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$^*(\hat{g}r)_h$ - HOMEOMORPHISMS IN HEXA TOPOLOGICAL SPACES

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Abstract

A new-class of homeomorphism called $^*(\hat{g}r)_h$ - homeomorphism and $^*(\hat{g}r)_h$ c - homeomorphism are introduce and $^*(\hat{g}r)_h$ c - homeomorphism are grouped together under the composition of maps operation. Also, their relation with few homeomorphism in hexa topological space is presented.

Keywords : $^*(\hat{g}r)_h$ -closed maps, $^*(\hat{g}r)_h$ -open maps, $^*(\hat{g}r)_h$ - homeomorphism and $^*(\hat{g}r)_h$ c - homeomorphism.

1. Introduction

Levine [9] introduced the concept of semi-open sets and its attributes in 1963. Bi-topological space, tri-topological space, quad-topological space, pentatopological space, and hexa-topological space (a space with six topologies $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6$) are all extensions of the single topology throughout this time. Kelly [6] introduced the bitopological concept, Kovar [8] initiated tri-topological space, Mukundan [11] investigated quad topological space, Khan and G. A. Khan [7] investigated penta topological space, and Chandra and Pushpalatha [4] investigated the concept of h - open sets in h -topological spaces, h - continuous. In hexa topological set Semi$_h$ - open, α_h - open, β_h- open, pre$_h$ -open, b_h-open sets are introduced by Asmaa S. Qaddoori[2]. J.Sathya Malar and C.Dhanapakyam[12, 13] introduced $^*(\hat{g}r)_h$ - closed sets and $^*(\hat{g}r)_h$ -continuous functions in hexa topological spaces. This study presents a novel class of homeomorphisms that we call $^*(\hat{g}r)_h$ - homeomorphism and study their properties. Also, we introduce another form of homeomorphism namely $^*(\hat{g}r)_h$ c - homeomorphism and its properties under composition of functions are analyzed.

2. Preliminaries

Definition 2.1 [5, 10]A subset A of space (M,τ_h) is said to be

1. hexa α-open set if $A \subseteq int_h(cl_h(int_h(A)))$

2. hexa semi-open set if $A \subseteq cl_h(int_h(A))$

3. hexa pre-open set if $A \subseteq int_h(cl_h(A))$

4. hexa β- open set if A ⊆ cl_h (int_h (cl_h (A)))

5. hexa b-open set if A ⊆ cl_h (int_h (A)) ∪ int_h (cl_h (A)).

Definition 2. 2 [12] A subset A of a hexa topological space (X, τ_h) is called a hexa-star generalised cap regular closed set [briefly $^*(\hat{g}r)_h$ -closed set], if $r_h cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}_h$ -open subset of (X, τ_h).

Definition 2.3 [13] A function f: (X, τ_h) → (Y, σ_h) is called $^*(\hat{g}r)_h$ -continuous if f^{-1}(V) is $^*(\hat{g}r)_h$ -closed set in (X, τ_h) for every hexa closed set V in (Y, σ_h).

Definition 2.4 [13] A function f: (X, τ_h) → (Y, σ_h) is said to be $^*(\hat{g}r)_h$ - irresolute, if f^{-1}(V) is $^*(\hat{g}r)_h$ - closed set in (X, τ_h) for every $^*(\hat{g}r)_h$ - closed set V in (Y, σ_h).

Definition 2.5 [3] A function f: (M, τ_{h1}) → (N, τ_{h2})is called

(i) h- continuous function if f^{-1} of any h-open set in N is an h - open set in M,

(ii) α_h - continuous function if f^{-1} of any h-open set in N is α_h - open set in M.

(iii) b_h-continuous function if f^{-1} of any h-open set in N is an b_h - open set in M.

(iv) β_h -continuous function if f^{-1} of any h-open set in N is an β_h - open set in M.

Definition 2.6 [3] A homeomorphism between two topological spaces X and Y is a bijective map

f: (X, τ) → (Y, σ) , provided that both f and f^{-1} are continuous .

