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A study of Bianchi type-l Bulk Viscous fluid String Dust and magnetic field in General Relativing

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Abstract :-

In this paper, we anelyze a Bianchi type–I cosmo logial model filled with bulk vicous fluid .string cloud with attached particals dust, and magnetic field wthin the framewomk,of,General Relativity. Balk viscocosity is considened to describe dissipative proceses in the early universe, whil cosmic strings represent topological defects strings represent topological defects formed during phase transitions. the presenc of a magnetic field equations are formulated and solved under suitable physical conditions. the behavior of physical parameter's such as energy density, expansion scalar, shear scalar, and magnetic field play a significant role in the evolution of the universe, leading to isothopization of later times.

Keywords :- Bianchi type –I, Bulk visgosity , cosmic strings , dust matter, magneticfield , genenal relativity.

1. Introduction

The study of anisotropic cosmological models has gainal gonsidenable inpon tance in understanding the early stajes of the univense. Among these models, the Bionehi type – I space – time is the simplest anisotropic generalization of the Friedmann–Lema–Robertson- walken (FLRW) model . If provides a useful framework useful to study the effects of anisotropy in cosmic evolution.

The inc¡lusion of bulk viscosity is impontent in describing disspqtive phenomena in losmology, especially duning the eanly universe. Cosmic strngs are hypalogial defects that may have forned duning symmetric.breaking phase transitions. Additionally, magnetic field are belived to play a crytical role in the large-scale structure of the universe.

In this paper , we consider a cosmological model in corporating bulk viscous fluid , string dust distribution , and magnetic field within the theory of General Relativity.

The every point of this model space time there are two metrics

$$ds^2 = g_{ij}\, dx^i\, dx^j \qquad 01$$

and

$$d\eta^2 = \gamma_{ij}\, dx^i dx^j \qquad 02$$

The field equation of rosen s biometric theory of gravitation are $K_i^j = N_i^j - N\delta_i^j = -8\pi k T_i^j$ 03

We consider Bianchi Type I metric 4

Where A,B and C Are functions of t alone. The flat metric corresponding to metric (4) is

5

$$d\eta^2 = -dt^2 + dx^2 + dy^2 dz^2 .$$

The energy momentum tensor T_i^j for string dust is given by 6

$$T_i^j = \epsilon \cup_i \cup^i - \lambda x_i x^i - \varsigma v_l^l \left(g_i^j + v_i v^j\right) + E_i^j$$

With 7

$$v_i v^i = x_i x^i = -1$$

And 8

$$v^i x_i = 0.$$

In this mode , ∈ is the rest engrgy denergy density for a cloud of strings and is give by $\in = \in_p + \mu$ where $\in_p$ and μ denote the particle density and the string tension density of the system of strings respectively, the dercction of strings and ς is the coefficient of bulk viscosity .

The Electromanetic field E_{ij} is given by Lichnerowicz (1967) 9

$$E_{ij} \,\bar{\lambda} \left[|h|^2 \left(v_i v_j + \frac{1}{2} g_{ij}\right) - h_i h_j \right],$$

Where four velocity vector v_i is given by

$$g_{ij} v^i v^j = -1$$

10

And $\bar{\mu}$ is the magnetic permeability and the magnetic flux vector h_i defined by

$$h_i = \frac{\sqrt{-g}}{2\mu} F_{ijkl} F^{kj},$$ where F_{kl} is the electromagntensor

and $h_i \frac{\sqrt{-g}}{2\mu} \in_{ijkl} F^{kl} \cup^j$, 11

where F_{kl} is the electromagnetic field tensor and $\in_{ijkl}$ is the levi Civita tensor density.

Asume the commoving coordinates system . so that $u^1 = u^2 = u^3$=0$u^4 = 1$. Further , we assume that the incident magnetic field is taken along x – axis so that $h_I \neq 0$ and $h_2 = h_3 = h_4 = 0$. The first set of Maxwell ' s equation 12

$$F_{[ij,k]=0}.$$

Yields$F_{24=}$ constant H (say) . duw to the assumption of infinite electrical conductivity , we have

$$F_{14} = F_{24} = F_{34} = 0.$$

The only non – vanisg\hing component of $F_{ij}$is F_{23}. So that 13

$$h_1 = \frac{AH}{\mu Bc}$$

and 14

$$|h|^2 = \frac{H^2}{\mu^{-2} B^2 c^2}.$$

From equation (9) We obtain 15

$$-E_1^1 = E_2^2 = E_3^3 = -E_4^4 = \frac{H^2}{2\mu B^2 C^2}.$$

From equation (6) we obtain 16

$$T_1^1 = \left(-\lambda - \frac{H^2}{2\mu B^2 c^2} - \varsigma v^{\ell}_{;\ell}\right), T_2^2 = T_3^3 = \left(\frac{H^2}{2\bar{\mu} B^2 C^2} - \varsigma v^{\ell}_{\ell}\right),$$

$$T_4^4 = -\left(\epsilon + \frac{H^2}{2\bar{\mu} B^2 C^2}\right).$$

Substituting these values of T_i^J [equations (16)] in the Rosen's field quations (3), we right

$$\frac{-A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16ABC\left(\mu + \frac{H^2}{2\bar{\mu} B^2 C^2} + \varsigma v^{\ell}_{;\ell}\right) \quad 17$$

