# Optimal Day-Head Power Attainment With Non-**Conventional Energy Source And Demand Response**

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Abstract: This investigation proposes the request side power acquisition issue to ideally diminish consumer's energy cost. The inspiration originates from squeezing issues on an expansion of energy cost in a mechanical segment. From a energy customer's point of view, there exists a chance to decrease energy cost by changing buy and utilization of energy in light of time-differing power cost while using sustainable power source, which is called request reaction (or) demand response. For this situation, energy storage can be utilized to mitigate change of discontinuous inexhaustible supply furthermore, unstable power cost. Despite the fact that it is expected to serve a critical measure of energy utilization from sustainable power source furthermore, unstable power cost. Despite the fact that it is expected to serve a significant amount of energy utilization from sustainable power source furthermore, to stay away from peak electricity price, inconstancy and uncountable in power demand, sustainable supply, and electricity price, make it testing to decide an ideal power obtainment. The primary goal of this examination is to propose a basic leadership technique that empowers energy customers to ideally decide control acquirement against time-fluctuating and stochastic power cost and inexhaustible supply. In particular, this investigation details an ideal day-ahead power obtainment as a two stage stochastic blended number program and proposes an answer approach in view of Benders decomposition. The proposed procedure can be effectively connected to energy serious enterprises, for example, server farms.

Index Terms – Demand Response, Non Conventional Energy Sources, Energy Storage, Day-Head, Two Stage Stochastic Problem and **Blender Decomposition.** 

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## NOMENCLATURE

- Sets and Indices
  - T: Index set of time periods  $t \in T$
  - w: Index set of scenarios w  $\in \Omega$
- **Deterministic Parameters** 
  - Dt: Forecasted power demand at time t € T
  - **R**<sub>t</sub>: Forecasted renewable supply at time  $t \in T$
  - $C_t^{DA}$ : Day-ahead electricity price at time t  $\in$  T
  - M<sup>char</sup> M<sup>dis</sup>: charging and discharging rate of storage S<sup>max</sup>: maximum level of energy storage

  - $\eta^{char}, \eta^{dis}$ : charging and discharging inefficiency of storage
  - Pt<sup>loss</sup>: Penality cost for power loss at time  $t \in T$
  - L<sup>max</sup>: Allowed number of time periods for shifting demand
  - TW: Time window to meet shifted power demand
  - €∙ Maximum fraction of amount of shifted load
  - Stochastic Parameters (for each scenario  $w \in \Omega$ )
    - $D_t(w)$ : Actual power demand at time t  $\in T$
    - $R_t(w)$ : Actual renewable supply at time  $t \in T$
    - $C_t^{RT}(w)$ : Real –Time electricity price at time t  $\in T$
- First stage Decision Variables (Day Ahead Operations)
  - X<sub>t</sub>: Day-ahead purchase commitment at time t  $\in$  T
  - $U_t$ : Binary variable indicates whether demand load at time t  $\in$  T can be shifted by demand response.
  - $zc_t^{DA}$ ,  $zd_t^{DA}$ : Amount to be charged/discharged at time t  $\in T$
  - $S_t^{DA}$ : Level of Storage at the beginning of time t  $\in T$
  - Second Stage Decision Variables(Real-Time Operations)
  - yt: Real-time electricity purchase at period t € T
    - $y_t^{loss}$ : Power loss at period t  $\in T$
    - v<sub>u</sub>: Amount of load shifted from time  $t \in T$  will be satisfied at time  $l \in T(t < l)$
    - $w_t\!\!:$  Amount of shifted load at the beginning of time  $t \in T$
    - $zc_t^{RT}, zd_t^{RT}$ : Amount to be charged/discharged at time t  $\in T$
    - $s_t^{RT}$ : Level of storage at the beginning of time t  $\in T$

#### I. INTRODUCTION

In recent years numerous businesses have seen a enormous increment in energy utilization that has brought about tremendous costs and also carbon contamination. Data centers are energy-intensive facilities that support a diverse set of services such as Web, e-mail, data storage, and processing. They are operated around the clock, and are energy intensive. It has been reported that global data center emissions will grow 7% year-on-year through 2020 [34]. Over the last decade India has witnessed increased demand in data because of explosive growth in smart phones and widespread use of social media apps, banking and e-commerce transactions, and multimedia storage needs, providing an impetus to the large growth in data center markets in India. According to studies, Indian data center spending on storage, server, and network equipment reached \$2.2 billion in 2012, and this market is expected to grow at a compound annual growth rate of 8.5% to reach USD 3 billion by 2016 [35]. Energy represents one of the most significant operating costs in data centers. Rising energy costs increase their operational expenses. In India, where coal is the primary source of electricity generation, it is necessary for data centers to adopt sustainable operations. In power-deficit India, energy efficiency offers the following benefits to data centers: (a) increased reliability of electricity supply; (b) reduction in operating costs; and (c) enhanced efficiency in design and operations. What's more, the measure of modern energy utilization spared by sustainable power source has been ceaselessly expanding; what's more, this pattern is required to proceed later on. Likewise, from the energy purchasers' point of view, there exists an open door for businesses to change buy and utilization of energy in light of time-differing cost in the energy advertise. Customarily, control buyers utilize power with a level rate offered by service organizations or energy advertises for their use.