Definition 2.7 [1] A function f: (X, τ_h) → (Y, σ_h)is named a Hexa homeomorphism, when f are a bijection h- open and h- continuous.

3.$^*(\hat{g}r)_h$ - Closed Maps

Definition 3.1 A map f: (X, τ_h) → (Y, σ_h) is said to be a $^*(\hat{g}r)_h$ - closed map if the image of every hexa closed set in (X, τ_h) is $^*(\hat{g}r)_h$ - closed in (Y, σ_h).

Theorem 3.2 For a hexa topological space (X, τ_h), the following hold.

(i) Every hexa closed is $^*(\hat{g}r)_h$ -closed .

(ii) Every r_h - closed map is $^*(\hat{g}r)_h$- closed.

(iii) Every $^*(\hat{g}r)_h$- closed map is g_h - closed.

(iv) Every $^*(\hat{g}r)_h$-closed map is $\hat{g}_h$ - closed.

Proof

(i) Let f: (X, τ_h) → (Y, σ_h) be a hexa closed map. Let F be a hexa closed subset of X. Since f is hexa closed, f(F) is hexa closed set in Y. Then f(F) is an $^*(\hat{g}r)_h$ - closed set in Y. Hence f is $^*(\hat{g}r)_h$ - closed.

Proof of (ii) to (iv) are similar to (i).

Remark 3.3

The converse of the above theorem need not be true as seen from the following examples.

(i) Let X =Y= {p, q, r } , $\tau_1 = \tau_3 =\{X, \varphi, \{q\}\}$, $\tau_2 = \tau_4 =\{X, \varphi, \{q, r\}\}$, $\tau_5 =\{X, \varphi, \{q\}, \{q, r\}\}$, $\tau_6 =\{X, \varphi \}$and $\sigma_h =\{Y, \varphi, \{r\}\}$. Let f be an identity map such that f: (X, τ_h) → (Y, σ_h) then f is $^*(\hat{g}r)_h$ -closed but not hexa closed map.

(ii) Let X = Y= {p, q, r }, $\tau_1 = \tau_5 =\{X, \varphi, \{p\}\}$, $\tau_2 = \{X, \varphi, \{p, q\}\}$, $\tau_3 =\tau_4 = \{X, \varphi, \{p\}, \{p, q\}\}$, $\{X, \varphi, \{q\}, \{q,r\}\}$, $\tau_6 = \{X, \varphi \}$and $\sigma_1 = \sigma_3 =\{X, \varphi, \{q\}\}$, $\sigma_2 = \sigma_4 =\{X, \varphi, \{p, q\}\}$, $\sigma_5 =\{X, \varphi, \{q\}, \{p, q\}\}$, $\sigma_6 = \{X, \varphi \}$.. Let f be an identity map such that f: (X, τ_h) → (Y, σ_h) then f is a $^*(\hat{g}r)_h$ - closed but not r_h - closed map.

(iii) Let X = Y={p, q, r } , $\tau_1 =\tau_3 =\{X, \varphi, \{q\}\}$, $\tau_2 = \tau_4 =\{X, \varphi, \{p, r\}\}$, $\tau_5 = \{X, \varphi, \{q\}, \{p, r\}\}$, $\tau_6 = \{X, \varphi \}$, $\sigma_1 = \sigma_3 =\{X, \varphi, \{r\}\}$, $\sigma = \sigma_4 =\{X, \varphi, \{p, q\}\}$, $\sigma_5 = \{X, \varphi \}$, $\sigma_6 =\{X, \varphi, \{r\}, \{p, q\}\}$.Let f be an identity map such that f: (X, τ_h) → (Y, σ_h) then f is a g_h - closed but not $^*(\hat{g}r)_h$-closed map.

