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Structured Grid Generation of a 2 Dimensional Domain with Semi-Circle Using Elliptic Partial Differential Equations

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Abstract: In fluid flow the governing equations are non-linear partial differential equation. In some cases using the similarity transformations the set of partial differential equations are transformed to non-linear ordinary differential equations. Numerical solutions to set of ordinary differential equations are computed using Runge-Kutta methods. In many cases the transformation of ordinary differential equations are not possible, there is need to compute solution in the physical domain using finite difference, finite volume and finite element methods. All these methods operate on the grid, breaking the domain into sub-domains or control volumes. The structured grid the neighbouring nodes are known using index. For simpler domains algebraic grid generation is used to compute grid. For complex domains the elliptic grid generation is used to compute grid. In this paper a 2-D grid with enclosed semi-circle is considered for grid generation.

Index Terms - Elliptic grid, structured grid, (ξ, η) plane, (x, y) plane

I. Introduction

In fluid flow the governing equations are non-linear partial differential equation. In some cases using the similarity transformations the set of partial differential equations are transformed to non-linear ordinary differential equations. Numerical solutions to set of ordinary differential equations are computed using Runge-Kutta methods. In many cases the transformation of ordinary differential equations are not possible, there is need to compute solution in the physical domain using finite difference, finite volume and finite element methods. All these methods operate on the grid, breaking the domain into sub-domains or control volumes. The structured grid the neighboring nodes are known using index. For simpler domains algebraic grid generation is used to compute grid. For complex domains the elliptic grid generation is used to compute grid. In this paper a 2-D grid with enclosed semi-circle is considered for grid generation.

II. MATHEMATICAL ANALYSIS

The Laplace Equation is transformed into ξ, η coordinate system. The resulting partial differential equations, which are elliptic in nature by enforcing the boundary conditions at the boundaries the numerically the grid is computed. E Hemalatha et al., (2016), (2017) , E Hemalatha et al., (2015), worked on the partial differential equations solutions based on similarity variables for vertical plate, horizontal plate respectively. The solution is computed using the Newton- Raphson shooting method. For numerical solution on physical domain requires computing grid. To compute solution using finite difference method structured grid is required. At each grid point the solution variable is computed.

$$a x_{\xi\xi} - 2b x_{\xi\eta} + c x_{\eta\eta} = 0$$
$$a y_{\xi\xi} - 2b y_{\xi\eta} + c y_{\eta\eta} = 0$$
$$a = x_\eta^2 + y_\eta^2$$
$$b = x_\xi x_\eta + y_\xi y_\eta$$

(ξ, η) Plane

- These partial differential equations are solved in the (ξ, η) plane to get corresponding x, y coordinates.
- Here, the coefficients a, b, and c are functions of dependent variables.
- By taking a, b, and c as constants, the partial differential equations become linear.
- The partial differential equations are discretized at every point in the (ξ, η) plane.
- These coefficients are calculated from the previous iteration in the Gauss–Seidel method.

Boundary Conditions for x-Equation

- Let the side ABC DE in the (x, y) plane correspond to the side abcde in the (ξ, η) plane.
- Let the sides EF, F G, and AG in the (x, y) plane correspond to the sides ef , f g, and aging the (ξ, η) plane respectively.
- On side abcde, x varies from [0, 1].
- On side f g, x varies from [0, 1].
- On side ag, x = 0.
- On side ef , x = 1.

Boundary Conditions for y-Equation

- On ae, y = 0.
- On ab, $Y = \sqrt{\left(\frac{1}{6}\right)^2 - X^2}$ with respect to the o′ coordinate system.
- Transforming this equation to global coordinate system: $y = \sqrt{\left(\frac{1}{6}\right)^2 - (x - .5)^2}$ on bd
- On de y = 0
- On f g, y = 1
- On ag, y varies from [0, 1]

- On ef , y varies from [0, 1]

Initialization

- Let ξ = constant and η = constant lines correspond to interpolated lines from the boundary in the (x, y) plane.
- Taking these values as initial values of x and y corresponding to (ξ, η) coordinates.