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4^2}{A} - + \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16\pi ABC\left(-\frac{H^2}{2\mu B^2 C^2} + \varsigma v^{\ell}_{.\ell}\right) \quad 18$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{A_4^2}{A} - \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} = 16\pi ABC\left(-\frac{H^2}{2\mu B^2 C^2} + \varsigma v^{\ell}_{.\ell}\right) \quad 19$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{A_4^2}{A}-\frac{B_4^2}{B^2}-\frac{C_4^2}{C^2}=16\pi ABC\left(-\frac{H^2}{2\mu B^2C^2}+\varsigma v^{\ell}_{.\ell}\right) \qquad 20$$

where

$$A_{4}=\frac{dA}{dt}, B_4=\frac{dB}{dt}, C_4=\frac{dC}{dt}, etc.$$

Equation (17) to (20) are four equation in five unknowns A, B, C, λ and $\in$. Therefore , to deduce a determinate solution ; we assume a supplementary condition

$$A = BC^n, \ n = 0 , \qquad 21$$

for which the shear (σ) is proportional to the sclar of expansion θ .

The universe is filled with Zel' dovich matter string dust and therefore we are using Zel' dovich (1980) condition

$$\in = \lambda \qquad 22$$

in our model . From equations (19) and (20) , we obtain

$$2\frac{C_4^2}{C^2}-2\frac{C_{44}}{C}= 16\pi ABC\left(\varsigma v^{\ell}_{\ell}-\in --= \frac{H^2}{\mu B^2C^2}\right). \qquad 23$$

Adding equations (17) and (23) together and using the condition $\varepsilon =$, we get

$$\frac{B_{44}}{B}-\frac{A_{44}}{A}-\frac{C_{44}}{C}+\frac{A_2^4}{A^2}-\frac{B_4^2}{B^2}+\frac{C_4^2}{C^2}= 16\pi ABC\left(\varsigma v^{\ell}_{\ell}-\frac{H^2}{\mu B^2C^2}\right). \qquad 24$$

From quations (21) and (24), we write

$$(n-1)\frac{B_4^2}{B^2}+(n+1)\frac{C_4^2}{C^2}+(1-n)\frac{B_{44}}{B}-(n+1)\frac{C_{44}}{C}=-16\pi K-16\pi K(BC)^{n-1}+32\pi(BC)^{n+1}\varsigma v^{\ell}_{;\ell}, \qquad 25$$

where $K=\frac{H^2}{2\mu}$.

From equations (18) and (19), we obtain

$$\frac{C_{44}}{C}-\frac{B_{44}}{B}=\frac{C_4^2}{C^2}-\frac{B_4^2}{B^2}. \qquad 26$$

On simplifying above equation, we get

$$\frac{(CB_4-BC_4)_4}{(CB_4-BC_4)}=\frac{(BC)_4}{BC}, \qquad 27$$

which on integrating , yield

$$C^2\left(\frac{B}{C}\right)_4=LBC, \qquad 28$$

whre L is the constant of integration.

Using assumptions BC = μ and $\frac{B}{C}=v$, $equation\ (28) leation\ (28) leads\ to$

$\frac{v_4}{v}$ = L . 29

Now using equation (21) and the condition BC = λ and $\frac{B}{C}$ = V , the equation (25) gives

$$-\frac{n}{\mu^{(n-1)}}\left(\frac{\gamma_{44}}{\gamma}\right)+\frac{n}{\gamma^{n+1}}\frac{\gamma_4^2}{\gamma^2}=-\frac{16\pi}{\mu^2}+32\,\varsigma v_\ell^\ell\,. \qquad 30$$

Applying the condition $\varsigma\,\theta =$ constant to the above equation , we get

$$\mu_{44}-\frac{\mu_4^2}{\gamma}+\beta\,\mu^{n+2}=\frac{16\pi k}{n}\mu^n\,, \qquad 31$$

where

$$\beta=\frac{32\pi}{n}\varsigma v_\ell^\ell,$$

where reduces to

$$\frac{d}{d\mu}[f^2]+\left(-\frac{2}{\mu}\right)f^2=\left[\frac{16\pi k}{n}-\beta\mu^2\right]\mu^n, \qquad 32$$

where $\mu_4 = f\,(\mu)$.