(iv) Let X = Y= {p, q, r }, $\tau_1 =\tau_3 =\{X, \varphi, \{p\}\}$, $\tau_2 = \tau_4 =\{X, \varphi, \{q, r\}\}$, $\tau_5 = \{X, \varphi, \{p\}, \{q, r\}\}$, $\tau_6 = \{X, \varphi \}$ and $\sigma_1 = \sigma_3 =\{X, \varphi, \{q\}\}$, $\sigma_2 = \sigma_4 =\{X, \varphi, \{p, r\}\}$, $\sigma_5 =\{X, \varphi, \{q\}, \{p, r\}\}$, $\sigma_6 = \{X, \varphi \}$. Let f be an identity map such that f: (X, τ_h) → (Y, σ_h) then f is a $\hat{g}_h$ - closed but not $^*(\hat{g}r)_h$ - closed map.

Theorem 3.4

A function f: (X, τ_h) → (Y, σ_h) is$^*(\hat{g}r)_h$ - closed, if and only if for each subset S of (Y, σ_h) and for each hexa open set U in (X, τ_h) containing f^{-1}(S), there exists a $^*(\hat{g}r)_h$ - open set V of (Y, σ_h) containing S such that f^{-1}(V) ⊂ U.

Proof : Necessity: Suppose f is $^*(\hat{g}r)_h$ - closed, let S be a subset of (Y, σ_h) and U be a hexa open set of (X, τ_h) such that f^{-1}(S) ⊂ U. Now, X \ U is hexa closed in (X, τ_h). Since f is $^*(\hat{g}r)_h$ -closed, f(X \U) is $^*(\hat{g}r)_h$ -closed in (Y, σ_h). Take V = (f(X \U))^c . Then V is $^*(\hat{g}r)_h$ - open in (Y, σ_h). Since f^{-1}(S) ⊂ U implies S ⊂ V and f^{-1}(V) = f^{-1}(f(X \U))^c ⊂ U.

Sufficiency: Let F be a hexa closed subset of (X, τ_h). Then F^c is hexa open in (X, τ_h) and f^{-1}(f(F))^c ⊂ F^c. By hypothesis, there exists an $^*(\hat{g}r)_h$ - open set V of (Y, σ_h), such that (f(F))^c ⊂V and f^{-1}(V) ⊂ F^c and so F ⊂ (f^{-1}(V))^c. Then V^c ⊂ f(F) ⊂f ((f^{-1}(V)) ^{c} = f(f^{-1}(V^c) ⊂ V^c. That is, f(F) = V^c. Hence f(F) is $^*(\hat{g}r)_h$ -closed in (Y, σ_h) and hence f is $^*(\hat{g}r)_h$ - closed.

Theorem 3.5 If f: (X, τ_h) → (Y, σ_h) is hexa closed map and g: (Y, σ_h) → (Z, η_h) is $^*(\hat{g}r)_h$ - closed map. Then the composition $g \circ f$: (X, τ_h) → (Z, η_h) is $^*(\hat{g}r)_h$ - closed map.

Proof Let F be any hexa closed set in (X, τ_h). Since f is a hexa closed map, f(F) is hexa closed set in

(Y, σ_h). Since g is $^*(\hat{g}r)_h$ - closed map, g(f(F)) = $g \circ f$ (F) is $^*(\hat{g}r)_h$ - closed set in (Z, η_h). Thus $g \circ f$ is $^*(\hat{g}r)_h$ - closed map.

Theorem 3.6 Let f: $(X, \tau_h) \to (Y, \sigma_h)$ and g: $(Y, \sigma_h) \to (Z, \eta_h)$ be two functions such that $g \circ f$: $(X, \tau_h) \to (Z, \eta_h)$ is $^*(\hat{g}r)_h$ - closed. Then if

(i) f is hexa continuous and surjective then g is $^*(\hat{g}r)_h$ - closed.

(ii) g is $^*(\hat{g}r)_h$ -irresolute and injective, then f is $^*(\hat{g}r)_h$ - closed.

