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SOME DISTANCE-BASED TOPOLOGICAL INDICES OF CORONA PRODUCT OF COMPLETE WITH CYCLE GRAPHS

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Abstract: In this paper, we study some distance based topological indices, such as Wiener Index(W), Hyper-Wiener Index (WW), Harary Index (H), Reciprocal Complementary Wiener Index (RCW), Sum Connectivity Index (SCI), Batschelet Index (B), Reverse Wiener Index $\wedge(G)$, Reciprocal Reverse Wiener Index R ∧ (G) of $K_m \odot C_n$.

MSC : 05C76, 05C10, 05C38

Keywords: Wiener index, Hyper-Wiener Index, Harary Index, Reciprocal Complementary Wiener Index, Sum Connectivity Index, Batschelet Index, Reverse Wiener Index, Reciprocal Reverse Wiener Index and Corona Product.

I. INTRODUCTION

In this study, we consider only **finite, simple, connected, and undirected graphs**. For a graph $G = (V, E)$, the number of vertices and edges are denoted by $|V(G)|$and $|E(G)|$, respectively. When the graph under consideration is clear from the context, we simply write $d(u, v)$for the distance between vertices $u$and v. For any two vertices $u, v \in V(G)$, the shortest path length between them in $G$is denoted by $d_G(u, v)$. The eccentricity of a vertex $u$in $G$is defined as

$$e(u) = \max\{d(u, v): v \in V(G)\}.$$

The radius of $G$is given by

$$r = \text{rad}(G) = \min\{e(v): v \in V(G)\},$$

while the diameter of $G$is defined as

$$d = \text{diam}(G) = \max\{e(v): v \in V(G)\}.$$

Topological indices derive from graph theory, which represents molecules as graphs. Atoms correspond to vertices, whereas bonds correspond to edges. Distance-based topological indices are topological indices that are calculated from the distances between vertices (atoms) in a molecular network. These indices concentrate on the graph's structure by measuring the distance between atoms, which can offer information about numerous chemical characteristics and behaviors.

In [3,4,5], 1947 Harry Wiener introduced the Wiener index and is defined as,

$$W(G) = \sum_{u,v \in V(G)} d(u,v)$$

In [7], 1993 Milan Randić introduced Hyper-Wiener index and is defined as,

$$WW(G) = \frac{1}{2} \sum_{u,v \in V(G)} [d(u,v) + d(u,v)^2]$$

In [8,9] Plavšić et. al., and Ivancine et. al., introduced Harary index independently and defined as,

$$H(G) = \sum_{u,v \in V(G)} \frac{1}{d(u,v)}$$

In [10,11] Ivancineet.al., introduced the Reciprocal Complementary Wiener index, denoted by $RCW(G)$ and given by,

$$RCW(G) = \sum_{u,v \in V(G)} \frac{1}{d + 1 - d(u,v)}$$

In [13], 1975 Milan Randić introduced the Sum Connectivity index and is defined as,

$$SCI(G) = \sum_{u,v \in V(G)} \frac{1}{d(u,v) + 1}$$

In [14], 1981 Eduard Batschelet introduced the Batschelet index and is defined as,

$$B(G) = \sum_{u,v \in V(G)} d(u,v)^2$$

In [1], 2000 Balaban introduced the Reverse Wiener index and is defined as,

$$\wedge(G) = \frac{n(n-1)d}{2} - W(G)$$

where n = |V (G)| and d is the diameter of G.

In [12], 2010 the Reciprocal Reverse Wiener (RRW) index R ∧ (G) of a connected graph G is defined as

$$R\wedge(G) = \sum_{u,v \in V(G)} \frac{1}{d - d(u,v)}$$

Where d is the diameter of a graph G.

A graph is said to be a complete graph if every vertex is adjacent to every other vertex. A complete graph of n vertices is denoted by K_n.

