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N$\alpha\pi$g Connectedness in Neutrosophic Topological Spaces

K. Kavipriya[1*] *and* K. Sakthivel[2]

[1*]Research scholar, Department of Mathematics, Government Arts College, Udumalpet,Tamilnadu, India.
[2] Assistant professor, Department of Mathematics, Government Arts and science College, Pollachi, Tamilnadu, India.

ABSTRACT:

The aim of this paper is to introduce a new class of functions called Neutrosophic $\alpha\pi$g connectedness. Also, we have analyzed this connected space with the existing connectedness in Neutrosophic topological spaces.

Key words:

Neutrosophic Topological spaces, Neutrosophic open sets, Neutrosophic Closed sets, Neutrosophic α Closed set, Neutrosophic $\alpha\pi g$ Closed sets and Neutrosophic $\alpha\pi g$ connected.

I. Introduction

In 1965, L.A.Zadeh [17] introduced the notion of fuzzy sets [FS]. It shows the degree of membership of the element in a set X. Later, fuzzy topological space was introduced by C.L. Chang [3] in 1968. In 1986, K.Atanassov [2] introduced the notion of intuitionistic fuzzy sets [IFS], where the degree of membership and degree of non-membership are discussed. Later, Intuitionistic fuzzy topological spaces was introduced by Coker [4] in 1997. In 2010, Florentin Smarandache [6] defined the Neutrosophic set on three components, namely Truth (membership), Indeterminacy, Falsehood (non-membership). In 2012, A. A Salama and S. A. Alblowi [14] introduced the concept of Neutrosophic topological space by using Neutrosophic sets. A. A. Salama [15] introduced Neutrosophic generalized pre regular closed sets in Neutrosophic topological spaces. Wadei Al-Omeri and Saeid Jafari [16] introduced Generalized Closed Sets, Generalized Pre-Closed, Generalized connectedness and Generalized compactness in Neutrosophic Topological Spaces. Parimala M et al[12]

we introduce the concept of Neutrosophic $\alpha\pi g$ closed sets in Neutrosophic topological spaces. Further, we extend our study to the concept of Neutrosophic $\alpha\pi g$ connectedness in Neutrosophic topological spaces.

II. Preliminaries

Definition 2.1:

Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x) and \nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A.

A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ can be identified as an ordered triple $\langle \mu_A, \sigma_A, \nu_A\rangle$ in $]^-0,1^+[$ on X.

Definition 2.2:

For any two neutrosophic sets $A=\{\langle x,\mu_A(x),\sigma_A(x),\nu_A(x)\rangle : x\in X\}$ and $B=\{\langle x,\mu_B(x),\sigma_B(x),\nu_B(x)\rangle : x\in X\}$ we may have,

1. $A\subseteq B \Leftrightarrow \mu_A(x)\le\mu_B(x),\sigma_A(x)\le\sigma_B(x),\nu_A(x)\ge\nu_B(x)\forall x\in X$
2. $A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\sigma_A(x)\wedge\sigma_B(x),\nu_A(x)\vee\nu_B(x)\rangle \mid x\in X\}$
3. $A\cup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\sigma_A(x)\vee\sigma_B(x),\nu_A(x)\wedge\nu_B(x)\rangle \mid x\in X\}$
4. $A^C=\{\langle x,\nu_A(x),1-\sigma_A(x),\mu_A(x)\rangle \mid x\in X\}$

Definition 2.3:

A Neutrosophic topology (NT) on a non-empty set X is a family τ_N of neutrosophic subsets in X satisfies the following axioms:

(NT_1) $0_N,1_N\in\tau_N$

(NT_2) $W_1\cap W_2\in\tau_N$ for any $W_1,W_2\in\tau_N$

(NT_3) $\cup W_j\in\tau_N\forall\{W_j : j\in I\}\subseteq\tau_N$

In this case the pair (X,τ_N) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (NOS) in X. A neutrosophic set A is a neutrosophic Closed set (NCS) if and only if complement A^C is a neutrosophic open set in X.