Proof (i) Let V be a hexa closed set in (Y, σ_h). Then $f^{-1}(V)$ is hexa closed in X, since f is hexa continuous. And $g \circ f(f^{-1}(V))$ is $^*(\hat{g}r)_h$ - closed (Z, η_h), since $g \circ f$ is $^*(\hat{g}r)_h$ - closed. But $g \circ f(f^{-1}(V))$ = $g(f(f^{-1}(V)))$ = g(V), since f is surjective. Hence g(V) is $^*(\hat{g}r)_h$ - closed in (Z, η_h) and hence g is $^*(\hat{g}r)_h$ - closed.

(ii) Let U be a hexa closed set in (X, τ_h). Since $g \circ f$ is $^*(\hat{g}r)_h$ - closed, $(g \circ f)(U)$ is $^*(\hat{g}r)_h$ - closed in (Z, η_h). Since g is $^*(\hat{g}r)_h$ -irresolute, $g^{-1}((g \circ f)(U)) = g^{-1}((g(f(U)))$ is $^*(\hat{g}r)_h$ - closed in (Y, σ_h). Since g is injective, $g^{-1}((g(f(U))) = f(U)$. Hence f(U) is $^*(\hat{g}r)_h$ - closed in (Y, σ_h) and hence f is $^*(\hat{g}r)_h$ - closed.

4. $^*(\hat{g}r)_h$ - Open Maps

Definition 4.1 A map f: $(X, \tau_h) \to (Y, \sigma_h)$ is said to be a $^*(\hat{g}r)_h$ -open map if the image of every hexa open set in (X, τ_h) is $^*(\hat{g}r)_h$ - open in (Y, σ_h).

Theorem 4.2

For any bijection map f: $(X, \tau_h) \to (Y, \sigma_h)$, the following statements are equivalent:

(1) $f^{-1}: (Y, \sigma_h) \to (X, \tau_h)$ is $^*(\hat{g}r)_h$ -continuous.

(2) f is $^*(\hat{g}r)_h$ - closed.

(3) f is $^*(\hat{g}r)_h$ - open.

Proof (1) ⇒ (2): Let U be a hexa closed set in (X, τ_h). By assumption, $(f^{-1})^{-1}(U)$= f(U) is $^*(\hat{g}r)_h$ - closed in (Y, σ_h) and so f is $^*(\hat{g}r)_h$ - closed.

(2) ⇒ (3): Let F be a hexa open set in (X, τ_h). Then F^c is hexa closed in X. By assumption, $f(F^c) = (f(F))^c$ is $^*(\hat{g}r)_h$ - closed in Y and hence f(F) is $^*(\hat{g}r)_h$ - open in (Y, σ_h). Hence f is $^*(\hat{g}r)_h$ - open.

(3) ⇒ (1): Let F be a hexa open in (X, τ_h). By assumption, f(F) is $^*(\hat{g}r)_h$ - open in Y. But f(F) = $(f^{-1})^{-1}(F)$. So $(f^{-1})^{-1}(F)$ is $^*(\hat{g}r)_h$ - open in (Y, σ_h). Hence f^{-1} is $^*(\hat{g}r)_h$ -continuous.

5. $^*(\hat{g}r)_h$ - Homeomorphism

Definition 5.1 A bijective function f : $(X, \tau_h) \to (Y, \sigma_h)$ is called

(i) g_h - homeomorphism if f is both g_h - continuous and g_h - open.

(ii) r_h - homeomorphism if f is both r_h - continuous and r_h - open.

(iii) $\hat{g}_h$ - homeomorphism if f is both $\hat{g}_h$ - continuous and $\hat{g}_h$ - open.

Definition 5.2

A bijective function f : $(X, \tau_h) \rightarrow (Y, \sigma_h)$ is called $^*(\hat{g}r)_h$ - homeomorphism if f is both $^*(\hat{g}r)_h$ - continuous and $^*(\hat{g}r)_h$ - open.

Theorem 5.3 Every hexa homeomorphism is $^*(\hat{g}r)_h$ -homeomorphism but not conversely.