III. METHOD OF SOLUTION

$$\left(x_{\xi\xi}\right)_{i,\ j} = \frac{x_i + 1,\ j - 2x_{i,\ j} + x_{i-1,\ j}}{\Delta\xi^2}$$

$$\left(x_{\xi\xi}\right)_{i,\ j} = \left[\frac{\delta}{\delta\xi}\left(\frac{\delta x}{\delta\eta}\right)\right]_{i,\ j}$$

$$= \frac{\left(\frac{\delta x}{\delta\eta}\right)_{i+1,\ j} - \left(\frac{\delta x}{\delta\eta}\right)_{i-1,\ j}}{2\Delta\xi}$$

$$= \frac{x_{i+1,\ j+1} - x_{i+1,\ j-1} - x_{i-1,\ j+1} + x_{i-1,\ j-1}}{4\Delta\eta\Delta\xi}$$

$$\left(x_{\eta\eta}\right)_{i,\ j} = \frac{x_{i,\ j+1} - 2x_{i,\ j} + x_{i,\ j-1}}{\Delta\eta^2}$$

$$\left(y_{\eta\eta}\right)_{i,\ j} = \frac{y_{i+1,\ j} - 2y_{i,\ j} + y_{i-1,\ j}}{\Delta\xi^2}$$

$$\left(y_{\xi\xi}\right)_{i,\ j} = \frac{y_{i+1,\ j} - 2y_{i,\ j} + y_{i-1,\ j}}{\Delta\xi^2}$$

$$\left(y_{\xi\eta}\right)_{i,\ j} = \left[\frac{\delta}{\delta\xi}\left(y_\eta\right)\right]_{i,\ j}$$

$$= \frac{\left(y_\eta\right)i+1,\ j - \left(y\eta\right)_{i-1,\ j}}{2\Delta\xi}$$

$$= \frac{y_{i+1,\ j+1} - y_{i+1,\ j-1} - y_{i-1,\ j+1} + y_{i-1,\ j-1}}{4\Delta\eta\Delta\xi}$$

$$\left(y_{\eta\eta}\right)_{i,\,j} = \frac{y_{i,\,j+1} - 2y_{i,\,j} + y_{i,\,j-1}}{\Delta\eta^2}$$

$$\left(x_{\xi}\right)_{i,\,j} = \frac{x_{i+1,\,j} - x_{i-1},\ j}{2\Delta\xi}$$

$$\left(x_{\eta}\right)_{i,\,j} = \frac{x_{i,\,j+1} - x_{i,\,j-1}}{2\Delta\eta}$$

$$\left(y_{\xi}\right)_{i,\,j} = \frac{y_{i+1,\,j} - y_{i-1,\,j}}{2\Delta\xi}$$

$$\left(y_{\eta}\right)_{i,\,j} = \frac{y_{i,\,j+1} - y_{i,\,j-1}}{2\Delta\eta}$$

$a_{i,\,j}, b_{i,\,j}, and\ c_{i,\,j}$ are calculated at every point in the $(\xi,\,\eta)$ plane from the previous iteration of the Gauss-Seidel method.

Discretized x-Equation

$$y1 = \frac{y_{i+1,\,j} + y_{i-1,\,j}}{\Delta\xi^2}$$

$$y2 = \frac{y_{i,\,j+1} + y_{i,\,j-1}}{\Delta\eta^2}$$

$$y3 = \frac{y_{i+1,\,j+1} - y_{i+1,\,j-1} - y_{i-1,\,j+1} + y_{i-1,\,j-1}}{4\Delta\xi\Delta\eta}$$

$$yk = 2\left(\frac{a_{i,\,j}}{\Delta\xi^2} + \frac{c_{i,\,j}}{\Delta\eta^2}\right)$$

$$y_{i,\,j} = \frac{a_{i,\,j}y1 - 2b_{i,\,j}y3 + c_{i,\,j}y2}{yk}$$

Gauss–Seidel Method

- ai,j , bi,j , and ci,j are calculated from the previous iteration.
- If ai,j , bi,j , and ci,j are taken from the previous iteration, the system of equations becomes linear.
- Now the Gauss–Seidel method is used to solve the system of equations.
- Convergence of the solution is known by summing up all the absolute values of the errors.