Nonetheless, as of late, it is getting to be plainly basic for numerous utilities to offer day-ahead and ongoing costs for keen evaluating [3], and some autonomous framework administrators, for example, ERCOT [4] and California ISO [5], have as of late permitted customers to buy power straightforwardly frame the market while giving value data. Along these lines, ventures get an opportunity to get energy by taking an interest in the market while being completely mindful of the time-shifting cost, and they may have a chance to decide the measure of their energy utilization relying upon the power cost. This opportunity is called demand response. In addition, considering a chance to utilize sustainable power source, demand response can additionally be effectively actualized to use sustainable power source by devouring more sustainable power source when it is accessible. In expansion, by applying request reaction to energy acquirement, energy stockpiling can be utilized to moderate change of discontinuous inexhaustible supply and unpredictable power cost. Information focuses are one of promising application zones for request reaction, since they have sensible and adaptable workloads [6] and are right now utilizing sustainable power source to supply control request by introducing nearby sustainable age office or make contracts with sunlight based or wind ranches [7]. Applying request reaction popular side power framework administration is considered under the idea of "Virtual Power Plant" [8], [9], and [10]. To understand the previously mentioned openings, professionals are emphatically urged to grow new innovations for arranging, plan, control, and operations of energy frameworks against fluctuation and vulnerability in sustainable power source and power cost.

At the end of the day, since the ordinary frameworks what's more, procedures have not been outlined while considering combination of sustainable power source and request reaction into control framework operations, irregular inexhaustible age what's more, unpredictable power value challenge control framework architects' basic leadership. In this unique circumstance, momentum look into in the power framework has been centered around coordinating advancement systems to yield solid and hearty energy age and obtainment. It is foreseen that utilization of advancement procedures will significantly affect arranging, plan, control, and operations of energy frameworks. Therefore, this examination concentrates on building up a basic leadership philosophy for request side power acquirement with sustainable energy, stockpiling, and request reaction utilizing a stochastic streamlining method. In particular, this investigation considers a two stage control obtainment made out of day-ahead and real time acquisitions.



Fig. 1.1. Demand-side Power Procurement

Note that there is a collection of writing on request side power acquisition in light of Markov choice process, [11], [12], [13], [14], [15], and Lyapunov advancement While the majority of the previously mentioned writing centers around displaying the successive stochastic control issue and outlining ideal arrangement custom fitted to constant control acquisition, this examination proposes a two-organize stochastic improvement issue customized to day-ahead power acquisition also, proposes an answer approach in light of Benders decay.

To the best of our insight, the two-organize stochastic improvement approach for day-ahead power acquisition issue with sustainable power source, stockpiling, and request reaction has not been tended to in the writing. Along these lines, this study would be a decent beginning stage to think about request side control obtainment issue in light of the system of two stage stochastic program. Whatever is left of this paper is composed as takes after: Section II gives a definite description and supposition of the proposed two-organize control acquirement issue and details the issue as a scientific model. Area III presents a calculation in light of Benders disintegration and recommends techniques intended to enhance the calculation. Segment IV breaks down the outcomes got by numerical analyses, furthermore, Section V closes the paper with finishing up comments and future research headings.

#### **II. PROBLEM DESCRIPTION**

#### 1.1.Scenario and Assumption

In view of situation considered in this examination, shopper's control request can be met by the accompanying sources: (I) buy from energy showcase, (ii) sustainable power source, and (iii) release from energy stockpiling as delineated in Figure 1. Practically speaking, energy advertises incorporates day-ahead and continuous markets that cooperate as takes after:

Day-ahead energy advertise gives members a chance to resolve to purchase power one day before the working day to help evade value unpredictability. Real-time energy showcase enables members to purchase power over the span of the working day to adjust confound between day-ahead buy responsibility real request stack. Considering the operations of energy showcase, we consider a two-organize system that comprises of day-ahead and real time control acquirement, and propose day-ahead acquisition issue. In light of a two-organize stochastic program, the proposed day-ahead power acquisition issue is composed so that the main stage issue decides day-ahead buy responsibility (without a moment's hesitation choices) in view of the estimated request stack and sustainable supply, while the second-organize decides the constant buy (plan of action choices) to modify the crisscross between buy duties and the genuine power request and inexhaustible supply. We accept that day-ahead power cost, power demand and inexhaustible supply are known in the primary stage, however continuous power cost, real power request and inexhaustible supply are time-differing and stochastic. Note that estimating power request and sustainable supply are out of the extent of this think about. Moreover, we consider energy stockpiling operations with limited limit, most extreme charging and releasing rates, and wastefulness in charging and releasing.

