# *Cube Roots*

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**ABSTRACT:**

Finding the cube root of large numbers was known to the Indian mathematician “Aryabatta”.This method is explained in “Ganithapada” the mathematical section of the “Aryabatiya” , in Ganithapada explained tha pattern of prime factorization method . A cube number is obtained when a number is multipiled by itself for three times . Therefore finding the number whose cube is known is called finding the cube root. It is the inverse operation of cubing.

For a large numbers prime factors method is lengthy and difficult. So to overcome this problem. I invented division method on 07-10-2014.

**Perfect cubic number :**

***I proved that if a number is in the form of (n^2k+1)( n^k+2) is a perfect cubic number and conversely all perfect cubic numbers are in the form of (n^2k+1)(n^k+2) where k is 0,1,2,3,…….all whole numbers where as n is all natural numbers.***

**Keywords:**

***Cube, cube root, perfect cube root, prime factor.***

## Extraction of Cube Roots of Real Numbers

**Introduction:** you have learnt the method of finding the Cube roots of numbers by prime factorization method. Now will learn the method of finding cube roots of numbers by division method. Cube roots are widely used not only in computers but also in Mathematics, Physics, Chemistry and Biology Subjects. By applying this new concept and technic, we should solve the number of problems quite easy.

**Definition of cube and cube roots:** Consider the numbers 64 and 216 resolving 64 and 216 into prime factors.

$64 = \underline{2x2x2}x\underline{2x2x2}$

$216=\underline{2x2x2}x\underline{3x3x3}$

In both these cases each factor appears three times that is the prime factors can be grouped in triples, thus if a number can be expressed as a product of three equal factors. Then it is said to be a perfect cube or cubic number.

So a cube number is obtained when a number is multiplied by itself for three times.

That is cube of a number is A x A x A = $A^3$ suppose a cube is formed with 125 unit cubes what could be side of the cube?

Let us assume, the length of the side to be ‘ x ’ .

$\therefore 125 = x^3$

To find the side of a cube it is necessary to find a number whose cube is 125.

Therefore, finding the number whose cube is known is called finding the cube root. It is the inverse operation of cubing.

As $5^3$= 125 then 5 is called cube root of 125.We write $\sqrt[3]{125}$=5 the symbol e $\sqrt[3]{\phantom{x}}$ denotes cube root. Hence a number ‘x’ is the cube root of another number y.

If $y=x^3$ then $x = \sqrt[3]{y}$

### Cube and Cube Root:

If a number is multiplied by itself for three times then the product is said to be the cube of that number.

**Eg:** 125 is the cube of 5 since 5x5x5=125 the number 5 is called the cube root of the product.

**Perfect Cube Number or Perfect Cube:**

If a number can be written as the product of three equal factors then the number is said to be a perfect cube.

**Eg:** 343 can be written as 343 =7x7x7 (Three equal factors)

So 343 is perfect cube.

**Finding The Cube Roots of Numbers By Factorization Method:**

## Let us find the cube root of 4096

Resolving 4096 into prime factors,
we get

4096 = $\underline{2x2x2}x\underline{2x2x2}x\underline{2x2x2}x\underline{2x2x2}$

$\sqrt[3]{4096}$ = =2x2x2 x2 =16

| | |
|---|---|
| 2 | 4096 |
| 2 | 2048 |
| 2 | 1024 |
| 2 | 512 |
| 2 | 256 |
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

For a large numbers it becomes lengthy and difficult. So to overcome this problem I invented division method on 07-10-2014.

If the given number is a cube number then to find its cube root even though that number contains more than 16 digits, we can do it easily by this division method. Even if it is a non-cubic number also We can find cube root of that number by using this method. Nobody has ever proved before such an easy and entirely different method.