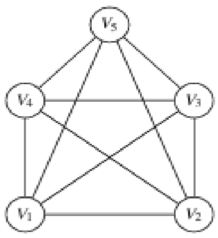

Figure 1: Complete Graph with 5 vertices

An alternating sequence of vertices and edges starting with a vertex and ending with a vertex is such that every edge in the sequence is incident on the proceeding and succeeding vertices is called a walk in the graph.

A closed walk in which no vertex except the end vertices is repeated(revisited) is called a cycle or a circuit. It maybe denoted by C_n.

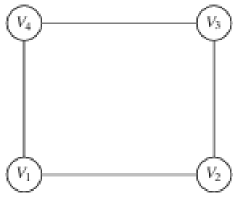

Figure 2: Cycle Graph with 4 vertices

In this paper we calculate W(G), WW(G), H(G), RCW(G), SCI(G), B(G), ʌ(G) and R ʌ (G) of corona product of complete
with cycle graphs.

Definition 1.1:

The corona product of two graphs G and H is defined as the graph obtained by taking one copy of G and |V (G)| copies of H and joining the i^{th} vertex of G to every vertex in the j^{th} copy of H. See Figure 1

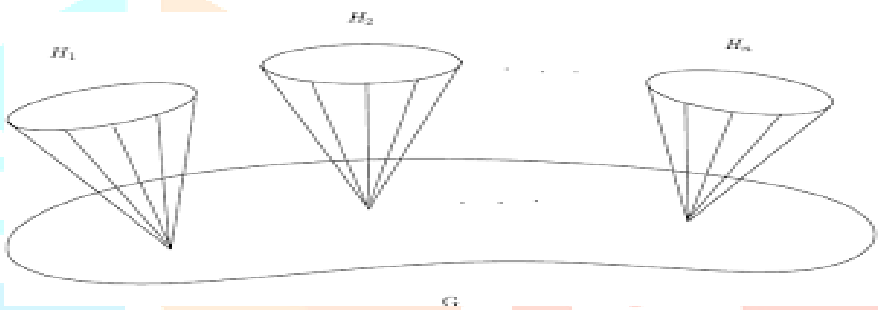

Figure3: Corona Product of two graphs G and H

II. MAIN RESULTS

In this section, we compute some distance based topological indices of corona product of $K_m \odot C_n$.

Definition 2.1:

The graph that results from taking m copies of C_n, one copy of K_m and linking each vertex in the i^{th} copy
of C_n, to every vertex in the i^{th} copy of K_m is known as the corona product of complete and cycle graphs, K_m
and C_n.
See Figure 3