Actually, the successive cycle of charging or releasing causes the debasement of the energy stockpiling as far as lifetime and productivity. Be that as it may, this examination does not consider the corruption since it is accepted to be immaterial inside one-day operations. Additionally, we actualize request reaction into the proposed day-ahead control acquirement so shopper doles out eras in day-ahead and enables requests to be moved progressively at doled out eras, yet ought to be met by the due date continuously operation. As indicated by the proposed two-organize control acquirement system, in view of day-ahead buy duty, control misfortune (i.e. secured control that proved unable be utilized to neither serve control request nor charge stockpiling) may be happened relying upon real request stack and inexhaustible ages. In our investigation, we characterize a punishment cost charged for control misfortune to guarantee that both day-ahead buy duty and sustainable power source are completely utilized as a part of continuous operations.

#### **1.2.Mathematical Model**

We define the proposed day-ahead power obtainment issue as a two-arrange stochastic blended whole number programming (SMIP) issue. The principal arrange issue decides the buy responsibility and dole out periods for moving request in view of the day-ahead power cost, forecasted demand and sustainable supply considering capacity operation to limit day-ahead buy cost and the normal response cost caused by the constant obtainment for every conceivable situation. In the second-organize, the sub problem is characterized to modify jumble caused by estimating mistakes against genuine control request and sustainable supply by acquiring power from an ongoing business sector and moving customers request in light of operations of energy stockpiling (charging/releasing). Our proposed day-ahead power obtainment issue can be planned as a two-arrange SMIP as takes after:

$\operatorname{Min} \sum_{t \in T} C_t^{DA} x_t + \mathbb{E}[f(x, u, \tilde{\omega})]$	(1)
s.t. $x_t + zd_t^{DA} - zc_t^{DA} = \overline{D}_t - \overline{R}_t  \forall t \in T$	(2)
$\sum_{t \in T} u_t \le L^{max}$	(3)
$zc_t^{DA} \leq \min\{M^{char}, S^{max} - s_t^{DA}\}  \forall t \in T$	(4)
$zd_t^{DA} \leq \min\{M^{dis}, s_t^{DA}\}  \forall t \in T$	(5)
$s_{t+1}^{DA} - s_t^{DA} - \eta^{char} z c_t^{DA} + \frac{1}{\eta^{dis}} z d_t^{DA} = 0  \forall t \in T$	(6)
$x_t, s_t^{DA}, zc_t^{DA}, zd_t^{DA} \geq 0  \forall t \in T$	(7)
$u_t \in \{0,1\}  \forall t \in T$	(8)
where for each scenario $\omega \in \Omega$	
$f(x, u, \omega) = \sum_{t \in T} \left( C_t^{RT}(\omega) y_t + P_t^{loss} y_t^{loss} \right)$	(9)
s.t. $y_t - y_t^{loss} + zd_t^{RT} - zc_t^{RT} + \sum_{\ell=t+1}^{t+TW} v_{t\ell} - \sum_{\ell=t-TW}^{t-1} v_{t\ell}$	let
$= D_t(\omega) - R_t(\omega) - x_t  \forall t \in T$	(10)
$\sum_{\ell=1}^{t+TW} v_{t\ell} \le D_t(\omega) u_t  \forall t \in T$	(11)

$$w_{t+1} - w_t - \sum_{\ell=t+1}^{t+TW} v_{t\ell} + \sum_{\ell=t-TW}^{t-1} v_{\ell t} = 0 \quad \forall t \in T$$
 (12)

$$w_t \le \epsilon \sum_{\ell=1}^{t-1} D_\ell(\omega) \quad \forall t \in T$$
(13)

$$zc_t^{RT} \le \min\{M^{char}, S^{max} - s_t^{RT}\} \quad \forall t \in T$$
(14)

$$zd_t^{RT} \le \min\{M^{dis}, s_t^{RT}\} \quad \forall t \in T$$
(15)

$$s_{t+1}^{RT} - s_t^{RT} - \eta^{char} z c_t^{RT} + \frac{1}{\eta^{dis}} z d_t^{RT} = 0 \quad \forall t \in T \quad (16)$$

$$y_t, y_t^{loss}, v_{\ell t}, w_t, s_t^{RT}, zc_t^{RT}, zd_t^{RT} \ge 0 \quad \forall \ell, t \in T.$$
(17)

In the above plan, the goal work (1) is com- postured of day-ahead power obtainment costs and the normal plan of action cost for continuous power acquisition in the second-arrange relating to the one-day operation cycle. Constraint (2) is the powers adjust condition for day-ahead power procurement design guaranteeing that day-ahead buy duty is resolved with the goal that anticipated power request is completely satisfied by considering forecasted sustainable supply and energy stockpiling operations. Requirement (3) doles out eras for moving interest continuously with a most extreme permitted number of eras. Limitations (4)- (6) are for day-ahead capacity operations. Limitation (7) is the non-cynicism confinements and imperative (8) gives the parallel limitations on the principal arrange choice factors. In the second-arrange, the target capacity of the sub problem for every situation is detailed to limit ongoing operations cost, which is made by constant buy cost and punishment cost for control misfortune as (9). Limitation (10) is the power adjusts condition for ongoing force acquisition operation including moving and serving power request (for request reaction) relating to the real power request and wind control supply given day-ahead buy responsibility. Note that, control misfortune may happen when the measure of aggregate influence acquirement is surpassing the real influence request and the most extreme charging sum. Imperatives (11)- (13) are for request reaction.