**Activity:** Examine the table given below

| Integer(N) | Cube of the Integer($N^3$) | Cube of the Integer($\sqrt[3]{N}$) |
|---|---|---|
| 1 | $1^3$ =1x1x1 =1 | $\sqrt[3]{1}$=1 |
| 2 | $2^3$ =2x2x2 =8 | $\sqrt[3]{8}$=2 |
| 3 | $3^3$ =3x3x3 =27 | $\sqrt[3]{27}$=3 |
| 4 | $4^3$ =4x4x4 =64 | $\sqrt[3]{64}$=4 |
| 5 | $5^3$ =5x5x5 =125 | $\sqrt[3]{125}$=5 |
| 6 | $6^3$ =6x6x6 =216 | $\sqrt[3]{216}$=6 |
| 7 | $7^3$ =7x7x7 =343 | $\sqrt[3]{343}$=7 |
| 8 | $8^3$ =8x8x8 =512 | $\sqrt[3]{512}$=8 |
| 9 | $9^3$ =9x9x9 =729 | $\sqrt[3]{729}$=9 |
| 10 | $10^3$ =10x10x10 =1000 | $\sqrt[3]{1000}$=10 |
| 11 | $11^3$ =11x11x11 =1331 | $\sqrt[3]{1331}$=11 |
| 12 | $12^3$ =12x12x12 =1728 | $\sqrt[3]{1728}$=12 |
| 13 | $13^3$ =13x13x13 =2197 | $\sqrt[3]{2197}$=13 |
| 14 | $14^3$ =14x14x14 =2744 | $\sqrt[3]{2744}$=14 |
| 15 | $15^3$ =15x15x15 =3375 | $\sqrt[3]{3375}$=15 |
| 16 | $16^3$ =16x16x16 =4096 | $\sqrt[3]{4096}$=16 |
| 17 | $17^3$ =17x17x17 =4913 | $\sqrt[3]{4913}$=17 |
| 18 | $18^3$ =18x18x18 =5832 | $\sqrt[3]{5832}$=18 |
| 19 | $19^3$ =19x19x19 =6859 | $\sqrt[3]{6859}$=19 |
| 20 | $20^3$ =20x20x20 =8000 | $\sqrt[3]{8000}$=20 |

**Conclusions :**

**From the above table you have observed.**

- The unit's place of the cube numbers is ends with 1 so cube root also ends with 1.
- If the unit's place of the cube number is ends with 2 its cube root ends with 8.
- If the unit's place of the cube number is ends with 3 its cube root ends with 7.
- If the unit's place of the cube number is ends with 4 its cube root ends with 4.
- If the unit's place of the cube number is ends with 5 its cube root ends with 5.
- If the unit's place of the cube number is ends with 6 then cube root ends with 6.
- If the unit's place of the cube number is ends with 7 then cube root ends with 3.
- If the unit's place of the cube number is ends with 8 then cube root ends with 2.
- If the unit's place of the cube number is ends with 9 then cube root ends with 9.
- If the unit's place of the cube number is ends with 0 then cube root ends with 0.

## Finding cube root by division method

Let us find cube root of 1331 by division method

**Ex 2: Find the cube root of 4096**

Step 1: 4 096

Step 2: 1 | 4 096 | 1
1

Step 3: 1 | 4 096 | 1
- 1
3 096

Step 4: 1 | 4 096 | 1 6
- 1
36 | 3 096
-3 096
0

$\therefore \sqrt[3]{4096} = 16.$

4 096

**Step 1:** Start making groups of three digits starting from the unit place.

1 | 4 096 | 1
1

**Step 2:** Find the largest number whose cube is less than or equal to the first group of digits from left (i.e : 1) take this number as the division and quotient.

1 | 4 096 | 1
- 1
3

**Step 3:** subtract cube of the number from first group ( i.e: 4-1 = 3) .

1 | 4 096 | 1
- 1
3 096

**Step 4:** Bring down the second group (i.e: 096 )

To right of the remainder this become new dividend ( i.e: 3096 )

1 | 4 096 | 1
- 1
3 | 3 096

**Step 5:** From the next possible divisor triple the quotient (ie 1x3=3) and write a box on its right.

1 | 4 096 | 1 6
- 1
36 | 3 096
-3 096

**Step 6:** Guess the largest possible digit to fill box in such a way take the cube of the new divisor take the unit value of cube number in unit digits (ie $6^3$=216) then the product of this number and previous divisor multiply with the new quotient (ie 3x6=18 then 18x16=288) add the remain digts we left from cube of new divisor is equal to or less then the new dividend.

| | | |
|---|---|---|
| 1 | 4 096 | 1 6 |
| | - 1 | |
| 36 | 3 096 | |
| | -3 096 | |
| | 0 | |

**Step 7:** By subtracting we get the remaindar zero. The final quotient 16 is the cube root

$\therefore \sqrt[3]{4096}=16.$

## Think , Discuss And Observe:

| | | |
|---|---|---|
| 1 | 4 096 | 1 6 |
| | - 1 | |
| 36 | 3 096 | |
| | -3 096 | |
| | 0 | |

- In the first group the number is 4.
- The cube of 1 is 1 and cube of 2 is 8 , it is bigger then first group so we take 1.
- In the step 5, we are trippling the quotient so we get 1x3=3.
- In the step 6, unit digit is 6, so we cant take 6 as new number , its cube is 216, in this 216 we can substitue the 6 in units place, then 3x6 = 18 and 18x16=288 then add 21 to 288 we get 309, if you subtract we will get ‘0’.