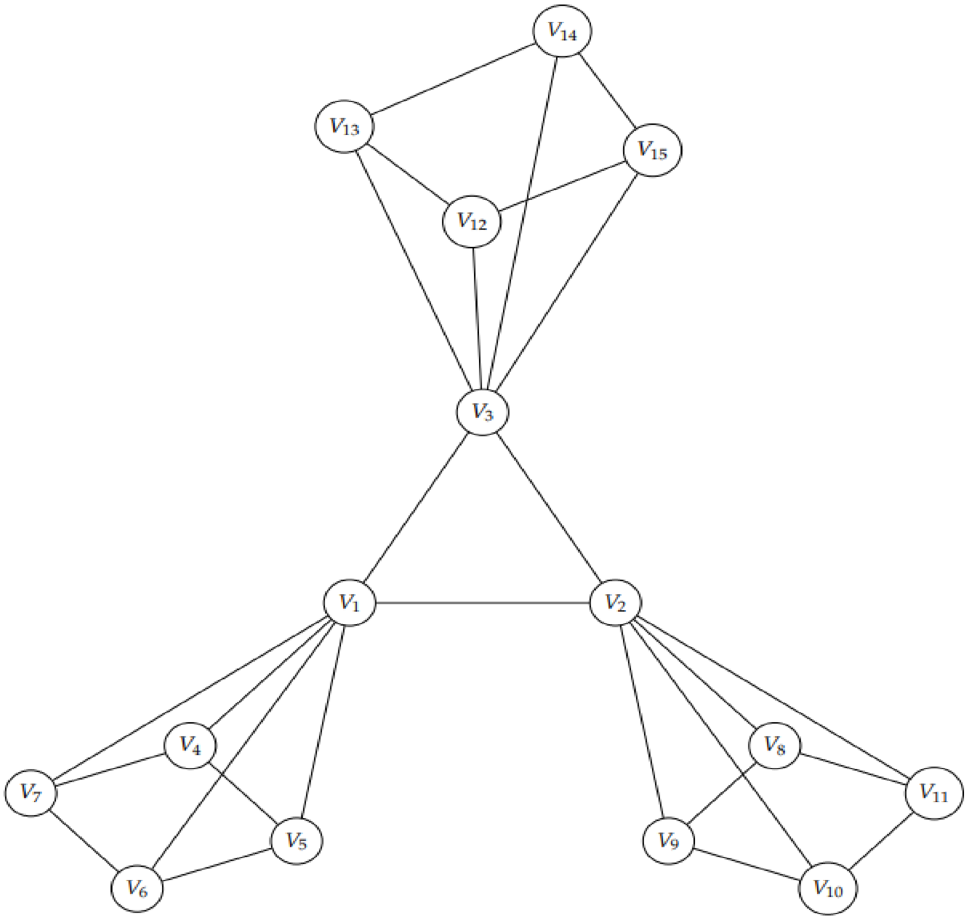

Figure 4: $K_3 \odot C_4$

Here diam(G) =3 and the distance between any pair of vertices varies from 1, 2, . . . , diam(G).

The number of 1 distance, pair of vertices is $\frac{m(m-1)}{2} + 2mn$.
The number of 2 distance, pair of vertices is $mn(m-1) + \frac{mn(n-3)}{2}$.
The number of 3distance, pair of vertices is $n^2(\frac{m(m-1)}{2})$, for $m \geq 2$ and $n \geq 3$.
By using the above, we derive the following theorems, Then,

Theorem 2.2: Let K_m *and* C_n be complete and cycle graphs with m ≥ 2 and n ≥ 3. Then

(i) $$W(K_m \odot C_n) = \frac{m(m-1)}{2}(3n^2 + 4n + 1) + mn(n-1)$$

(ii) $$WW(K_m \odot C_n) = \frac{m(m-1)}{2}(1 + 6n + 6n^2) + \frac{mn}{2}(3n-5)$$

Proof: Let K_m *and* C_n be complete and cycle graphs with m ≥ 2 and n ≥ 3. Then V$(K_m \odot C_n) = V_1 \cup V_2 \cup \ldots\ldots V_n$,
where $V_i = \{v_{i,0}, v_{i,1}, \ldots \ldots v_{i,n+1}\}$, for 1 ≤ i ≤ m.

(i)By the definition of Wiener index

$$W(K_m \odot C_n) = \sum_{u,v \in V(G)} d(u,v)$$

$$= \left[\frac{m(m-1)}{2} + 2mn\right](1) + \left[mn(m-1) + \frac{mn(n-3)}{2}\right](2) + n^2\left[\frac{m(m-1)}{2}\right](3)$$

$$= \frac{m(m-1)}{2} + 2mn + 2mn(m-1) + mn(m-1) + \frac{3n^2m(m-1)}{2}$$

$$= \frac{m(m-1)}{2}(1 + 4n + 3n^2) + mn(2 + n - 3)$$

$$W(K_m \odot C_n) = \frac{m(m-1)}{2}(3n^2 + 4n + 1) + mn(n-1).$$

(ii)By the definition of Hyper − Wiener index

$$WW(K_m \odot C_n) = \frac{1}{2}\sum_{u,v \in V(G)}[d(u,v) + d(u,v)^2]$$

$$= \frac{1}{2}\left\{\left[\frac{m(m-1)}{2} + 2mn\right](1 + 1^2) + \left[mn(m-1) + \frac{mn(n-3)}{2}\right](2 + 2^2) + n^2\left[\frac{m(m-1)}{2}\right](3 + 3^2)\right\}$$