Imperative (11) characterizes a condition that power request can be moved just at pre-appointed eras, and limitation (12) is the adjust condition for request moving under request reaction. We characterize the nature of use limitation as (13) so that the portion of the measure of moved request (however not yet served) to the aggregate sum of energy request can't be surpassed a pre-concurred level. Imperatives (14)- (16) are for ongoing energy capacity operations, and imperative (17) are the non-antagonism confinements on the second-arrange choice factors. Note that "min{}" work utilized as a part of imperatives (4), (5), (14), what's more, (15) can essentially be linearized by two separate imperatives.

For example, constraint (4) is equivalent to 
$$zc_t^{DT} \leq M^{char}$$
  
and  $zc_t^{DT} + s_t^{DT} \leq S^{max} \ \forall t \in T$ .

We might want to stress that in the two-arrange SMIP definition, just the clench hand organize issue incorporates number factors and the sub problem is detailed with no number factors, and in this manner, the proposed two-organize SMIP issue has ceaseless plan of action. What's more, the two-organize SMIP has moderately total plan of action [20] such that each arrangement got by taking care of the ace issue continuously brings about a plausible sub problem.

#### **III. SOLUTION APPROACH**

As depicted in Section II-B, our proposed day-ahead obtainment issue is detailed as a two-arrange SMIP issue with nonstop plan of action where just the ace issue incorporates twofold choice factors. Note that the two-organize SMIP issue can be displayed as a deterministic equivalent problem (DEP) that is defined as a vast blended number programming issue with a limited number of situations. In general, comprehending a DEP of the two-arrange SMIP issue is wasteful with a substantial number of situations, and for this situation, deterioration methods can be utilized to take care of the issue productively. In particular, for the consistent plan of action, the L-shaped calculation [21] and the multicut L-molded calculation [22] can be utilized to comprehend the two-arrange stochastic programming issue in light of Benders decay [23].

The primary thought of the L-molded calculation and the multicut calculation is to comprehend the decayed ace and sub problems independently by approximating a plan of action work by including Benders cuts inside the course of taking care of the ace issue. Be that as it may, the two calculations in view of Benders decay may lead the ease back merging to get an ideal arrangement depending on issue structure and additionally situation information. For these reasons, there has been an assortment of writing that the proposed systems to produce more grounded Benders cuts that quicken the joining of the calculation [24], [25], [26], and [27].

In this study, we propose cut generation strategy (Section-III-A) that introduces valid inequalities to generate stronger Benders cuts and define valid optimality cuts that can be added to the master problem in addition to Benders cuts during the course of the multicut L-shaped algorithm. Notwithstanding cut age system, we recommend cut collection methodology (Area III-B) in light of the relative exchange off between the single cut and multicut techniques [28], while examining the ideal accumulation level of Benders cuts. Give us a chance to rethink choice factors utilized as a part of plan (1)- (17) as an arrangement of vectors, x; u and y, to such an extent that x means (i.e. $x_t$ , $s_t^{DA}$ , $zc_t^{DA}$ , $zd_t^{DA}$  for all t  $\in$  T) vectors of nonstop factors indicates paired factors (i.e. $u_t$  for all t $\in$ T) in the main stage, furthermore, y signifies vectors of consistent factors in the second organize (i.e.  $y_t$ , $y_t^{loss}$ , $u_{lt}$ , $w_t$ , $s_t^{RT}$ , $zc_t^{RT}$ , $zd_t^{RT}$  for all 1, t  $\in$  T)At that point, with reasonable networks, A,T,D,W,H(w) and vectors, b, e, q(w), r(w), our proposed two-organize day-ahead power acquisition issue (1)- (17) can be characterized as takes after,

$$\begin{split} \operatorname{Min} \mathbf{c}^{\top} \mathbf{x} + \mathbb{E}[f(\mathbf{x}, \mathbf{u}, \tilde{\omega})] & (18) \\ \text{s.t.} & \mathbf{A}\mathbf{x} \leq \mathbf{b} & (19) \\ & \mathbf{D}\mathbf{u} \leq \mathbf{e} & (20) \\ & \mathbf{x} \geq 0, \mathbf{u} \in \{0, 1\}^n & (21) \\ \text{where for each scenario } \omega \in \Omega \\ & f(\mathbf{x}, \mathbf{u}, \omega) = \operatorname{Min} \mathbf{q}(\omega)^\top \mathbf{y} & (22) \\ & \text{s.t.} & \mathbf{W}\mathbf{y} \leq \mathbf{r}(\omega) - \mathbf{T}\mathbf{x} - \mathbf{H}(\omega)\mathbf{u} & (23) \\ & \mathbf{y} > 0, & (24) \end{split}$$

Where w is a multivariate random variable defined on a probability space with outcome models  $w \in \Omega$ . Let s denote index of models such that  $s=1,\ldots,S(S=|\Omega|<\infty)$  and  $p_s$ 

