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LEFT ANNIHILATOR AND RIGHT ANNIHILATOR OF BCK-ALGEBRA

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ABSTRACT

The study of BCK-algebra was initiated by Imai Iseki in 1966. Some related concepts have been applied to many branches of mathematics, such as Group theory, Functional analysis, Probability theory and Topology. The concept of left annihilator and right annihilator of BCK-algebra studied by eminent authors. Here we have studied left annihilator, Right annihilator Ideal & Prime ideal of BCK-algebra and epimorphism, monomorphism.

Keywords : *BCK-algebra, Left Annihilator, Right Annihilator Ideal and Prime ideal etc.*

INTRODUCTION :

Definition (1.1) : A system (X,*,0) of type (2, 0) consisting of a non empty set X, a binary operation * and a fixed element 0 is called a BCK-algebra if the following conditions are satisfied.

1. (BCK-1) $((x * y) * (x * z) * (z * y)) = 0$
2. (BCK-2) $(x * (x * y)) * y = 0$
3. (BCK-3) $x * x = 0$
4. (BCK-4) $0 * x = 0$
5. (BCK-5) $x * y = 0; y * x = 0$ imply $x = y$

For all $x, y, z \in X$.

An order relation $\leq$ in X is defined as $x \leq y$ if $x * y = 0$ ….

Definition (1.2) : A BCK-algebra X is said to be bounded if there exists an element $1 \in x$ such as $x \leq 1$ for all $x \in X$.

The element 1 is called unity of X.

Let (X, *, 0) be a BCK-algebra

Definition (1.3.) : For $x, y \in X$, we define

$x \wedge y = y * (y * x)$

Definition (1.4) : A BCK-algebra (X, *, 0) is said to be commutative if $x \wedge y = y \wedge x$ for all $x, y \in X$

Definition (1.5) : In a bounded BCK-algebra (X, *, 0) N_x and $x \vee y$ are defined as

$N_x = | * x$

$x \vee y = N (N_x \wedge Ny)$ for all $x, y \in X$

on the basis of theorem 6[Iseki and Tanaka, 1978] we note that

NOTE (1.1) : It is important to note that a bounded commutative BCK-algebra X is a distributive lattice with respect to relation $\wedge$ and $\vee$ more over, we have

$N_x \vee N_y = N (x \wedge y)$

$N_x \wedge N_y = N (x \vee y)$

Let X be a Bounded commutative BCK-algebra

Definition (1.6) : An element $x \in X$ is called zero divisor if there exists a non zero element $y \in X$ such that $x \wedge y = 0$

If X has no nontrivial zero divisors then X is said to be cancellative. In other words, X is cancellative iff $x \wedge y = 0$ implies $x = 0$ or $y = 0$ for all $x, y \in X$.

Definition (1.7) : A non empty set I of a BCK-algebra X is said to be

(a) an ideal if $0 \in I$ and $x, y * x \in I$ imply $y \in I$ for all $y \in X$.

(b) An ideal I in a commutative BCK-algebra X is a prime ideal if $x \wedge y \in I$ implies $x \in I$ or $y \in I$.

Proposition (1.1) : An ideal I of a commutative BCK-algebra X is prime if and only if for any ideals A and B of X, $A \cap B \subseteq I$ implies $A \subseteq I$ or $B \subseteq I$.

Definition (1.8) : For any non empty set A of a BCK-algebra X, the set of all $x \in X$, for which

$(\ldots((x * a_1) * a_2) * \ldots * a_n) = 0$ for some $a_1, a_2, \ldots a_n \in A$, is the minimal ideal containing A [Iseki and Tanaka, 1976 theorem 3]. This ideal is denoted by <A> and is called the induced ideal generated by A.

An ideal generated by $x \in X$ is denoted by <a>.

Definition (1.9) : A subset A of a BCK-algebra (X, *, 0) is called a sub algebra of X if $x, y \in A \Rightarrow x * y \in A$.

Iseki and Tanka [1978] have established some properties if this system as follows.

Theorem (1.1) : Let (X, *, 0) be a BCK- algebra then

(i) $x * y = 0 \Rightarrow (z * y) * (z * x) = 0$

(ii) $x * y = 0$ and $y * z = 0 \Rightarrow x * z = 0$

(iii) $(x * y) * z = (x * z) * y$

(iv) $\mu * (z * y) \leq \mu * ((x * y) * (x * z))$

(v) $((x * \mu) * y) * (z * \mu) \leq (x * z) * y))$

(vi) $x * y \leq z \Rightarrow (x * z) \leq y$

(vii) $(x * y) * x = 0$

(viii) $x * 0 = x$

(ix) $x \leq y \Rightarrow x * z \leq y * z$ and $z * y \leq z * x$

(x) $(x * y) * z \leq (x * z) * (y * z)$

For all $x, y, z, \mu \in X$

Some special classes of BCK-algebras have been also studied.