$$= \frac{1}{2}\left\{\left[\frac{m(m-1)}{2} + 2mn\right](1) + \left[mn(m-1) + \frac{mn(n-3)}{2}\right](6) + n^2\left[\frac{m(m-1)}{2}\right](12)\right\}$$

$$= \frac{1}{2}[(m(m-1) + 4mn) + 6mn(m-1) + 3mn(n-3) + 6n^2m(m-1)]$$

$$= \frac{m(m-1)}{2} + 2mn + 3mn(m-1) + \frac{3}{2}mn(n-3) + 8n^2m(m-1)$$

$$= \frac{m(m-1)}{2}(1 + 6n + 6n^2) + mn\left(2 + \frac{3}{2}(n-3)\right)$$

$$= \frac{m(m-1)}{2}(1 + 6n + 6n^2) + mn\left(\frac{3}{2}n + 2 - \frac{9}{2}\right)$$

$$WW(K_m \odot C_n) = \frac{m(m-1)}{2}(1 + 6n + 6n^2) + \frac{mn}{2}(3n-5).$$

Theorem 2.3: Let K_m *and* C_n be complete and cycle graphs with m ≥ 2 and n ≥ 3. Then

$$\text{(i)}H(K_m \odot C_n) = \frac{m(m-1)}{2}\left(\frac{n^2}{3} + n + 1\right) + mn\left(\frac{n+5}{4}\right)$$

$$\text{(ii)}RCW(K_m \odot C_n) = \frac{m(m-1)}{6}(n^2 + n + 1) + mn\left(\frac{3n-1}{12}\right)$$

Proof: Let K_m *and* C_n be complete and cycle graphs with m ≥ 2 and n ≥ 3. Then

(i) By the definition of Harary index

$$H(K_m \odot C_n) = \sum_{u,v \in V(G)}\frac{1}{d(u,v)}$$

$$= \left[\frac{m(m-1)}{2} + 2mn\right]\left(\frac{1}{1}\right) + \left[mn(m-1) + \frac{mn(n-3)}{2}\right]\left(\frac{1}{2}\right) + \left[\frac{n^2m(m-1)}{2}\right]\left(\frac{1}{3}\right)$$

$$= \frac{m(m-1)}{2} + 2mn + \frac{mn(m-1)}{2} + \frac{mn(n-3)}{4} + \frac{n^2m(m-1)}{6}$$

$$= \frac{m(m-1)}{2}\left(1 + n + \frac{n^2}{3}\right) + mn\left(2 + \frac{n-3}{4}\right)$$

$$H(K_m \odot C_n) = \frac{m(m-1)}{2}\left(\frac{n^2}{3} + n + 1\right) + mn\left(\frac{n+5}{4}\right).$$

(ii)By the definition of Reciprocal Complementary Wiener index

$$RCW(K_m \odot C_n) = \sum_{u,v \in V(G)} \frac{1}{d+1-d(u,v)}$$

$$= \left[\frac{m(m-1)}{2} + 2mn\right]\left(\frac{1}{3+1-1}\right) + \left[mn(m-1) + \frac{mn(n-3)}{2}\right]\left(\frac{1}{3+1-2}\right) + \left[\frac{n^2 m(m-1)}{2}\right]\left(\frac{1}{3+1-3}\right)$$

$$= \frac{m(m-1)}{6} + \frac{2mn}{3} + \frac{mn(m-1)}{2} + \frac{mn(n-3)}{4} + \frac{n^2 m(m-1)}{2}$$

$$= \frac{m(m-1)}{6}\left(\frac{1}{3} + n + n^2\right) + mn\left(\frac{2}{3} + \frac{n-3}{4}\right)$$

$$= \frac{m(m-1)}{6}(n^2 + n + 1) + mn\left(\frac{3n-9+8}{12}\right)$$

$$RCW(K_m \odot C_n) = \frac{m(m-1)}{6}(n^2 + n + 1) + mn\left(\frac{3n-1}{12}\right).$$