Denote the probability of occurrence for each model, then based on multicut L-shaped algorithm, we solve the following master problem iteratively

$$\begin{array}{ll} \operatorname{Min} \mathbf{c}^{\top} \mathbf{x} + \sum_{s=1}^{S} p_{s} \eta_{s} & (25) \\ \text{s.t.} \ \mathbf{A} \mathbf{x} \leq \mathbf{b} & (26) \\ \mathbf{D} \mathbf{u} \leq \mathbf{e} & (27) \\ \beta_{\mathbf{t}(\mathbf{s})}^{\top} \mathbf{x} + \gamma_{\mathbf{t}(\mathbf{s})}^{\top} \mathbf{u} + \eta_{s} \geq \alpha_{t(s)} & t(s) = 1, ..., u(s), \\ s = 1, ..., S & (28) \\ x \geq 0, u \in \{0, 1\}^{n}, \eta_{s} \text{ free, } s = 1, \dots, S, & (29) \end{array}$$

Where t(s) is an index of Benders optimality cuts generated by solving the sub problem with scenarios  $s \in S$  and u(s) is the number of Benders optimality cuts added to the master problem during the course of algorithm. Note that Benders optimality cuts (28) are generated by solving the following dual sub problem for each possible model,

$$f_s(x) = \operatorname{Max} \ \pi_{\mathbf{s}}^\top (\mathbf{r}_{\mathbf{s}} - \mathbf{Tx} - \mathbf{Hu})$$
(30)  
s.t.  $\pi_{\mathbf{s}}^\top \mathbf{W} \le \mathbf{q}$ (31)  
 $\pi_{\mathbf{s}} \le 0,$ (32)

With  $\alpha_s = p_s(\pi_s^*)r_s, \beta_s^T = p_s(\pi_s^*)^T T, and \gamma_s^T = p_s(\pi_s^*)^T H_s with \pi_s^*(\mathbf{x})$  an optimal solution of the dual subproblem. We might want to stress that our proposed two-arrange SMIP issue has generally total plan of action, and therefore, as it were optimality cuts (28) are created and added to the ace issue in light of the multicut L-formed calculation. In this investigation, we execute the Benders disintegration in light of single inquiry tree alluded to as "Branch-and- Drinking sprees cut" (B&BC) calculation [29] by utilizing the sluggish requirements pool gave by CPLEX Concert Technology (IBM ILOG CPLEX [30]). The primary preferred standpoint of B&BC is that drinking sprees slices can be added to the ace issue amid the course of branch-and-cut calculation (i.e. single inquiry tree) instead of re-taking care of the ace issue as another issue at every cycle when Benders cuts are produced and included by unraveling the sub problems. This can assist fathoming the ace program. Notwithstanding, there additionally may be impediments of utilizing the apathetic imperatives pool because of the accompanying reasons. Over the span of branch-and-cut calculation, Benders cuts are generated and added each time when the integer (and fractional) solutions are encountered, and the algorithm check the lazy constraint pool for the fractional solution. This might take longer computational time than the classical implementation of Benders decomposition. Therefore, we conducted preliminary experiments, and results showed that the B&BC algorithm using the lazy constraints pool outperforms the classical implementation for solving the proposed problem. Hence, we implement the multicut L-shaped algorithm by using the lazy constraints pool. The details of our proposed cut generation and aggregation strategies are described in the following Sections III-A and III-B, respectively.

#### 3.1 Cut Generation Strategy

1) Valid Inequalities: The key idea for improving performance of the multicut L-shaped algorithm is to generate stronger Benders cuts so that the solution space of the master problem can be significantly restricted. For the purpose of generating stronger Benders cuts, we propose the following

valid inequalities (33) and (34). By adding valid inequalities (33) and (34), and projecting them into the solution space of the sub problem, the additional effects of the master problem's solution can be reflected in the sub problem's solution, and thus, stronger Benders cuts can be generated and added.

$$\sum_{\ell=t+1}^{t+TW} v_{t\ell} \leq \epsilon \left( \sum_{\ell=1}^{t-1} D_{\ell}(\omega) \right) - w_t + \sum_{\ell=t-TW}^{t-1} v_{\ell t} + \epsilon D_t(\omega) u_t \quad \forall t \in T$$
(33)  
$$w_{t+1} \leq \epsilon \sum_{\ell=1}^{t-1} D_{\ell}(\omega) + \epsilon D_t(\omega) u_t \quad \forall t \in T.$$
(34)

For demand response, the two set of valid inequalities ensure that the amount of shifted demand at time period t€ T does not exceed the actual allowable limit that is restricted by the quality of usage constraint (13). We have the following propositions and proofs to show the validity of the proposed

Inequalities (33). Proposition 1: The following inequality,

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$$\sum_{\ell=t+1}^{t+TW} v_{t\ell} = \epsilon \left( \sum_{\ell=1}^{t-1} D_{\ell}(\omega) \right) - w_t + \sum_{\ell=t-TW}^{t-1} v_{\ell t} + \epsilon D_t(\omega) \quad \forall t \in T$$
(35)

is valid for problem (1)-(17).