Here the empty set 0_N and the whole set 1_N may be defined as follows:

(0_1) $0_N=\{\langle x,0,0,1\rangle : x\in X\}$

(1_1) $1_N=\{\langle x,1,1,0\rangle : x\in X\}$

Definition 2.4:

Let (X,τ_N) be a NTS and $A=\{\langle x,\mu_A(x),\sigma_A(x),\nu_A(x)\rangle : x\in X\}$ be a NS in X. Then the neutrosophic interior and the neutrosophic closure of A are defined by

$NInt(A)=\cup\{H : H$ is an NOS in X $H\subseteq A\}$

$NCl(A)=\cap\{M : M$ is an NCS in X $A\subseteq M\}$

Definition 2.5:

An NS $A=\langle x,\mu_A,\sigma_A,\nu_A\rangle$ in an NTS (X,τ_N) is said to be an

(i) Neutrosophic regular closed set(NRCS in short) if $A=Ncl(N\operatorname{int}(A))$,

(ii) Neutrosophic α closed set(NαCS in short) if $Ncl(N\operatorname{int}(Ncl(A)))\subseteq A$,

(iii) Neutrosophic semi closed(NSCS in short) if $N\operatorname{int}(Ncl(A))\subseteq A$,

(iv)Neutrosophic Pre closed(NPCS in Short) if $Ncl(N\operatorname{int}(A))\subseteq A$,

(v) Neutrosophic Generalized closed set(NGCS in short) if $Ncl(A)\subseteq U$ whenever $A\subseteq U$ and U is an NOS in (X,τ_N),

(vi) Neutrosophic Generalized Semi closed set(NGSCS in short) if $Nscl(A)\subseteq U$ whenever $A\subseteq U$ and U is an NOS in (X,τ_N),

(vii) Neutrosophic Generalized pre closed set(NGPCS in short) if $Npcl(A)\subseteq U$ whenever $A\subseteq U$ and U is an NOS in (X,τ_N),

The complements of the above mentioned Neutrosophic closed sets are called their Neutrosophic Open sets.

Definition 2.6:

An NS $A=\langle x,\mu_A,\sigma_A,\nu_A\rangle$ in an NTS (X,τ_N) is said to be an Neutrosophic π Open set (NπOS in short) if A is a finite union of Neutrosophic Regular open sets.

Definition 2.7:

Let A be an NTS (X,τ_N). Then the α-interior of A ($\alpha\text{int}(A)$ in short) and the α-closure of A ($\alpha cl(A)$ in short) are defined as

$$N\alpha\text{int}(A) = \cup\{\{H : H \text{ is an } N\alpha \text{ OS in X } H \subseteq A\},$$

$$N\alpha cl(A) = \cap\{M : M \text{ is an } N\alpha \text{ CS in X } A \subseteq M\}.$$

Definition 2.8:

Let $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ in an NTS (X,τ_N) is said to be an

i. $N\alpha cl(A) = A \cup Ncl(N\text{int}(Ncl(A)))$,

ii. $N\alpha\text{int}(A) = A \cap N\text{int}(Ncl(N\text{int}(A)))$.

Definition 2.9:

An NS A in (X,τ_N) is said to be an Neutrosophic $\alpha\pi$ Generalized closed set (N$\alpha\pi$ GCS in short) if $N\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\pi OS$ in (X,τ_N). Here the family of all N$\alpha\pi$ GCS of an NTS (X,τ_N) is denoted by N$\alpha\pi$ GCS (X,τ_N).

Definition 2.10:

Let (X,τ_N) be NTS and $A = \{\langle x,\ \mu_A(x), \sigma_A(x),\ \nu_A(x)\rangle : x \epsilon X\}$ be a NS in X. Then the Neutrosophic $\alpha\pi g$ closure and Neutrosophic $\alpha\pi g$ interior of A are defined by

$$N\alpha\pi g\,\text{int}(A) = \cup\{\{H : H \text{ is an } N\alpha\pi g\text{OS in X } H \subseteq A\},$$

$$N\alpha\pi g\, cl(A) = \cap\{M : M \text{ is an } N\alpha\pi g\text{CS in X } A \subseteq M\}.$$

III. Neutrosophic $\alpha\pi g$ connected space in Neutrosophic Topological Spaces

Definition 3.1:

An NTS (X,τ) is said to be an Neutrosophic $\alpha\pi g$ connected space ($N\alpha\pi g$ connected space for brief) if the only NTS$_s$ which are both an N$\alpha\pi g$OS and an N$\alpha\pi g$CS are 0_N and 1_N.