Theorem 2.4: Let K_m and C_n be complete and cycle graphs with m ≥ 2 and n ≥ 3. Then

$$\text{(i)}SCI(K_m \odot C_n) = \frac{m(m-1)}{2}\left(\frac{3n^2 + 8n + 6}{12}\right) + mn\left(\frac{n+3}{6}\right)$$

$$\text{(ii)}B(K_m \odot C_n) = \frac{m(m-1)}{2}(9n^2 + 8n + 1) + 2mn(n-2)$$

Proof: Let K_m and C_n be complete and cycle graphs with m ≥ 2 and n ≥ 3. Then

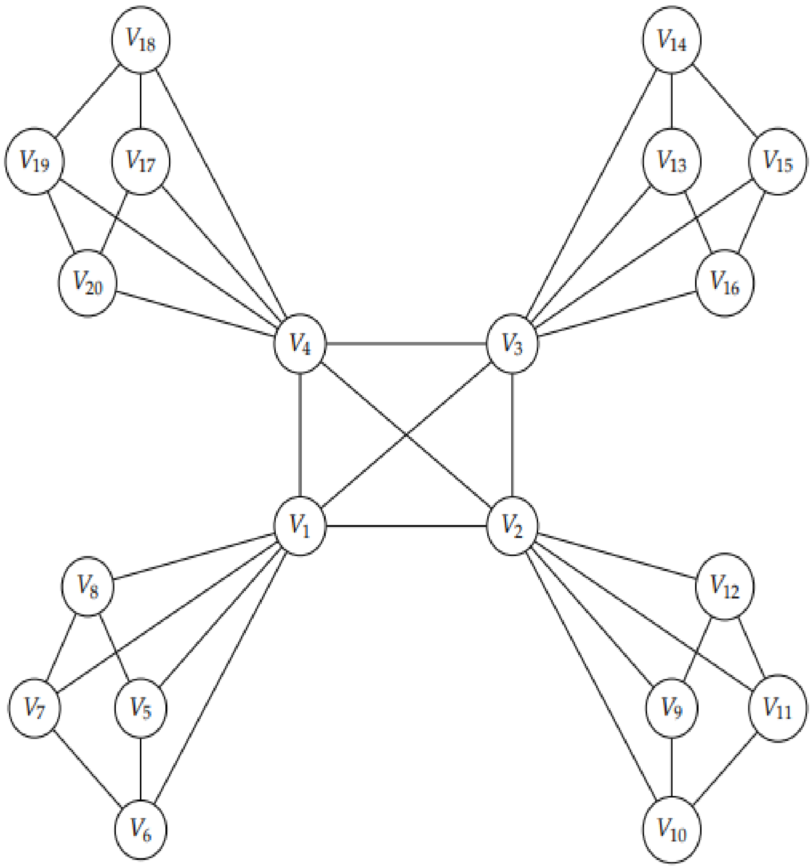

Figure 3: $K_4 \odot C_4$

(i)By the definition of Sum Connectivity index

$$SCI(K_m \odot C_n) = \sum_{u,v\in V(G)} \frac{1}{d(u,v)+1}$$

$$= \left[\frac{m(m-1)}{2} + 2mn\right]\left(\frac{1}{1+1}\right) + \left[\mathrm{mn}(\mathrm{m}-1) + \frac{\mathrm{mn}(\mathrm{n}-3)}{2}\right]\left(\frac{1}{2+1}\right) + \left[\frac{\mathrm{n}^2\mathrm{m}(\mathrm{m}-1)}{2}\right]\left(\frac{1}{3+1}\right)$$

$$= \frac{m(m-1)}{4} + mn + \frac{mn(m-1)}{3} + \frac{mn(n-3)}{6} + \frac{n^2m(m-1)}{8}$$

$$= \frac{m(m-1)}{2}\left(\frac{1}{2} + \frac{2n}{3} + \frac{n^2}{4}\right) + mn\left(1 + \frac{n-3}{6}\right)$$

$$SCI(K_m \odot C_n) = \frac{m(m-1)}{2}\left(\frac{3n^2+8n+6}{12}\right) + mn\left(\frac{n+3}{6}\right).$$