Proof: By plugging (13) into (12), we can show that

$$\sum_{\ell=t+1}^{t+1} w v_{t\ell} = w_{t+1} - w_t + \sum_{\ell=t-TW}^{t-1} v_{\ell t}$$
  
$$\leq \epsilon \left( \sum_{\ell=1}^{t} D_\ell(\omega) \right) - w_t + \sum_{\ell=t-TW}^{t-1} v_{\ell t}$$
  
$$= \epsilon \left( \sum_{\ell=1}^{t-1} D_\ell(\omega) \right) - w_t + \sum_{\ell=t-TW}^{t-1} v_{\ell t} + \epsilon D_t(\omega) \quad \forall t \in T,$$

which proves the result.

Proposition 2: The inequality,

$$\sum_{\ell=t+1}^{t+TW} v_{t\ell} \leq \epsilon \left( \sum_{\ell=1}^{t-1} D_{\ell}(\omega) \right) - w_t + \sum_{\ell=t-TW}^{t-1} v_{\ell t} + \epsilon D_t(\omega) u_t \quad \forall t \in T$$

is valid for problem (1)-(17).

*Proof:* Considering the value of decision variable  $u_t$  for all  $t \in T$ , we have the following two cases:

Case 1: If  $u_t = 0$ , then

$$\sum_{\ell=t+1}^{t+TW} v_{t\ell} = 0, \tag{37}$$

by constraint (11). Now, for  $u_t = 0$ , we have the inequality (36) as,

$$\sum_{\ell=t+1}^{t+TW} v_{\ell\ell} \le \epsilon \Big( \sum_{\ell=1}^{t-1} D_{\ell}(\omega) \Big) - w_t + \sum_{\ell=t-TW}^{t-1} v_{\ell t}.$$
 (38)

Note that the RHS of (38) would be positive by constraint (13), and  $u_{lt} \ge 0$  for all  $l, t \in T$  Hence, inequality (36) is valid for  $u_t = 0$  for all  $t \in T$ **Case 2:** If  $u_t = 1$ , then inequality (36) is equivalent to inequality (35) which is valid for the proposed problem (1)-(17). Hence, inequality (36) is valid for  $u_t = 1$  for all  $t \in T$ 

In addition, the following proposition shows the validity of inequality (34). Proposition 3: The inequality,

$$w_{t+1} \leq \epsilon \sum_{\ell=1}^{t-1} D_{\ell}(\omega) + \epsilon D_t(\omega) u_t \quad \forall t \in T,$$
(39)

#### is valid for problem (1)-(17).

Proof: By plugging (12) into valid inequality (36), we obtain inequality (39). Note that our proposed valid inequalities (36) and (39) are equivalent because of equation (12), however, their contribution to improve the performance of Benders decomposition might be different. In Section IV, we will compare performance improvement by applying each of valid inequalities (36) and (39). 2) Valid Optimality Cuts: In addition to Benders optimality cuts, we introduce a set of valid optimality cuts designed to be added to approximate the recourse function in the first stage problem of the two-stage SMIP problem. Note that Laporte and Louveaux [31] developed the optimality cut for approximating the expected recourse function with the binary first-stage problem (i.e. the first-stage problem includes only binary decision variables). In this

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study, we extend their optimality cut so that it can be used to approximate the expected continuous recourse F(x,u)=E[f(x,u,w)] for the mixedbinary first-stage problem where x is continuous and u is binary decision variables. To introduce the proposed valid optimality cuts, we assume that a lower bound L on E[f(x,u,w)] is known, that is,

$$L \le \min_{\mathbf{u}} \{ \mathbb{E}[f(\mathbf{x}, \mathbf{u}, \tilde{\omega})] | \mathbf{A}\mathbf{x} \le \mathbf{b}, \mathbf{D}\mathbf{u} \le \mathbf{e}, \mathbf{x} \ge \mathbf{0}, \mathbf{u} \in \{0, 1\}^n \}.$$

Let  $x^k$  and  $u^k$  denote the master problem's solution at  $k^{th}$  iteration during the course of the multicut L-shaped algorithm, then we have recourse function for  $x^k$  and  $u^k$  as,  $F(x^k, u^k) = E[f(x^k, u^k, w)]$ ,

and define the set Sk for kth binary decision variables as,  $S^{k} = \{t|u_{t}^{k}=1\}$ . We summarize our proposed optimality cut in the following theorem. Theorem 1: The following cut is a valid cut for F(x, u):

$$\eta \ge (F(\mathbf{x}^{\mathbf{k}}, \mathbf{u}^{\mathbf{k}}) - L) \left( \sum_{t \in S^k} u_t - \sum_{t \notin S^k} u_t - |S^k| + 1 \right) + L - \mathbf{c}^\top (\mathbf{x} - \mathbf{x}^{\mathbf{k}}).$$
(40)

1) If  $u=u^k$ ,  $(\sum_{t\in S^k} u_t - \sum_{t\notin S^k} u_t - |S^k|+1)=1$ . • If  $\mathbf{x}=\mathbf{x}^k$ , then the cut  $\eta \ge F(\mathbf{x}^k, \mathbf{u}^k)$  is tight (i.e. active).