## Ex: Find the cube root of 1367631

Step 1: 1 367 631

| | | | |
|---|---|---|---|
| Step 2: | 1 | 1 367 631 | 1 |
| | | 1 | |
| Step 3: | 1 | 1 367 631 | 1 1 |
| | | -1 | |
| | 31 | 0 367 | |
| | | - 331 | |
| Step 4: | 1 | 1 367 631 | 111 |
| | | -1 | |
| | 31 | 0 367 | |
| | | - 331 | |
| | 331 | 36631 | |
| | | 36631 | |
| | | 0 | |

$\therefore \sqrt[3]{1367631}=111.$

## CUBE ROOTS OF DECIMAL NUMBERS USING DIVISION METHOD

Let us begin with the example $\sqrt[3]{\mathbf{553.387661.}}$

**Step 1:** Start making the number into groups of three digits each starting from the unit place on the integral part of the number i.e. 553 in the usual manner and group of three digits of decimal part from left to right.

**Step2:** Find the largest number (i.e. 8) whose cube is less than or equal to the first group of integral part (i.e. 553). Take this number 8 as a divisor and the first group 553 as the dividend. Get the remainder as 41.

**Step3 :**Write the next group (i.e. 387) to the right of the remainder to get 41387, which become the new dividend.

**Step4:** Triple the quotient (3x8=24) and write it as 24 in the bon on its right. Since 387 is the decimal part so. Put a decimal point in the quotient (ie.8).

**Step5:**Guess the largest possible digit to fill the box in such a way take the cube of the new divisor, take the unit value of cube number in units digits ($2^3$=8) than the product of this number and the previous divisor multiply with the new quotient

I.e. (24x2 = 48 them 48x82=3936).

| | | |
|---|---|---|
| 8 | 553. 387 661 | 8.2 |
| | -512 | |
| 24 [2] | 41 387 | |
| | 39 368 | |
| | 2 019 | |

Add the remaining digits we left from cube of new devisor if any)
Is equal to or less than the new dividend.

**Step 6**: By subtracting we get the remainder 2019.

**Step 7**: Again triple the quotient (3x82=246) and write it as 246 in the box on its right.

**Step 8**: Write down the next group (i.e. 661) to the right of the remainder to get 2019661 is the new dividend.

| | | | |
|---|---|---|---|
| 8 | | 553. 387 661 | 8.2 |
| | | -512 | |
| 24 | 2 | 41 387 | |
| | | 39 368 | |
| | | 2 019 661 | |

**Step 9**: Guess the largest possible to fill the box in such a way take the cube of the new divisor take the unit value of cube number in units.

Digits ($1^3$=1) than the product of this number and the previous divisor multiply with the new quotient (i.e.246x1 =246 then 246x821=201966) is equal to or less than the new dividend.

| | | | |
|---|---|---|---|
| 8 | | 553. 387 661 | 8.21 |
| | | -512 | |
| 24 | 2 | 41 387 | |
| | | 39 368 | |
| 246 | 1 | 2 019 661 | |
| | | 2 019 661 | |
| | | 0 | |

**Step 10**: by subtracting we get the remainder as zero since the remainder is zero

$\therefore \sqrt[3]{553.387661}$=**8.21.**

I . Find the cube root of 42.25 using division method.

Sol:

| | | | |
|---|---|---|---|
| 3 | | 42. 250 000 000 000 | 3.4827 |
| | | -27 | |
| 9 | 4 | 15 250 | |
| | | 12 304 | |
| 102 | 8 | 2 946 000 | |
| | | 2 840 192 | |
| 1044 | 2 | 105 808 000 | |
| | | 72 704 168 | |
| 10446 | 7 | 33 103 832 000 | |
| | | 25 466 199 283 | |

$\therefore \sqrt[3]{42.25} = 3.483.$

**Finding the cube roots of numbers which are not perfect cubic numbers.**

Find the cube root of 2 using division method.