(**ii**)By the definition of Batschelet index

$$B(K_m \odot C_n) = \sum_{u,v\in V(G)} d(u,v)^2$$

$$= \left[\frac{m(m-1)}{2} + 2mn\right](1)^2 + \left[mn(m-1) + \frac{mn(n-3)}{2}\right](2)^2 + \left[\frac{n^2m(m-1)}{2}\right](3)^2$$

$$= \frac{m(m-1)}{2} + 2mn + 4mn(m-1) + 2mn(n-3) + \frac{9n^2m(m-1)}{2}$$

$$= \frac{m(m-1)}{2}(1 + 8n + 9n^2) + 2mn(1 + n - 3)$$

$$B(K_m \odot C_n) = \frac{m(m-1)}{2}(9n^2+8n+1) + 2mn(n-2).$$

Theorem 2.5: Let K_m and C_n be complete and cycle graphs with m ≥ 2 and n ≥ 3. Then

$$\text{(i)} \wedge (K_m \odot C_n) = \frac{m}{2}[3(n+1)(m(n+1)-1) - (m-1)(3n^2+4n+1) - 2n(n-1)]$$

$$\text{(ii)} R \wedge (K_m \odot C_n) = \frac{m(m-1)}{2}\left(\frac{4n+1}{2}\right) + mn\left(\frac{n-1}{2}\right)$$

Proof: Let K_m and C_n be complete and cycle graphs with m ≥ 2 and n ≥3. Then

(i)By the definition of Reverse Wiener index

$$\wedge (K_m \odot C_n) = \frac{n(n-1)d}{2} - W(G)$$

$$= \frac{m+mn((m+mn)-1)3}{2} - \frac{m(m-1)}{2}[3n^2+4n+1] - mn(n-1)$$

$$= \frac{3[m^2+m^2n-m+m^2n+m^2n^2-mn]}{2} - \frac{m(m-1)}{2}(3n^2+4n+1) - mn(n-1)$$

$$= \frac{3[m^2(1+2n+n^2)-m(1+n)]}{2} - \frac{m(m-1)}{2}(3n^2+4n+1) - mn(n-1)$$

$$= \frac{m}{2}\left[3\left(m(n^2+2n+1)-(n+1)\right) - (m-1)(3n^2+4n+1) - 2n(n-1)\right]$$

$$= \frac{m}{2}\left[3\left(m(n+1)^2-(n+1)\right) - (m-1)(3n^2+4n+1) - 2n(n-1)\right]$$

$$\wedge (K_m \odot C_n) = \frac{m}{2}[3(n+1)(m(n+1)-1) - (m-1)(3n^2+4n+1) - 2n(n-1)].$$

(ii)By the definition of Reciprocal Reverse Wiener index

$$R \wedge (K_m \odot C_n) = \sum_{u,v \in V(G)} \frac{1}{d-d(u,v)}$$

$$= \left[\frac{m(m-1)}{2} + 2mn\right]\left(\frac{1}{3-1}\right) + \left[mn(m-1) + \frac{mn(n-3)}{2}\right]\left(\frac{1}{3-2}\right) + \left[\frac{n^2m(m-1)}{2}\right]\left(\frac{1}{3-3}\right)$$

$$= \left[\frac{m(m-1)}{2} + 2mn\right]\left(\frac{1}{2}\right) + \left[mn(m-1) + \frac{mn(n-3)}{2}\right]\left(\frac{1}{1}\right)$$

$$= \frac{m(m-1)}{4} + mn + mn(m-1) + \frac{mn(n-3)}{2}$$

$$= \frac{m(m-1)}{2}\left(\frac{1}{2} + 2n\right) + mn\left(1 + \frac{n-3}{2}\right)$$

$$R \wedge (K_m \odot C_n) = \frac{m(m-1)}{2}\left(\frac{4n+1}{2}\right) + mn\left(\frac{n-1}{2}\right).$$

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