If  $\mathbf{x} \neq \mathbf{x}^{\mathbf{k}}$ , then the cut  $\eta \geq F(\mathbf{x}^{\mathbf{k}}, \mathbf{u}^{\mathbf{k}}) - c^{\top}(\mathbf{x} - \mathbf{x}^{\mathbf{k}})$  is valid for  $\mathbf{x} \in \{\mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \in \mathbb{R}^{n}_{+}, \mathbf{x} \neq \mathbf{x}^{\mathbf{k}}\}$ .

In addition, for incumbent solution xk and uk obtained during the course of branch and- cut algorithm, the following inequality is valid,

$$\mathbf{c}^{\top}\mathbf{x} + F(\mathbf{x}, \mathbf{u}^{\mathbf{k}}) \ge c^{\top}\mathbf{x}^{\mathbf{k}} + F(\mathbf{x}^{\mathbf{k}}, \mathbf{u}^{\mathbf{k}}), \tag{41}$$

for all 
$$\mathbf{x} \in \{\mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$$
. Note that  $\mathbf{c}^{\top}\mathbf{x} + F(\mathbf{x}, \mathbf{u})$ 

Represents objective function value of overall problem (i.e. including the first and second-stage objective function value) decision variable u is fixed as  $u=u^k$  in both left hand- side and right-hand-side of the above inequality (41). Now we have the following inequality:

 $\eta \ge F(\mathbf{x}, \mathbf{u}^{\mathbf{k}}) \ge F(\mathbf{x}^{\mathbf{k}}, \mathbf{u}^{\mathbf{k}}) - c^{\top}(\mathbf{x} - \mathbf{x}^{\mathbf{k}}).$ (42) This shows that the cut  $\eta \ge F(\mathbf{x}^{\mathbf{k}}, \mathbf{u}^{\mathbf{k}}) - c^{\top}(\mathbf{x} - \mathbf{x}^{\mathbf{k}})$  is valid. 2) If  $\mathbf{u} \ne \mathbf{u}^{\mathbf{k}}$ , then  $(\sum_{t \in S^{k}} u_{t} - \sum_{t \notin S^{k}} u_{t} - |S^{k}| + 1) \le 0$ . And let  $M = (F(\mathbf{x}^{\mathbf{k}}, \mathbf{u}^{\mathbf{k}}) - L)(\sum_{t \in S^{k}} u_{t} - \sum_{t \notin S^{k}} u_{t} - |S^{k}| + 1)$ , then  $M \le 0$  since  $F(x^{k}, u^{k}) \ge L$ . • If  $\mathbf{x} = \mathbf{x}^{\mathbf{k}}$ , then the cut is  $\eta \ge M + L$  and it must be valid. • If  $\mathbf{x} \ne \mathbf{x}^{\mathbf{k}}$ , then the cut  $\eta \ge L + M - \mathbf{c}^{\top}(\mathbf{x} - \mathbf{x}^{\mathbf{k}})$  is valid since,  $\eta \ge F(\mathbf{x}, \mathbf{u}^{\mathbf{k}}) \ge M + L - F(\mathbf{x}^{\mathbf{k}}, \mathbf{u}^{\mathbf{k}}) + F(\mathbf{x}, \mathbf{u}^{\mathbf{k}})$  $> M + L - \mathbf{c}^{\top}(\mathbf{x} - \mathbf{x}^{\mathbf{k}})$ ,

based on inequality (41).

We would like to emphasize that optimality cut (40) is weak, therefore it should be used together with Benders cuts to improve the performance. In addition, optimality cut (40) can be implemented into the cut aggregation scheme based on the multicut L-shaped algorithm. Let  $j \in J$  be the index of cut aggregate and define the expected recourse function for the subset of scenarios corresponding to each cut aggregate as

$$F_{j}(\mathbf{x}, \mathbf{u}) = \mathbb{E}[f(\mathbf{x}, \mathbf{u}, \tilde{\omega}_{j})].$$
(43)  
$$L_{j} \leq \operatorname{Min}_{x, u} \{ \mathbb{E}[f(\mathbf{x}, \mathbf{u}, \tilde{\omega}_{j})] | \mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{D} \leq \mathbf{e}, \mathbf{x} \geq \mathbf{0}, \mathbf{u} \in \{0, 1\}^{n} \}.$$
(44)

Assuming that a lower bound Lj is known, that is, Then, the following cut is a valid optimality cut for Fj(x, u):

$$\eta_j \ge \left(F(\mathbf{x}^{\mathbf{k}}, \mathbf{u}^{\mathbf{k}})_j - L_j\right) \left(\sum_{t \in S^k} u_t - \sum_{t \notin S^k} u_t - |S^k| + 1\right) + L_j - \mathbf{c}^\top(\mathbf{x} - \mathbf{x}^{\mathbf{k}}).$$
(45)

Optimality cut (45) can be added to the master problem together with Benders optimality cuts for the approximated recourse function of each cut aggregate,  $\eta_j$ . To implement optimality cuts (45), lower bound Lj can be determined by solving the following relaxed problem:

$$L_{j} = \operatorname{Min} \sum_{\omega \in \Omega_{j}} p(\omega) \mathbf{q}(\omega)^{\top} \mathbf{y}(\omega)$$
  
s.t.  $\mathbf{W}\mathbf{y}(\omega) \leq \mathbf{r}(\omega) - \mathbf{T}(\omega)\mathbf{x} - \mathbf{H}(\omega)\mathbf{u} \quad \forall \omega \in \Omega$   
 $\mathbf{A}\mathbf{x} \leq \mathbf{b}$  (46)  
 $\mathbf{x} > 0, \mathbf{u} \in [0, 1]^{n}, y(\omega) > 0 \quad \forall \omega \in \Omega.$ 

Where  $\Omega j$  represents subset of scenarios corresponding to cut aggregate  $j \in J$ . Note that problem (46) is relatively easy to solve with relaxed binary decision variables.