$$f^2=\frac{32\pi k}{n(n-1)}\,\mu^{n+1}-\frac{2\beta}{n+1}\,\mu^{n+3}+p\mu^2, \qquad 33$$

where p is the constant of integration. From equation (29) we write

$$\log v = f\frac{Ldu}{\sqrt{\frac{32\pi k}{n(n-1)\mu^{1-\frac{2\beta}{n+1\,\mu^{n+3}+p\mu^2}}}}+\log b.} \qquad 34$$

Using $\mu_4 = f\,(\mu)$ and expression (33), the metric (4) will be

$$ds^2=-\frac{d\mu^2}{\left[\frac{32\pi k}{n(n-1)\mu^{(n+1)-\frac{2\beta}{n+1}+\mu^{(n+3)}+p\mu^2}}\right]}+\mu^{2n}dx^2+\mu v dy^2+\frac{\mu}{v}+\frac{\mu}{v}dz^2,$$

35

whre v is determined by equation (34) . After suitable transformation of courdinates i. e., putting

$$\mu = T\,, x = X\,, y\,, z = Z$$

$$ds^2 = -\frac{dT^2}{\left[\frac{32\pi K}{n(n-1)T^{n+1} - \frac{2\beta}{n+1}T^{n+3}PT^2}\right]} + T^{2n}\,dX^2 + TvdY^2.$$

36

where v is determition (34) . After suitable transformation of coordinates i. e.. putting

$$\mu = T, x = X, Y = Y, z = Z$$

This is the Bianchi Type-I bulk viscous fluid steing dust cosmological model with magnetic field in biometric of gravitation.

In the absence of magnetic field i. e. K = o , the metric (36) have the from

$$\boldsymbol{ds^2 = \frac{-dT^2}{\left[PT^2 - \frac{2\beta}{n+1}T^{(n+3)}\right]} + T^{2n}dx^2 + Tvdx^2 + \frac{T}{v}\,dz^2} .$$

37

In the absence of viscosity , i. e., $\beta = 0$, the metric (36) takes the from

$$ds^2 = \frac{-dt^2}{\frac{32\pi k}{n(n-1)T^{n+1} + PT^2}} + T^{2n}dx^2 + TvdY^2 + \frac{T}{v}dz^2$$

38

Some Physical and Geometrical Features

The density (ε), the string tendion density (λ) = $\left[\frac{(5n-4n^2-1)5n-4n^2-1}{n(n-1)}\frac{K}{T^2} + \frac{(2n^2+n-1)\beta}{16\pi(n+1)}\right]$.

39 Now, the expansion θ is given by

$\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$, which has the value θ = $\frac{(n+1)f}{T}$, or θ = (n+1) $\left[P + \frac{32\pi K}{n(n-1)}T^{n+1}\right]^{\frac{1}{2}}$.

40

The components of shear tensor σ_i^j are given byType equation here.

$$\sigma = \frac{1}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right]$$

or

$$\sigma_i^j = \frac{(2n-1)}{3}\left[p + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{n+1}\right]^{\frac{1}{2}}.$$

41

Likkewise, we obtainthe othercomponents of σ_i^j as $\sigma_2^2 = \frac{(1-2n)}{6}\left[p + \frac{32\pi k}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{n+1}\right]^{\frac{1}{2}} + \frac{L}{2},$

42

$$\sigma_3^3 = \frac{(1-2n)}{6}\left[p + \frac{32\pi k}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{n+1}\right]^{\frac{1}{2}} + \frac{L}{2},$$

43

and

$$\sigma_4^4 = 0.$$

44

Thus,

$$\sigma^2 = \frac{1}{2}(\sigma_{ij}\sigma^{ij}) = \frac{(2n-1)^2}{12}\left[p + \frac{32\pi k}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right] + \frac{L^2}{4},$$

45

and the spatial volume is

$$R^3 = \alpha T^{(n+1)},$$

where $\alpha = \frac{1}{f} = \frac{1}{\left[\frac{32\pi k}{n(n+1)}T^{(n+1)} - \frac{2\beta}{(n+1)} + PT^2\right]^{\frac{1}{2}}}$.

46

From these results , ir is learned that in the presence of magnetic field and bulk viscosity , the rest energy density ∈ and string tensity λ both are infinite initially, whereas both are depends on viscosity coefficient β decreases and it becomes maximumfor $\beta = 0$. For very large value of T , the expansion θ , the shear σ *and* the spatial volume R^3 , are infinite andour model (36) does not approach isotropy.