Proof : Let f : $(X, \tau_h) \rightarrow (Y, \sigma_h)$ be a hexa homeomorphism. Then f and f^{-1} are hexa continuous and f is bijection. Since every hexa continuous function is $^*(\hat{g}r)_h$ -continuous , f and f^{-1} are $^*(\hat{g}r)_h$ - continuous. Hence f is $^*(\hat{g}r)_h$ -homeomorphism.

Example 5.4

Let X = Y= {p, q, r } with topologies $\tau_1 = \tau_3 = \{X, \varphi, \{q\}\}$, $\tau_2 = \tau_4 = \{X, \varphi, \{p, r\}\}$, $\tau_5 = \{X, \varphi, \{q\}, \{p, r\}\}$, $\tau_6 = \{X, \varphi\}$ and $\sigma_1 = \sigma_2 = \{X, \varphi, \{q\}\}$, $\sigma_3 = \sigma_4 = \{X, \varphi, \{q, r\}\}$ $\sigma_5 = \{Y, \varphi, \{q\}, \{q, r\}\}$, $\sigma_6 = \{X, \varphi\}$ .Let the function f : $(X, \tau_h) \rightarrow (Y, \sigma_h)$ be an identity map. Then f is $^*(\hat{g}r)_h$ - homeomorphism. But it is not a hexa homeomorphism. Since the inverse image of the hexa closed set {q} in (X, τ_h) is {q} is not hexa closed in (Y, σ_h).

Theorem 5.5 Every r_h - homeomorphism is $^*(\hat{g}r)_h$ -homeomorphism.

Proof: The proof follows from the definition and fact that every r_h - closed set is $^*(\hat{g}r)_h$ - closed.

Example 5.6

Let X = Y= {p, q, r } with topologies $\tau_1 = \tau_3 = \{X, \varphi, \{q\}\}$, $\tau_2 = \tau_4 = \{X, \varphi, \{p, q\}\}$, $\tau_5 = \{X, \varphi, \{q\}, \{p, q\}\}$, $\tau_6 = \{X, \varphi\}$ and $\sigma_1 = \sigma_2 = \{X, \varphi, \{q\}\}$, $\sigma_3 = \sigma_4 = \{X, \varphi, \{p, r\}\}$, $\sigma_5 = \{Y, \varphi, \{q\}, \{p, r\}\}$, $\sigma_6 = \{X, \varphi\}$. Let the function f : $(X, \tau_h) \rightarrow (Y, \sigma_h)$ be an identity map. Then f is $^*(\hat{g}r)_h$ - homeomorphism. But it is not a r_h - homeomorphism. Since the inverse image of the hexa closed set {r} in (X, τ_h) is {r } is not r_h- closed in (Y, σ_h).

Theorem 5.7 Every $^*(\hat{g}r)_h$ -homeomorphism is g_h - homeomorphism.

Proof: Let f : $(X, \tau_h) \rightarrow (Y, \sigma_h)$ be a $^*(\hat{g}r)_h$ - homeomorphism. Then f and f^{-1} are $^*(\hat{g}r)_h$ - continuous and f is bijection. Since every $^*(\hat{g}r)_h$ - continuous function is g_h - continuous , f and f^{-1} are g_h -continuous. Hence f is g_h - homeomorphism.

Example 5.8

Let X = Y= {p, q, r } with topologies $\tau_1 = \tau_3 = \{X, \varphi, \{q\}\}$, $\tau_2 = \tau_4 = \{X, \varphi, \{p, r\}\}$, $\tau_5 = \{X, \varphi, \{q\}, \{p, r\}\}$, $\tau_6 = \{X, \varphi\}$ and $\sigma_1 = \sigma_2 = \{X, \varphi, \{q\}\}$, $\sigma_3 = \sigma_4 = \{X, \varphi, \{q, r\}\}$ $\sigma_5 = \{Y, \varphi, \{q\}, \{q, r\}\}$, $\sigma_6 = \{X, \varphi\}$. Let the function f : $(X, \tau_h) \rightarrow (Y, \sigma_h)$ be an identity map. Then f is g_h - homeomorphism. But it is not a $^*(\hat{g}r)_h$ - homeomorphism. Since the inverse image of the hexa closed set {q} in (X, τ_h) is {q} is not $^*(\hat{g}r)_h$ -closed in (Y, σ_h).