Theorem 3.2:

Every N$\alpha\pi g$ connected space is Neutrosophic Connected space but not conversely.

Proof:

Let (X,τ) be a N$\alpha\pi$GCNS. Suppose (X,τ) is not Neutrosophic connected space. Then there exists a proper non-empty Neutrosophic set B of X which is both NOS and NCS in X. Since every NCS is N$\alpha\pi$GCS, B is both N$\alpha\pi g$OS and N$\alpha\pi g$CS in X. Hence (X,τ) is not N$\alpha\pi$GCNS which implies that our assumption is wrong. Therefore (X,τ) is Neutrosophic connected space.

Example 3.3:

Let $X = \{p,q\}$, and $\tau_N = \{0_N, W_1, 1_N\}$ be a NTS on(X,τ), where $W_1 = \langle x,(0.5,0.5,0.4),\ (0.6, 0.4, 0.5)\rangle$. Then (X,τ) is a Neutrosophic connected space. However, Neutrosophic set $W_2 = \langle x,(0.3, 0.4, 0.6),(0.4,0.3,0.5)\rangle$ is both a N$\alpha\pi g$OS and N$\alpha\pi g$CS in(X,τ). Therefore (X,τ) is not a $N\alpha\pi g$ connected space.

Theorem 3.4:

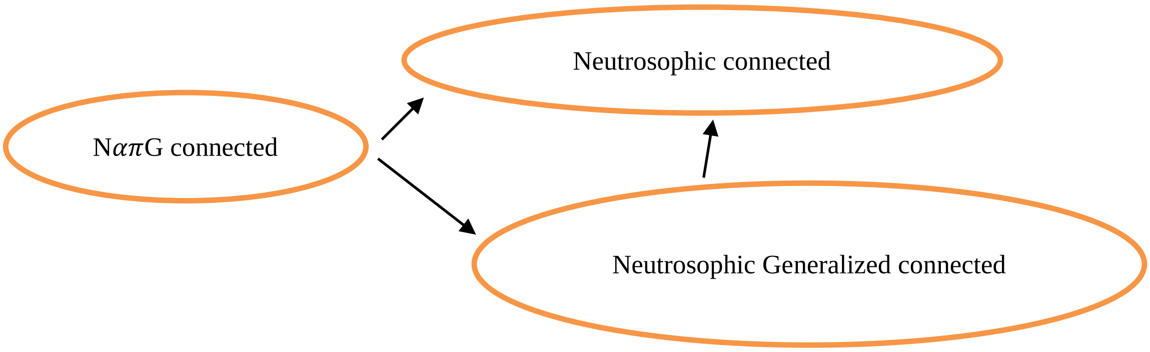
Every N$\alpha\pi g$ connected space is Neutrosophic Generalized Connected space but not conversely.

Proof:

Let (X,τ_N) be a N$\alpha\pi$GCNS. Suppose (X,τ_N) is not Neutrosophic generalized connected space. Then there exists a proper non-empty Neutrosophic set B of X which is both NGOS and NGCS in X. Since, every NGCS is N$\alpha\pi$GCS, B is both N$\alpha\pi g$OS and N$\alpha\pi g$CS in X. Hence (X,τ_N) is not N$\alpha\pi$GCNS which implies that our assumption is wrong. Therefore (X,τ_N) is Neutrosophic generalized connected space.