Sol:

| | | | |
|---|---|---|---|
| 1 | | 2. 000 000 000 000<br>-1 | 1.2599 |
| 3 | 2 | 1 000<br>728 | |
| 36 | 5 | 272 000<br>225 125 | |
| 375 | 9 | 46 875 000<br>42 491 979 | |
| 3777 | 9 | 4 383 021 000<br>4 282 778 879 | |

$\therefore \sqrt[3]{2} = 1.2599$.

Find the cube root of 3 using division method.

Sol:

| | | | |
|---|---|---|---|
| | 1 | 3. 000 000 000 000<br>- 1 | 1.4422 |
| 3 | 4 | 2 000<br>1 744 | |
| 42 | 4 | 256 000<br>241 984 | |
| 432 | 2 | 14 016 000<br>12 458 888 | |
| 4326 | 2 | 1 557 112 000<br>1 247 791 448 | |

$\therefore \sqrt[3]{3} = 1.4422.$

# Cube roots of larger numbers:

Eg. Find the cube root of 1 371 737 997 260 631.

Sol:

| | | | |
|---|---|---|---|
| 1 | | 1 371 737 997 260 631 | 111111 |
| | | -1 | |
| 3 | 1 | 0 371 | |
| | | 331 | |
| 33 | 1 | 40 737 | |
| | | 36 631 | |
| 333 | 1 | 4 106 997 | |
| | | 3 699 631 | |
| 3333 | 1 | 407 366 260 | |
| | | 370 329 631 | |
| 33333 | 1 | 37 036 629 631 | |
| | | 37 036 629 631 | |

$$\therefore \sqrt[3]{\mathbf{1371737997260631}} = 111111.$$

# SOME APPLICATIONS OF CUBE ROOTS :

EX 1: - Find the surface area of cube whose volume is 29791cm$^3$

Sol:- here let edge of a cube is "a" cm

Volume of the cube = $a^3$ cubic units

$a^3$ = 29791cm$^3$

a = $\sqrt[3]{29791}$

we can find a volume using division method we get

| | | |
|---|---|---|
| 3 | 29 791 | 31 |
| | -27 | |
| 91 | 2 791 | |
| | 2 791 | |
| | 0 | |

Therefore a = 31cm

Surface area of the cube= 6a$^2$sq.units

= 6x31$^2$

= 6x31x31

= 5766sq.units.

**EX 2:** -A water tank is in the form of cube, it can hold 79507000cm$^3$ of water find the length of the cube?

Sol:-capacity of the cube =volume of the tank

Volume of the tank = 79507000cm$^3$

= 79507000cm$^3$/1000 litres(1000cm$^3$ =1lt)

= 79507litres

Therefore L$^3$=79507litres, L = $\sqrt[3]{79507}$

We can find a length using division method we get

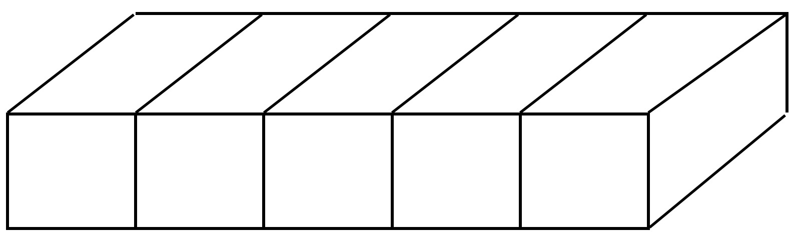
| | | |
|---|---|---|
| 4 | 79507 | 43 |
| | -64 | |
| 123 | 15 507 | |
| | 15 507 | |
| | 0 | |

Therefore L = 43cm

Length of the cube = 43cm

**EX 3:** - five equal cubes are placed adjacent to each other, its volume is $6655cm^3$ find the side of the each cube?

Sol:-



Volume of five cubes = $6655cm^3$

Volume of each cube = $6655cm^3/5$

= $1331cm^3$

Side of each cube = $\sqrt[3]{1331}$

| | | |
|---|---|---|
| 1 | 1 331 | 11 |
| | - 1 | |
| 31 | 331 | |
| | 331 | |
| | 0 | |

Length of the each cube = 11cm

**EX 4**: -The dimensions of a metallic cuboid or 216cm x 125cm x 320cm it is melted and recast into a cube find the surface area of a cube?