Theorem 5.9 Every $^{*}(\hat{g}r)_h$ - homeomorphism is $\hat{g}_h$ - homeomorphism

Proof: Let $f : (X, \tau_h) \rightarrow (Y, \sigma_h)$ be a $^{*}(\hat{g}r)_h$ - homeomorphism. Then f and f^{-1} are $^{*}(\hat{g}r)_h$ - continuous and f is bijection. Since every $^{*}(\hat{g}r)_h$ - continuous function is $\hat{g}_h$ - continuous , f and f^{-1} are $\hat{g}_h$ -continuous. Hence f is $\hat{g}_h$ - homeomorphism.

Example 5.10

Let X = Y= {p, q, r } with topologies $\tau_1 = \tau_3 = \{X, \varphi, \{r\}\}$, $\tau_2 = \tau_4 = \{X, \varphi, \{p, r\}\}$, $\tau_5 = \{X, \varphi, \{r\}, \{p, r\}\}$, $\tau_6 = \{X, \varphi\}$ and $\sigma_1 = \sigma_2 = \{X, \varphi, \{r\}\}$, $\sigma_3 = \sigma_4 = \{X, \varphi, \{q, r\}\}$, $\sigma_5 = \{Y, \varphi, \{r\}, \{q, r\}\}$, $\sigma_6 = \{X, \varphi\}$. Let the Function $f : (X, \tau_h) \rightarrow (Y, \sigma_h)$ be an identity map. Then f is $\hat{g}_h$ - homeomorphism. But it is not a $^{*}(\hat{g}r)_h$ - homeomorphism. Since the inverse image of the hexa closed set {q} in (X, τ_h) is {q} is not $^{*}(\hat{g}r)_h$ -closed in (Y, σ_h).

Remark 5.11

The Composition of two $^{*}(\hat{g}r)_h$ -homeomorphism need not be $^{*}(\hat{g}r)_h$ - homeomorphism.

Example 5.12

Consider X=Y=Z {p, q, r, s} with topologies $\tau_1 = \{X, \varphi, \{p\}\}$, $\tau_2 = \tau_3 = \{X, \varphi, \{p, q\}\}$ } , $\tau_4 = \tau_5\{X, \varphi, \{p\}, \{p, q\}\}$, $\tau_6 = \{X, \varphi\}$, $\sigma_1 = \{X, \varphi, \{q\}\}$ $\sigma_2 = \sigma_4 = \{Y, \varphi, \{q\}, \{p, q\}\}$, $\sigma_3 = \sigma_5 = \{Y, \varphi, \{p,q\}\}$, $\sigma_6 = \{Y, \varphi\}$, and $\eta_1 = \eta_2 = \{Z, \varphi, \{p\}\}$, $\eta_3 = \{Z, \varphi, \{q, r\}\}$, $\eta_4 = \eta_5 = \{Z, \varphi, \{p\}, \{q, r\}\}$, $\eta_6 = \{Z, \varphi\}$. Define $f : (X, \tau_h) \rightarrow (Y, \sigma_h)$ by f(p) = p, f(q) = q, f(r) =r, f(s) =s and g: $(Y, \sigma_h) \rightarrow (Z, \eta_h)$ by g(p) =p, g(q) = q, g(r) = r, g(s) =s. Here f and g are $^{*}(\hat{g}r)_h$ -homeomorphism. But their composition $g \circ f : (X, \tau_h) \rightarrow (Z, \eta_h)$ is not $^{*}(\hat{g}r)_h$ -homeomorphism as the hexa open set {p, q} in (X, τ_h) is not $^{*}(\hat{g}r)_h$ -open in (Z, η_h).

Theorem 5.13

Assume that $f : (X, \tau_h) \rightarrow (Y, \sigma_h)$ is bijective and $^{*}(\hat{g}r)_h$ -continuous.