#### **3.2 Cut Aggregation Strategy**

The inspiration of cut accumulation comes from the relative points of interest of the L-molded (single cut) calculation and the multicut L-molded calculation. When all is said in done, the multicut L-formed calculation has less significant emphases by means of passing more data by taking into consideration slices up to the quantity of situations than the L-molded calculation, be that as it may, taking care of the ace issue requires more calculation time. Then again, when we total cuts and less number of optimality cuts are included to the ace issue, the calculation may have more major emphases because of loss of data caused by accumulation. In any case, the ace issue can be understood simpler than when the multicut L-formed calculation is utilized. In view of the exchange off as far as computational time, creators of [28] proposed a versatile optimality multicut technique that progressively changes the level of accumulation of the optimality cuts in the ace issue over the span of the calculation. The numerical aftereffects of [28] demonstrate that the ideal computational time is accomplished on some center level of total, however this level isn't known from the earlier and relies upon issue structure. In a comparable design, we endeavor to explore a fitting accumulation levels in light of the exchange off of calculation execution in terms of computational time.

In this study, we propose a cut aggregation strategy that assigns Benders optimality cuts to be aggregated for the given aggregation level during the course of the algorithm. The fundamental idea of our suggested strategy is to aggregate Benders cuts while minimizing loss of information caused by cut aggregation. This can be accomplished by aggregating Benders optimality cuts obtained from the sub problem defined by "similar" scenario data. Each scenario consists of three-dimensional vectors, power demand, renewable supply, and electricity prices, respectively. These vectors show time varying patterns across 24 hours periods corresponding to one day time horizon. We would like to emphasize those relations among power demand, renewable supply, and electricity prices have a significant impact on the solution of the sub problem due to the problem structure. For example, if there exists a negative correlation between power demand and electricity price, then the optimal solution of sub problem is determined so that storage is charged and discharged more frequently as

well as more power demand is shifted to minimize expense. In this context, we characterize the structure of each scenario data using pair wise correlations between power demand, renewable supply, and electricity prices and measure similarity of scenario data based on those correlations. For example correlation between series of  $D_t(w)$  and  $C_t^{RT}(w)$  across time periods  $t \in T$  for each scenario : $w \notin \Omega$ ,  $\dot{\rho}DC(w)$  can be computed as follows

$$, \dot{\rho}DC(\omega) = \frac{\sum_{t=1}^{24} (D_t(\omega) - D(\omega)) (C_t(\omega) - C(\omega))}{\sqrt{\sum_{t=1}^{24} (D_t(\omega) - D(\omega))^2 \sum_{t=1}^{24} (C_t(\omega) - C(\omega))^2}}$$

Where D(w) is the average power demand and C(w) is the average electricity prices for each scenario  $w \in \Omega$ 







(a) L-shaped algorithm

(b) Multicut algorithm

Fig. 3.2. Performance analysis of the proposed valid inequalities (33) and (34)



Fig. 3.3. Performance analysis of the proposed valid inequalities (33) and (34) combined with optimality cut (40)

# IV. CONCLUSION AND FUTURE WORK

This examination is roused by a chance to decrease the vitality cost and carbon contamination by using sustainable power source furthermore, embracing request reaction from the request side viewpoint. While using sustainable power source to meet power request, buyers might will to alter their request stack, which is called as request reaction, to keep away from top power cost and in addition ideally use sustainable power source to diminish obtainment cost. What's more, vitality stockpiling can be used to alleviate variances of irregular sustainable supply also, unstable power cost. Considering sustainable power source, request reaction, and vitality stockpiling, the primary target of this investigation is to propose basic leadership models that empower vitality purchasers to get vitality in a cost-proficient way because of fluctuation and vulnerability of inexhaustible supply and also power cost. In rundown, the principle commitments of this paper are: (I) propose day-ahead power acquirement issue and define it as a two-arrange SMIP issue; (ii) present cut age and cut accumulation procedures that can be coordinated with the course of the multicut L-molded calculation to enhance calculation performance. The proposed day-ahead power obtainment issue and arrangement approach can be connected to numerous enterprises (e.g. information focuses and fabricating) and furthermore reached out to lattice level control framework operations (e.g. miniaturized scale network) to reduce costs of obtaining vitality to take care of demand stack. We trust that this investigation would be a decent beginning stage to examine demand side control obtainment issue in light of the structure of two-arrange stochastic program and will have a huge affect on think about for the use of sustainable power source and execution of interest reaction.

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