Sol:-Volume of the metallic cuboid = 216cm x 125cm x 320cm

= $1728000cm^3$

Volume of the metallic cuboid = volume of the cube

Let the length of the each edge of the recast cube be “a”cm then

$$a^3 = 1728000cm^3$$

a= $\sqrt[3]{1728000}$

We can find a length using division method we get

| | | |
|---|---|---|
| 1 | 1 728 000 | 120 |
| | -1 | |
| 32 | 728 | |
| | 728 | |
| 360 | 0 000 | |
| | 0 000 | |
| | 0 | |

Therefore “a” = 120cm

Surface area of the cube = $6a^2cm^2$

= 6 x 120 x 120

= $86400cm^2$

**EX 5:** -The volume of a solid iron sphere is 19652/3 x $\pi cm^3$ find it radius

Sol: - Volume of the solid iron sphere = 4/3 x $\pi$ x $r^3$ $cm^3$

$$4/3 \times \pi \times r^3\ cm^3 = 19652/3 \times \pi cm^3$$

Therefore $r^3$= 4913

We can find a radius using division method we get

| | | |
|---|---|---|
| 1 | 4 913 | 17 |
| | - 1 | |
| 37 | 3 913 | |
| | 3 913 | |
| | 0 | |

So radius of the iron rad=17cm

**Ex 6:**- A right circular cone is of height 8.4cm and radius of its base is 16.8cm. it is melted and recasts into a sphere. Find the radius of the sphere

r = radius of the base of the cone = 16.8cm

h= height of the come = 8.4cm

Volume of the cone $1/3\pi r^2h = 1/3 \times \pi \times (16.8)^2 \times 8.4cm^3$

Let "R" cm be the radius of the sphere obtained by recasting the melted cone. Then,

Volume of the sphere = $4/3\pi R^3$

Since the volume of the material in the form of cone and spare remains the same

$\therefore 4/3\pi R^3 = 1/3\ \pi(16.8)^2 \times 8.4$

⇨ $R^3 = \frac{(16.8)^2 \times 8.4}{4}$

⇨ $R^3 = 282.24 \times 2.1 = 592.704$ ➔ $R = \sqrt[3]{592.704}$

We can find a radius of the sphere using division method we get

| | | |
|---|---|---|
| 8 | 592. 704 | 8.4 |
| | -512 | |
| 244 | 80 704 | |
| | 80 704 | |
| | 0 | |

So radius of the sphere = 8.4cm.

**Ex 7:-** A metallic sphere of radii 6cm,8cm and 10cm respectively are melted to form a single solid sphere . Find the radius of the resulting sphere.

Sol:-

Let the radius of the resulting sphere be "R" cm then

Volume of the resulting sphere=sum of the radii 6cm, 8cm and 10cm

- ⇨ $4/3\pi r^3 = 4/3 \times \pi \times 6^3 + 4/3 \times \pi \times 8^3 + 4/3 \times \pi \times 10^3$
- ⇨ $4/3\pi r^3 = 4/3\pi\ (6^3 + 8^3 + 10^3)$
- ⇨ $r^3 = 6^3 + 8^3 + 10^3$
- ⇨ $r^3 = 216 + 512 + 1000$
- ⇨ $r = \sqrt[3]{1728}$

We can find a radius of the sphere using division method we get

| | | |
|---|---|---|
| 1 | 1 728 | 12 |
| | -1 | |
| 32 | 728 | |
| | 728 | |
| | 0 | |

- ⇨ So radius of the sphere = 12cm.

**Ex 8:-** the dimensions of a metallic cuboid are 90cm x 210cm x 490cm.it is melted and recast into a cube. Find the surface area of the cube.

Sol:-Volume of the metallic cuboid = 90cm x 210cm x 490cm

$= 9261000 cm^3$

Since the metallic cuboid is melted and recast into a cube.

Volume of the metallic cuboid = volume of the cube

Let the length of each edge of the recast cube be "a" cm then

Volume of the cube = volume of the cuboid

- ⇨ $a^3 = 9261000$
- ⇨ $a = \sqrt[3]{9261000}$

We can find a length of the cube using division method we get

| | | |
|---|---|---|
| 2 | 9 261 000<br>-8 | 210 |
| 61 | 1 261<br>1 261 | |
| 630 | 0 000<br>0 000 | |
| | 0 | |

- ⇨ So length of the cube = 210cm.

