obtain \( G^{(6)} \). We use this block repeatedly to obtain \( G^{(6)} \). At this stage we have label numbers given by \( v_i(0,1) = (9x,9x+1) \), \( e_i(0,1) = (12x,12x) \), where \( k = 6x \), \( x = 1, \ldots \).

1) If \( k = 1+6x \), \( x = 0, 1, \ldots \) First obtain labeled block on 6x copies as above. Fuse it with a copy of type C label at vertex ‘a’ on it. We have label numbers given by \( v_i(0,1) = (9x+1,9x+2) \), \( e_i(0,1) = (12x+1,12x+2) \). 2) \( k = 6x+2, x = 0, 1, \ldots \) First obtain labeled block on 6x copies as above. Fuse it with a copy of Type B label at vertex ‘a’ on it. We have label numbers given by \( v_i(0,1) = (9x+4,9x+4) \), \( e_i(0,1) = (12x+5,12x+4) \). 3) \( k = 6x+3, x = 0, 1, \ldots \) First obtain labeled block on 6x copies as above. Fuse it with a copy of Type A label and Type C label at vertex ‘a’ on it. We have label numbers given by \( v_i(0,1) = (9x+5,9x+5) \), \( e_i(0,1) = (12x+6,12x+6) \). 4) \( k = 6x+4, x = 0, 1, \ldots \) First obtain labeled block on 6x copies as above. Fuse it with a copy of type A label and two copies of type C label on vertex ‘a’ on it. We have label numbers given by \( v_i(0,1) = (9x+6,9x+6) \), \( e_i(0,1) = (12x+7,12x+8) \). 5) \( k = 6x+5, x = 0, 1, \ldots \) First obtain labeled block on 6x copies as above. Fuse it with a copy of type A label, one copy of type B label and a copy of C label at vertex ‘a’ on it. We have label numbers given by \( v_i(0,1) = (9x+8,9x+9) \), \( e_i(0,1) = (12x+11,12x+10) \). This is not balanced graph. It is cordial graph.

4.6 Theorem: Mixed one point union of \( C_6 \) and house graph given by \( G^k = (C_6, \text{house})^k \) is cordial. Proof: Define \( f: V(G) \rightarrow \{0,1\} \) as follows. \( f \) gives different types of labeled copies of \( C_6 \) and house as given below.

In designing a labeled copy of \( G^k \), different label types are fused at point ‘a’ on it. When \( k = 1 \) we use Type C label. \( G^{(1)} \) is obtained by fusing \( x \) copies of Type A and \( x \) copies of Type B at vertex ‘a’. At this stage the label number distribution is \( v_i(0,1) = (9x,9x+1) \), \( e_i(0,1) = (12x,12x) \). Where \( k = 6x \), \( x = 1, 2, \ldots \) When \( k = 1+4x \), \( x = 0, 1, \ldots \) first obtain labeled block of \( G^{(4x)} \). To this fuse type C at vertex ‘a’. At this stage the label number distribution is \( v_i(0,1) = (9x+2,9x+3) \), \( e_i(0,1) = (12x+3,12x+3) \). When \( k = 4x+2, x = 0, 1, \ldots \) first obtain labeled block of \( G^{(4x)} \). To this fuse type A at vertex ‘a’. At this stage the label number distribution is \( v_i(0,1) = (9x+5,9x+5) \), \( e_i(0,1) = (12x+6,12x+6) \). When \( k = 4x+3, x = 0, 1, \ldots \) first obtain labeled block of \( G^{(4x+3)} \) as above. To this fuse type C at vertex ‘a’. At this stage the label number distribution is \( v_i(0,1) = (9x+2,9x+2) \), \( e_i(0,1) = (12x+9,12x+9) \).
The graph is balanced and cordial.

Conclusions. We have discussed mixed one point union on $C_3, C_5, C_6$ and house graph by taking two of them together for cordial labeling. It is necessary to investigate the cordial labeling for taking three or more copies together.

References:


[6] Harary, Graph Theory, Narosa publishing, New Delhi