(i) f is $^{*}(\hat{g}r)_h$ -open

(ii) f is $^{*}(\hat{g}r)_h$ -homeomorphism

(iii) f is $^{*}(\hat{g}r)_h$ -closed

Proof

(i) ⟹ (ii)

Let f be a $^{*}(\hat{g}r)_h$ -open map. By hypothesis, f is bijective and $^{*}(\hat{g}r)_h$ -continuous. Hence f is $^{*}(\hat{g}r)_h$ - homeomorphism.

(ii) ⟹ (iii)

Let f be a $^{*}(\hat{g}r)_h$ -homeomorphism. Let V be a hexa closed set in (X , τ_h). Then X \V is hexa open in (X , τ_h). Since f is $^{*}(\hat{g}r)_h$ -open, f (X \V) = Y \f (V) which is $^{*}(\hat{g}r)_h$ -open in (Y , σ_h). This implies that f (V) is $^{*}(\hat{g}r)_h$ -closed in (Y , σ_h). Hence f is $^{*}(\hat{g}r)_h$ -closed.

(iii) ⟹ (i)

Let V be hexa open in (X , τ_h). Then V^c is hexa closed in $(X , \tau h)$. By (iii), f (V) is $^{*}(\hat{g}r)_h$ - closed in (Y, σ_h). But $f(V^c) = (f(V))^c$. This implies that $(f(V))^c$ is $^*(\hat{g}r)_h$ - closed in (Y, σ_h) and so $f(V)$ is$^*(\hat{g}r)_h$ -open in (Y, σ_h). This proves (i), that is f is $^*(\hat{g}r)_h$ - open.

6. $^*(\hat{g}r)_h$ c -Homeomorphism

Definition 6.1

A bijective function $f : (X, \tau_h) \rightarrow (Y, \sigma_h)$ is called $^*(\hat{g}r)_h$ c -Homeomorphism if f is $^*(\hat{g}r)_h$ - irresolute and its inverse f^{-1} is also $^*(\hat{g}r)_h$ - irresolute.

Theorem 6.2

Every $^*(\hat{g}r)_h$ c -homeomorphism is a $^*(\hat{g}r)_h$ - homeomorphism.

Proof.

Let $f : (X, \tau_h) \rightarrow (Y, \sigma_h)$ be $^*(\hat{g}r)_h$ - homeomorphism. Since every $^*(\hat{g}r)_h$ - irresolute map is $^*(\hat{g}r)_h$ - continuous, by the hypothesis f is $^*(\hat{g}r)_h$ - homeomorphism.

Theorem 6.3

The composition of two $^*(\hat{g}r)_h$ c - homeomorphism is a $^*(\hat{g}r)_h$ c -homeomorphism.

Proof

Let V be a $^*(\hat{g}r)_h$ - open set in (Z, η_h). Now $(g\, o\, f)^{-1}$ $(V) = f^{-1}(g^{-1}(V)) = f^{-1}(U)$ where $U = g^{-1}(V)$. By hypothesis, U is $^*(\hat{g}r)_h$ - open in (Y, σ_h) and by hypothesis $f^{-1}(U)$ is $^*(\hat{g}r)_h$ - open in (X, τ_h). Therefore $g\, o\, f$ is $^*(\hat{g}r)_h$ - irresolute. Also for an $^*(\hat{g}r)_h$ - open set S in (X, τ_h), we have $(g\, o\, f)(S) = g(f(S)) = g(T)$ where $T = f(S)$. By hypothesis $f(S)$ is $^*(\hat{g}r)_h$ - open in (Y, σ_h) and again by hypothesis $g(f(S))$ is $^*(\hat{g}r)_h$ - open in (Z, η_h) (ie) $(g\, o\, f)(S)$ is $^*(\hat{g}r)_h$ - open in (Z, η_h) and therefore $(g\, o\, f)(V)^{-1}$ $^*(\hat{g}r)_h$ - irresolute. Hence $g\, o\, f$ is g $^*(\hat{g}r)_h$ c - homeomorphism.

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