Surface area of cube = $6a^2 cm^2$= 6 x 210 x 210**=** $264600cm^2$.

## Simlarity with algebraic indentity:

we know $(a+b)^3 = a^3+3ab(a+b)+b^3$

cube root of $\sqrt[3]{a3 + 3ab(a + b) + b3}$ = a+b

cube root of this division method satisfies the above identity.

Let us see the example cube root of 4096

| | | |
|---|---|---|
| 1 | 4 096<br>- 1 | 1 6 |
| 36 | 3 096<br>-3 096 | |
| | 0 | |

(10+6)

↓ ↓

a + b

| | | |
|---|---|---|
| a | $a^3+3ab(a+b)+b^3$<br>$- a^3$ | a+b |
| 3a [ b] | $3ab(a+b)+b^3$<br>$3ab(a+b)+b^3$ | |
| | 0 | |

M

**The Size of Atoms**

The size of atoms can be estimated with the use of Avogadro's number along with the atomic

mass and bulk density of a solid material. From these, the volume per atom can be determined.

$$\text{Atomic volume} = \frac{\text{Molar mass (gm)}}{(\text{density in gm/cm}^3)(\text{Avogadro's number})}$$

The cube root of the volume is an estimate of the diameter of the atom. For carbon, the molar mass

is exactly 12, and the density is about 2 gm/cm^3. The estimated volume is then

$$\text{Carbon atomic volume} \approx \frac{12 \text{ gm}}{(2 \text{ gm/cm}^3)(6.02 \times 10^{23})} \approx 9.97 \times 10^{-24} \text{ cm}^3$$

and the estimate of the carbon atomic diameter is the cube root of that.

$$\text{Carbon diameter} \approx 2.2 \times 10^{-8} \text{ cm} = 0.22 \text{ nm}$$

This estimate is a bit small. It can be refined somewhat by considering the atoms to be spheres and packing them in different ways. Carbon in diamond form has a different density than graphite because of its atomic lattice structure. But this estimate at least establishes the kind of atomic sizes expected. A typical atomic diameter is 0.3 nm.

## FORMULA – HISTORY OF ASTRONOMY

Kepler's Third Law

**Kepler's Third Law** shows the relationship between the period of and objects orbit and the average distance that it is from the thing it orbits. This can be used for anything naturally orbiting around any other thing.

**Formula : $P^2=ka^3$**

P= Period of the orbit, measured in units of time

a= Average distance of the object, measured units of distance

k=constant, which has various values depending upon what the situation is, who P and a are measured

**Typical Problems:**

An object is orbiting around the star Gumby with a period of 80 years. If "k"=2 units. In this is system what is the average distance of the object orbiting.

*Solution: You are trying to get "a" so you need to rearrange the formula*

$P^2=ka^3$

$a^3=p^2/k$

$a^3=(80)^2/2=3200$

Now take the cube root of the both sides

$a=(3200)^{1/3}$ = **15AU**

**Solutions of Algebraic and Transcendental Equations**

Compute a positive value of $(17)^{1/3}$ correct to four decimal places by Newton - Raphson method

Solution: For the $K^{th}$ root on N. i.e., we have Newtons iterative formula.

$$x_{n+1} = \frac{1}{K}\left[(K-1)x_n + \frac{N}{x_n^{k-1}}\right]$$

Putting K=3 we get

$$x_{n+1} = \frac{1}{3}\left[2x_n + \frac{N}{x_n^2}\right]$$

Now to find $\sqrt[3]{17}$ so it is clear that

$8 < 17 < 27$

$2^3 < 17 < 3^3$

$2 < \sqrt[3]{17} < 3$

For First Approximation

$$x_1 = \frac{1}{3}\left[2\times 2.5 + \frac{17}{2.5^2}\right] = 2.573333$$

For Second Iteration

$$x_2 = \frac{1}{3}\left[2\times 2.573333 + \frac{17}{(2.573333)^2}\right] = 2.571283$$

$$x_3 = \frac{1}{3}\left[2\times 2.571283 + \frac{17}{(2.571283)^2}\right] = 2.571281$$

$$\therefore x_3 = \frac{1}{3}\left[2\times 2.571283 + \frac{17}{(2.571283)^2}\right] = 2.571281$$

Hence $\sqrt[3]{17}$ =2.571281 Correct to up to four places $\sqrt[3]{17}$ =2.5713.

