IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Side-Angle-Side (SAS) Congruence Theorem: A Fundamental Criterion for Triangle Congruence

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Abstract

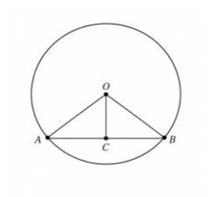
The Side-Angle-Side (SAS) Congruence Theorem is one of the most important principles in Euclidean geometry. It states that if two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, then the two triangles are congruent. This theorem provides a reliable method for establishing the equality of triangles and serves as a foundation for many geometric proofs and constructions. The present paper discusses the statement, significance, applications, and logical basis of the SAS Congruence Theorem in elementary geometry.

Introduction

Question:- The line drawn through the centre of a circle to bisect a chord is

Perpendicular to the chord.

First Solution:-



Given:- O is the centre of this circle.

C is middle point of AB chord. OC is joint.

We are to show that or prove that

$\angle OCA = \angle OCB = 90°$ or $OC \perp AB$.

Construction:- OA and OB are joined.

Proof:- With respect to

$\triangle AOC$ and $\triangle BOC$

$OA = OB$

[Radius of a circle]

$OC = OC$ [Common side]

and $AC = BC$ [C is middle point of AB]

$\therefore \triangle AOC \cong \triangle BOC$

$\therefore \angle OCA = \angle OCB$------------- (1)

But $\angle ACB = 180°$ [Straight angle]

$\Rightarrow \angle OCA + \angle OCB = 180°$

$\Rightarrow \angle OCA + \angle OCA = 180°$ ([From (1))

$\Rightarrow 2\angle OCA = 180°$

$\Rightarrow \angle OCA = 180°/2 = 90°$

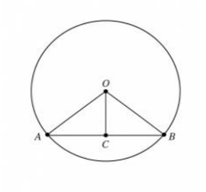
$\therefore \angle OCA = \angle OCB = 90°$

$\therefore OC \perp AB$.

Question:- The line drawn through the centre of a circle to bisect a chord is

Perpendicular to the chord.

Second Solution:-



Given: - O is the centre of this circle. C is middle point of AB chord.

We are to show that or prove that OC ⊥ AB.

Construction: - OA and OB are joined.

Proof: - In △AOB

OA = OB

[In a triangle two sides are equal then their opposite angles are equal]

∴ ∠OAB = ∠OBA

⇒ ∠OAC = ∠OBC ----------------- (1)

Now, with respect to

△AOC and △BOC

OA = OB

[Radius of a Circle]

∠OAC = ∠OBC [From (1)]

and AC = BC [Given]

∴ △AOC ≅ △BOC.

∴ ∠OCA = ∠OCB ---------- (1)

But ∠ACB = 180° [straight angle]

⇒ ∠OCA + ∠OCB = 180°

⇒ ∠OCA + ∠OCA = 180° [from (1)]

⇒ 2∠OCA = 180°

⇒ ∠OCA = 180°/2

⇒ ∠OCA = 90°

But ∠OCA = ∠OCB [from (1)]

Therefore, ∠OCA = ∠OCB = 90°

Now we can write that

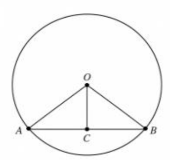
OC ⊥ AB

Proved.

Question:- The line drawn through the centre of a circle to bisect a chord is

Perpendicular to the chord.

Third solution



Given: - O is the centre of this circle. C is middle point of AB chord. We are to that or prove that OC ⊥ AB.

Construction:- OA and OB are joined.

Proof:-

In △AOB

OA = OB

[Radius of a circle]

∴ ∠OAB = ∠OBA

[A triangle whose two sides are equal then their opposite angles are equal]

⇒ ∠OAC = ∠OBC —— (1)

With respect to

△AOC and △BOC

OA = OB

[Radius of a circle]

∠OAC = ∠OBC [From (1)]

and OC = OC [common side]

∴ △AOC ≅ △BOC

∴ ∠OCA = ∠OCB —— (2)

But ∠ACB = 180° [Straight angle]

⇒ ∠OCA + ∠OCB = 180°

⇒ ∠OCA + ∠OCA = 180°

[From (2)]

⇒ 2∠OCA = 180°

⇒ ∠OCA = 180°/2

⇒ ∠OCA = 90°

But ∠OCA = ∠OCB [From (2)]

Therefore, ∠OCA = ∠OCB = 90°.