Definition (1.10) : A BCK-algebra $(X, *, 0)$ is said to be

(i) Implicative if $x * (y * x) = x$

(ii) Positive implicative if

$(x * z) * (y * z) = (x * y) * z$ for all $x, y, z, \in X$.

Iseki and Tanaka [1978] have established some useful results as follows.

Theorem (1.2) : A BCK-algebra X is positive implicative iff

$(x * y) = (x * y) * x$ for all $x, y \in X$

Theorem (1.3) : If $(X, *, 0)$ is a commutative BCK-algebra then the following equalities are equivalent.

(i) $x * (y * x) = x$

(ii) $(x * y) * y = x * y$

(iii) $(x * z) * (y * z) = (x * y) * z$

Theorem (1.4) : Every implicative BCK-algebra is commutative and positive implicative.

Definition (1.11) : Let $(X, *, 0)$ and $(Y, □, 0')$ be a BCK-algebras and $f : X \rightarrow Y$ be a mapping Then f is called a homomorphism if

$f(x * y) = f(x) □ f(y)$

for all $x, y \in X$

Thus we see that if f is a homomorphism and $x, y \in X$ then

$$\begin{aligned} f(x \wedge y) &= f(y * (y * x)) \\ &= f(y) □ (f(y) □ f(x)) \\ &= f(x) \wedge f(y) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1.A) \end{aligned}$$

Definition (1.12) : The mapping $f : X \rightarrow Y$ is called an epimorphism if it is a surjective homomorphism and monomorphism if it is an injective homorphism.

Note : (1.2) : (i) It is important to note that f(0) is the zero element of Y.

(ii) For any $x, y \in X$ with $x \leq y$ we have

$f(x) \leq f(y)$

So f is an isotone

Definition (1.13) : Let $f : X \to Y$ be a homomorphism then the kernel of *f*, denoted as ker(*f*), is defined as

$$Ker(f) = \{x \in X : f(x) = 0\}$$

Proposition (1.1) : The kernel of a homomorphism is an ideal.

Definition (1.14) : Let (X, *, 0) be a BCK-algebra and let $a \in X$. Then left annihilator and Right annihilator of a, denoted as L_a and R_a are defined as

$$L_a = \{ x \in X : x \wedge a = 0\}$$

$$R_a = \{ x \in X : a \wedge x = 0\}$$

The following properties of L_a and R_a are worth noting.

Proposition (1.2) : If (X, *, 0) is a BCK-algebra and $a \in X$, then

(i) $0 \in L_a \cap R_a$ for every $a \in X$

(ii) $a \notin L_a$, $a \notin R_a$

(iii) L_a is an ideal of X

(iv) $L_0 = X = R_0$

(v) $S \leq t \Rightarrow L_t \subseteq L_s$ for any $s, t \in X$

(vi) $x \in L_a$ (resp. $x \in R_a$) $\Rightarrow f(x) \in L_{f(a)}$ (resp. $f(x) \in R_{f(a)}$)

where f is a homomorphism

(This follows from the definitions and (1.A)

Definition (1.15) : Let (X, *, 0) be a BCK-algebra and $a \in X$. Then left co-set and Right co-set of a denoted as L c (a) and R c (a) are defined as

$$L\,c\,(a) = \{x : x \wedge a = a\}$$

$$R\,c\,(a) + \{x : a \wedge x = a\}$$

It is important to mention the following facts.

Proposition (1.4) : If (X, *. 0) is a BCK-algebra ad $a \in X$, then,

(i) $0 \notin L\,c\,(a)$, $0 \notin R\,c\,(a)$

(ii) $a \in L\,c\,(a) \cap R\,c\,(a)$

Let (X, *, 0) be a BCK-algebra and let $S \subseteq X$. The aS and S_a denoted the sets

$$aS = \{a * b : b \in S\}$$

$$S_a = \{b * a = b \in S\}$$

Now we have the following results

Proposition (1.5) : (i) $a[L\,c\,(a)] \subseteq La$ and

(ii) $aL_a \subseteq L\,c(a)$

Proof (i): Let $t \in a$ [L c (a)]. Then t = a * b for some $b \in$ L c (a)

Now $t \wedge a = (a * b) \wedge a$

$= a * (a * (a * b))$

$= a * a = 0 \Rightarrow t \in La$

So a [L c (a)] $\subseteq L_a$.

(ii) Again $t \in a La \Rightarrow t = a * s$ where $S \in L_a$

So $t \wedge a = (a * S) \wedge a$

$= a * (a * (a * S))$

$= a * 0 = a$

Which means that $t \in$ L c (a). so $aL_a \subseteq Lc(a)$.

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