Convergence Criteria

$$\text{Temp}1_{i,j} = x_{i,j} \text{ previous iteration value}$$

$$\text{Temp}2_{i,j} = y_{i,j} \text{ previous iteration value}$$

$$\text{Error } x = \sum_{i=1}^{n+1}\sum_{j=1}^{m+1}\left|x(i,\ j) - temp1(i,\ j)\right|$$

$$\text{Error } y = \sum_{i=1}^{n+1}\sum_{j=1}^{m+1}\left|y(i,\ j) - temp2(i,\ j)\right|$$

Data File

- If convergence criteria is satisfied then the data file is written.
- x and y values are written for ξ = constant lines.
- x and y values are written for η = constant lines.
- x and y values are plotted which correspond to ξ = constant and η = constant lines in the (ξ, η) plane.
- For plotting, Octave software is used.

IV. RESULTS

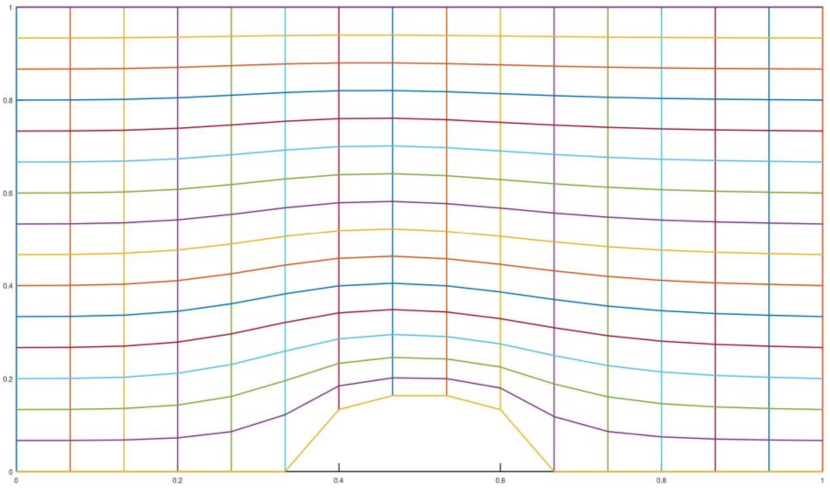