**Find the cube root of 17 using division method.**

| | | | |
|---|---|---|---|
| | 2 | 17. 000 000 000 000 | 2.5712 |
| | | -08 | |
| 6 | 5 | 09 000 | |
| | | 07 625 | |
| 75 | 7 | 01 375 000 | |
| | | 01 349 593 | |
| 771 | 1 | 25 407 000 | |
| | | 19 822 411 | |
| 7713 | 2 | 5 584 589 000 | |
| | | 3 966 333 128 | |

Hence $\sqrt[3]{17}$ =2.5712

## Miscellaneous:

- Cube roots also used to find the atomic nucleus using formula
  $R = r_0 A^{1/3}$ (where $r_0 = 1.2 \times 10^{-15}$m = 1.2fm)
  where "R" is the radius of the nucleus
- Cube roots used in Orbital Period Equation

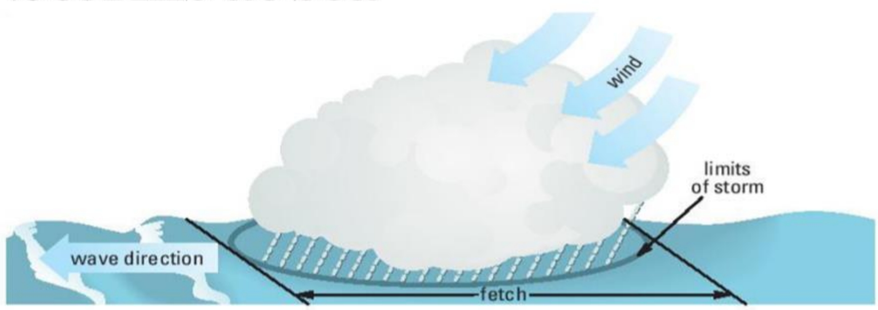
  $$\frac{T^2}{R^3} = \frac{4 \times \pi^2}{G \times M_{(central)}}$$

  Where T is the period of the satellite , R is the average radius of orbit and G is $6.673 \times 10^{-11}$ $N.m^2/kg^2$

- Cube roots are used in Murray's law ( $r^3_p = r^3_{d1} + r^3_{d2} + r^3_{d3} + \ldots\ldots\ldots + r^3_{dn}$) where $r_p$ is the radius of the parent branch and $r_{d1}, r_{d2}, r_{d3}$ are the radii of the respective daughter branches.
- Cube roots are used in Linear Approximations and de Moivre's formula
- Cube roots are used in mechanical of structures beams and bending stresses in beams.
- Biologists have discovered that the shoulder height of a male African elephant can be modelled by
  - ⇨ $H = 62.5 \sqrt[3]{t} + 75.8$

  t is the age of the elephant .
- Cube roots are used in car racing can be modelled by $s = 14.8 \times \sqrt[3]{p}$ where "p" is power (in horse power)
- Cube roots are used in

## Storms at sea




*The fetch f (in nautical miles) of the wind at sea is the distance over which the wind is blowing. The minimum fetch required to create a fully developed storm can be medeled by* $s = 3.1\sqrt[3]{f + 10} + 11.1$ *where* s *is the speed (in knots) of the wind. graph the model. Then determine the minimum fetch required to create a fully developed storm if the wind speemd is 25 knots.*

- Cube roots are used to find the height of geostationary satellite

  $$H = \sqrt[3]{\left(\frac{G \times M}{W \times W}\right)} - R$$

R (radius of Earth) $6.37 \times 10^{6}$ meter

w (angular speed of Earth) = $7.2921 \times 10^{-5}$ rad/s

M (mass of earth) = $5.97 \times 10^{24}$ kilogram

G (Gravitational Constant) = $6.67259 \times 10^{-11}$ $m^{3}*kg^{-1}*s^{-2}$

h (height of satellite) = ?

m (mass of satellite) = irrelevant to the answer

## References:


Solution of Algebraic And Transcendental Equations:

Engineering mathematics – III

Author : - Dr. Anil Kumar Guptha & Veerpal Singh

http://www.phy6.org/stargaze/kep3laws.htm.