3. Discussion

From the results of earlier section-2 , it is seen that in presence of magnetic field and bulk viscosity , our model (26) is expanding , when the coefficient of viscosity β decreasing , and the maximum expansion is (n+1) $\left[p + \frac{32\pi k}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}}$.The rest energy density $\in$ and string tension density λ both are infitially , whereas both aredepends on viscosity coefficient β at infinite time .The expansion θ in the model increases as the coefficient of viscosity β decreasses and it becomes maximum for $\beta = 0$. For very very large value of T . the expansion θ , the shear σ and the spatial volume R^3 are infite and our model (36) does not approach isotropy.

For n =$\frac{1}{2}$, these is shear σ =$\frac{L}{2}$. Hence . the model (36) does not approaches isotropy for large values of T.

In the absence of magnetic field , K , the equation (39)leads to

$$\in = \lambda = \frac{(2n^2+n-1)\beta}{16\pi(n+1)}, \tag{47}$$

from which we conclude that the rest energy density density ($\in$), string tension density (λ) depends only on viscosity coefficient β.

The expressions for θ , σ_i^i and R^3 , in the absence of magnetic field K are given by

$$\theta = (n+1) \quad \left[p - \frac{2\beta}{(n+1)} T^{n+1}\right]^{\frac{1}{2}} \tag{48}$$

$$\sigma_1^1 = \frac{(n-1)}{3}\left[p - \frac{2\beta}{n+1} T^{(n+1)}\right]^{\frac{1}{2}}, \tag{49}$$

$$\sigma_2^2 = \frac{(1-2n)}{6}\left[p - \frac{2\beta}{n+1} T^{(n+1)}\right]^{\frac{1}{2}} + \frac{L}{2}, \tag{50}$$

$$\sigma_3^3 = \frac{(1-2n)}{6}\left[p - \frac{2\beta}{n+1} T^{(n+1)}\right]^{\frac{1}{2}} + \frac{L}{2}, \tag{51}$$

$$\sigma_4^4 = 0, \tag{52}$$

$$\sigma^2 = \frac{(2n-1)^2}{12}\left[p - \frac{2\beta}{n+1} T^{(n+1)}\right]^{\frac{1}{2}} + \frac{L^2}{2}, \tag{53}$$

and

$$R^2 = \alpha_1 \, T^{(n+1)}, \qquad 54$$

where $\alpha_1 = \dfrac{1}{\left[PT^{2-\frac{2\beta}{(n+1)T^{(n+3)}}}\right]^{\frac{1}{2}}}$.

In the absence of viscosity , the rest energy density $\in$, string tension density λ for the model (36) is given by

$$\in = \lambda = \frac{(5n-4n^2-1)K}{n(n-1)} \frac{K}{T^2} \;, \qquad 55$$

and it becomes zero initially and infinite for very large value of T.

The expressions for θ, σ_i^j and R^3, in the absence of viscosity , are given by

$$\theta = (n+1) \left[p + \frac{32\pi K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}}, \qquad \sigma_1^1 = \frac{(2n-1)}{3} \left[p + \frac{32\pi K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}}, \qquad 56$$

$$\sigma_2^2 = \frac{(1-2n)}{6} \left[p + \frac{32\pi K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}} + \frac{L}{2} \;, \qquad 57$$

$$\sigma_3^3 = \frac{(1-2n)}{6} \left[p + \frac{32\pi K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}} - \frac{L}{2} \;, \qquad 58$$

$$\sigma_4^4 = 0 \;, \qquad 59$$

$$\sigma^2 = \frac{(2n-1)}{12} \left[p + \frac{32\pi K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}} + \frac{L^2}{2} \;, \qquad 60$$

and

$$R^3 = \alpha_2 \; T^{(n+1)}, \qquad 61$$

where $\alpha_2 = \dfrac{1}{\left[PT^2 + \frac{32\pi K}{n(n-1)} T^{(n-1)}\right]^{\frac{1}{2}}}$.

In the absence of magnetic field, our model (36) contracting as T and β increases, and it is expanding in the absence of bulk viscosity. The model does not approaches isotropy for large values of T. The spatially and it is infinite for very very large value of T.

We compzred between the case in the presence of magnetic field and bulk viscosity and the case in the absence of magnetic field and bulk viscosity and bulk viscosity and it is realized that our model is expanding as well as shearing in presence of magnetic field and bulk viscosity (for decreasing β) , whereas it is neither expanding nor shearing in the absence of both magnetic field and bulk viscosity.

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