∴ Now, we can write that

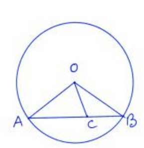
OC ⊥ AB.

Proved.

Now, I am going to another example – second and third solution are wrong their proof.

Question: O is the centre of a circle. C is a point on the AB chord, where AC > BC. Show that or prove that AC = BC.

Solution : -



Given: O is the centre of this circle. C is a point on the AB chord where AC > BC. We are to show that or prove AC = BC.

Construction: OA and OB are joined.

Proof: In △AOB

OA = OB

[Radius of a Circle]

∴ ∠OAB = ∠OBA

[In a triangle two sides are equal then their opposite angles are equal]

$\Rightarrow \angle OAC = \angle OBC$ ____ (1)

In $\triangle AOC$ and $\triangle BOC$

$OA = OB$

[Radius of a Circle]

$\angle OAC = \angle OBC$ [from (1)]

and $OC = OC$

[Common side]

$\therefore \triangle AOC \cong \triangle BOC$ [SAS]

$\therefore AC = BC$

Corrected Information:

N.B.: However, this is not possible. The first solution is completely incorrect. If the second and third solutions were correct, the conclusion would be valid. But upon careful examination, the last three solutions are also completely wrong. Therefore, all three solutions are incorrect and should be avoided by teachers and students.

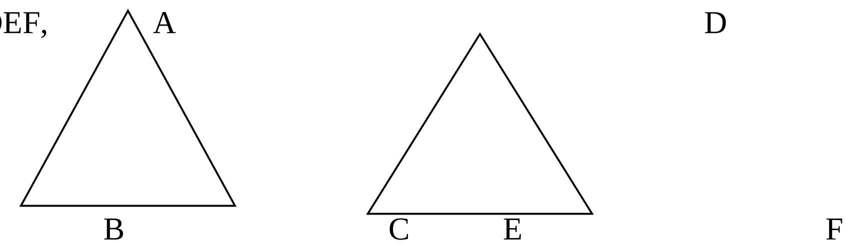
Congruence is a fundamental concept in geometry that describes figures having the same shape and size. Among the various criteria used to establish triangle congruence, the Side-Angle-Side (SAS) Congruence Theorem occupies a central position. The theorem enables mathematicians, teachers, and students to verify the congruence of triangles without measuring all sides and angles. It is widely applied in geometric proofs, engineering designs, architecture, and other fields requiring precise measurements. Understanding the SAS theorem helps learners develop logical reasoning and proof-writing skills.

Discussion

Statement of the SAS Congruence Theorem

If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, then the two triangles are congruent.

Symbolically, if in triangles ABC and DEF,



AB = DE,

AC = DF,

and ∠A = ∠D,

where the equal angle is included between the equal sides, then

△ABC ≅ △DEF.

Explanation

The theorem relies on the fact that two sides and the angle between them uniquely determine a triangle. Once these three elements are fixed, no other triangle with different dimensions can be formed. Therefore, the corresponding sides and angles of the two triangles must be equal, making the triangles congruent.

Importance of the Theorem

1. Provides a direct method for proving triangle congruence.

2. Forms the basis for many geometric constructions.

3. Supports the proof of other geometric theorems.

4. Develops logical and deductive reasoning skills.

5. Has practical applications in engineering, surveying, and architecture.

Applications

The SAS Congruence Theorem is frequently used in:

- Geometric proofs.
- Construction of figures.
- Measurement and surveying.
- Engineering design.
- Computer graphics and modeling.

Conclusion

The Side-Angle-Side (SAS) Congruence Theorem is a fundamental criterion for establishing the congruence of triangles. By requiring the equality of two corresponding sides and the included angle, the theorem guarantees that two triangles are identical in shape and size. Its simplicity, reliability, and wide applicability make it one of the most significant theorems in elementary geometry and mathematical reasoning.

References

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