Example 3.5:

Let $X = \{p,q\}$, and $\tau_N = \{0_N, W_1, 1_N\}$ be a NTS on(X,τ), where $W_1 = \langle x,(0.5,0.5,0.4),\ (0.6, 0.4, 0.5)\rangle$. Then (X,τ) is a Neutrosophic Generalized connected space. However, Neutrosophic set $W_2 = \langle x,(0.3, 0.4, 0.6),(0.4,0.3,0.5)\rangle$ is both a N$\alpha\pi g$OS and N$\alpha\pi g$CS in(X,τ). Therefore (X,τ) is not a $N\alpha\pi g$ connected space.



Theorem 3.6:

The NT (X,τ_N) is a $N\alpha\pi G$ connected space if and only if there exists no non-zero $N\alpha\pi GOS$ J and K in (X,τ_N) such that $J = K^c$.

Proof: Necessity: Assume that (X,τ_N) is an $N\alpha\pi G$-connected space. Let J and K be two $N\alpha\pi GOS$ in (X,τ_N) such that J$\neq 0_N \neq K$ and $J = K^c$. Therefore K^c is a $N\alpha\pi G$CS. Since $J \neq 0_N, K \neq 1_N$. This implies K is a proper NS which is both a $N\alpha\pi GOS$ and a $N\alpha\pi GCS$ in (X,τ_N). Hence (X,τ_N) is not a $N\alpha\pi G$ connected space. But this is a contradiction to our hypothesis. Thus there exists no non-zero $N\alpha\pi GOS$ J and K in (X,τ_N) such that $J = K^c$.

Sufficiency: Let J both a $N\alpha\pi GOS$ and $N\alpha\pi GCS$ in (X,τ_N) such that $1_N \neq J \neq 0_N$. Now let K=J^c. Then K is a $N\alpha\pi GOS$ and $K \neq 1_N$. This implies $K = J^c \neq 0_N$ which is a contradiction to our hypothesis. Therefore (X,τ_N) a $N\alpha\pi G$ connected space.

Theorem 3.7:

A NT (X,τ_N) is an $N\alpha\pi G$ connected space if and only if there exists no non-zero $N\alpha\pi GOSs$ J and K in (X,τ_N) such that J=K^c, $K = \left(N\alpha\pi cl(J)\right)^C, J = \left(N\alpha\pi cl(K)\right)^c$.

Proof: Necessity: Assume that there exist NSs J and K such that J$\neq 0_N \neq K$, K=J^c, $K = \left(N\alpha\pi cl(J)\right)^C, J = \left(N\alpha\pi cl(K)\right)^c$. Since $\left(N\alpha\pi cl(J)\right)^c$ and $J = \left(N\alpha\pi cl(K)\right)^c$ are $N\alpha\pi GOSs$ in (X,τ_N) J and K are $N\alpha\pi GOSs$ in (X,τ_N). This implies (X,τ_N) is not a $N\alpha\pi G$ connected space, which is a contradiction. Therefore there exists no non-zero $N\alpha\pi GOSs$ J and K in (X,τ_N) such that $J = K^c, K = \left(N\alpha\pi cl(J)\right)^C, J = \left(N\alpha\pi cl(K)\right)^c$.

Sufficiency: Let J be both a $N\alpha\pi GOS$ and $N\alpha\pi GCS$ in (X,τ_N) such that $1_N \neq J \neq 0_N$. Now by taking $K = J^c$, we obtain a contradiction to our hypothesis. Hence (X,τ_N) is a $N\alpha\pi G$ connected space.

IV. Neutrosophic $\alpha\pi G$ compact space in Neutrosophic Topological Spaces

Definition 4.1:

The Neutrosophic set A=$\langle x, \mu_A, \sigma_A, \gamma_A\rangle$ in a NTS (X,τ_N) is called Neutrosophic $\alpha\pi$G compact space (N$\alpha\pi GC_T$ in short) if and only if every N$\alpha\pi G$ open cover of A has a finite subcover.