Figure 1: Grid with 16 X 16 points

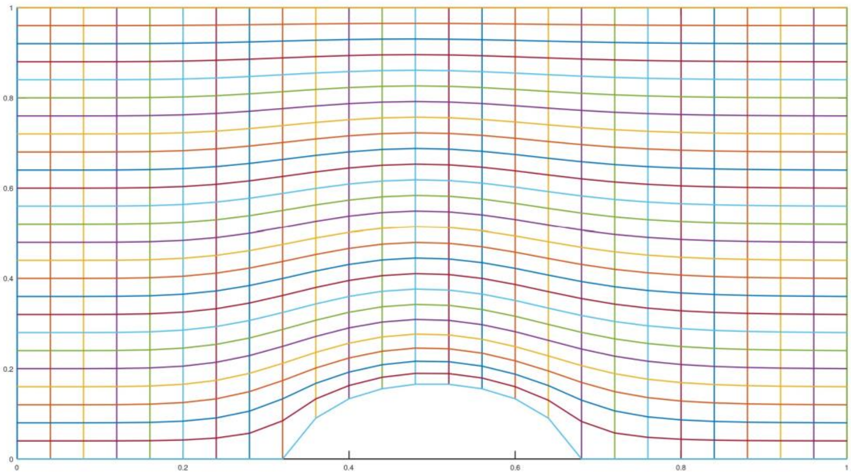

Figure 2: Grid with 26X 26 points

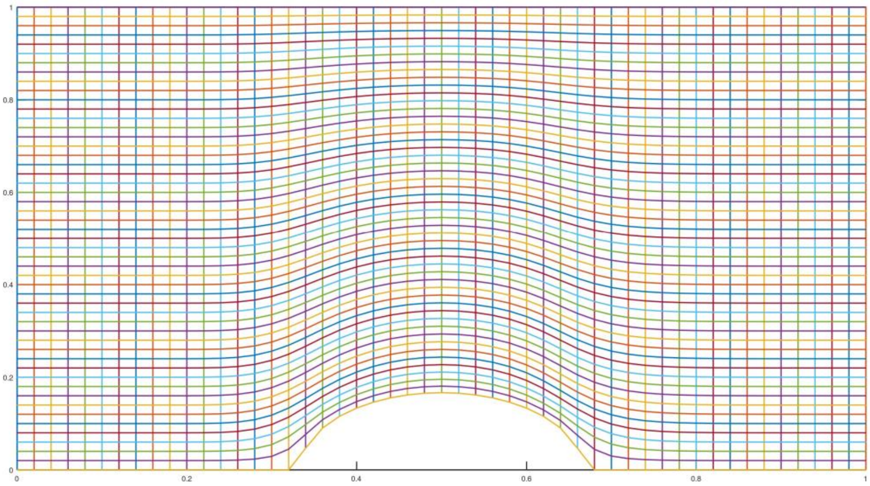

Figure 3: Grid with 51 X 51 points

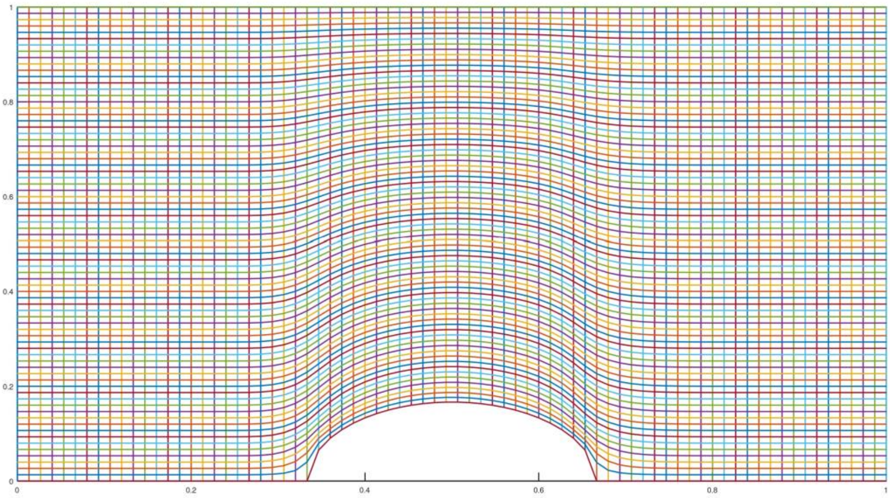

Figure 4: Grid with 76 X 76 points

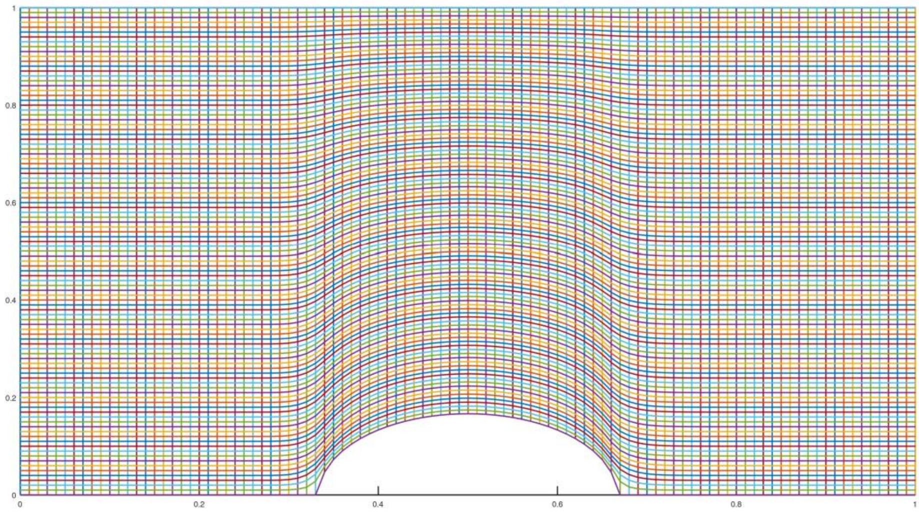

Figure 5: Grid with 101 X 101 points

V. CONCLUSION

The grid is computed using the elliptic partial differentia equations. For boundary condition each of the four sides of the computational domain $\xi - \eta$ plane is mapped to corresponding physical $x - y$ plane. The initialization plays key role in convergence. Coarse to fine grids are generated.

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