Definition 4.2:

Let (X,τ_N) be a NTS. If a family $\{\langle x, \mu_{G_i}(x),\ \sigma_{G_i}(x),\ \gamma_{G_i}(x)\rangle : i\epsilon J\}$ of N$\alpha\pi$G open sets in (X,τ_N) satisfies the condition $\cup\{\langle x, \mu_{G_i}(x),\ \sigma_{G_i}(x),\ \gamma_{G_i}(x)\rangle : i\epsilon J\} = 1_N$. Then it is called N$\alpha\pi$G open cover of (X,τ_N).

Definition 4.3:

Let (X,τ_N) be a NTS. A finite subfamily of a N$\alpha\pi$G open cover $\{\langle x, \mu_{G_i}(x),\ \sigma_{G_i}(x),\ \gamma_{G_i}(x)\rangle : i\epsilon J\}$ of (X,τ_N), which is also a N$\alpha\pi$G open cover of (X,τ_N) is called a finite subcover of $\{\langle x, \mu_{G_i}(x),\ \sigma_{G_i}(x),\ \gamma_{G_i}(x)\rangle : i\epsilon J\}$.

Theorem 4.4:

Every N$\alpha\pi GC_T$ is Neutrosophic compact space.

Proof:

Let (X,τ_N) be a $N\alpha\pi GC_T$ space. Let $\{U_i : i\epsilon J\}$ be a Neutrosophic open cover of (X,τ_N) by Neutrosophic open sets in (X,τ_N). Hence $\{U_i : i\epsilon J\}$ be a $N\alpha\pi$G open cover of (X,τ_N) by $N\alpha\pi$G open sets in (X,τ_N). Since (X,τ_N) be a $N\alpha\pi$G compact space, the $N\alpha\pi$G open cover $\{U_i : i\epsilon J\}$ has a finite subcover say $\{U_i : i = 1,2,3, \ldots\ldots, n\}$ of (X,τ_N). Hence (X,τ_N) is Neutrosophic compact space.

Theorem 4.5:

Every $N\alpha\pi$G compact space is NG compact space.

Proof:

Let (X,τ_N) be a $N\alpha\pi GC_T$ space. Let $\{U_i : i\epsilon J\}$ be a NG open cover of (X,τ_N) by NG open sets in (X,τ_N). Hence $\{U_i : i\epsilon J\}$ be a $N\alpha\pi$G open cover of (X,τ_N) by $N\alpha\pi$G open sets in (X,τ_N). Since (X,τ_N) be a $N\alpha\pi$G compact space, the $N\alpha\pi$G open cover $\{U_i : i\epsilon J\}$ has a finite subcover say $\{U_i : i = 1,2,3, \ldots\ldots, n\}$ of (X,τ_N). Hence (X,τ_N) is NG compact space.

Theorem 4.6:

Let (X,τ_N) and (Y,η_N) be any two NTS and $g_N : (X,\tau_N) \to (Y,\eta_N)$ be a $N\alpha\pi$G continuous mapping. If A is $N\alpha\pi$G compact in (X,τ_N) then $g_N(A)$ is Neutrosophic compact in (Y,η_N).

Proof:

Let $U_i = \{\langle y, \mu_{U_i}(y), \sigma_{U_i}(y), \gamma_{U_i}(y)\rangle : i\epsilon J\}$ be a Neutrosophic open cover of $g_N(A)$ in (Y,η_N). That is $g_N(A) \subseteq U_{i\epsilon J} U_i$. Since g_N is $N\alpha\pi$G continuous, $\{g_N^{-1}(U_i) : i\epsilon J\}$ is a $N\alpha\pi$G open cover of A in (X,τ_N). Now, A$\subseteq U_{i\epsilon J}\{g_N^{-1}(U_i)\}$. Since A in (X,τ_N) is $N\alpha\pi$G compact, there exist a finite subcover $J\epsilon K$ such that A$\subseteq U_{i\epsilon J} g_N^{-1}(U_i) = 1_N$. Hence $g_N(U_{i\epsilon K} g_N^{-1}(U_i)) = U_{i\epsilon K} U_i = 1_N$. Therefore $g_N(A)$ is Neutrosophic compact in (Y,η_N).

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