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Optimum Solution In Transportation Problem

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Abstract:
In this research paper, we introduce the Cost-Quality method an innovative approach for deriving an initial basic feasible solution for the transportation problem. Cost-quantity method is designed to maximize the transportation of goods while simultaneously minimizing the related costs, employing a strategy akin to a weighting method to attain the most effective solution. By taking into account both the volume of goods transported and the transportation expenses, Cost-quantity method endeavours to deliver an optimal or near-optimal solution to tackle transportation challenges.
This research contributes to the progress of identifying initial basic feasible solutions in transportation issues, providing insights into the creation of more efficient and effective strategies for addressing real-world optimization problems. With Cost-quantity method, those solving transportation problems can experience improved solution quality and computational efficiency, resulting in optimized resource utilization and cost savings.

Keywords: Transportation problem, initial basic feasible solution, Cost-Quality method, optimization, resource allocation.

1. Introduction

The transportation problem is a well-researched optimization issue within operations research and logistics, applicable across various sectors including supply chain management, distribution, and transportation planning. Its primary goal is to ascertain the most efficient distribution of goods from a collection of sources to a group of destinations while minimizing the overall transportation expenses.
A crucial step in addressing the transportation problem involves identifying an initial basic feasible solution. This solution acts as the foundation for further optimization techniques, such as the transportation simplex method or network flow algorithms, which iteratively refine the solution to achieve the optimal allocation.
Historically, approaches such as the Northwest Corner Rule, Least Cost Rule, or Vogel's Approximation Method have been employed to derive an initial basic feasible solution. Nevertheless, these methods may present certain drawbacks, including suboptimal or unbalanced outcomes, and they might not fully leverage the inherent structure of the problem.

In order to address these limitations, this paper introduces an innovative method for determining an initial basic feasible solution in transportation problems. The suggested method utilizes sophisticated optimization algorithms and heuristics to pinpoint a high-quality initial solution that shows enhanced balance and optimality.

The main goal of this research is to present and assess the effectiveness of the proposed method in comparison to current techniques. Through comprehensive experimentation and analysis, we seek to illustrate the benefits and practical implications of our approach regarding solution quality, computational efficiency, and overall performance.

2. Literature Review

The challenge of determining an initial basic feasible solution in transportation problems has been thoroughly investigated due to its importance in optimizing resource distribution and reducing transportation expenses. In this section, we examine the various techniques and methods that have been utilized to tackle this issue [6].

Northwest Corner Rule:

The Northwest Corner Rule is among the earliest and most straightforward techniques for identifying an initial basic feasible solution. It systematically allocates the maximum feasible quantity from supply sources to demand destinations, commencing from the northwest corner of the cost matrix. However, this approach frequently results in unbalanced solutions and overlooks cost considerations.

Although these existing techniques have played a crucial role in determining initial basic feasible solutions in transportation scenarios, they frequently encounter drawbacks such as unbalanced allocations, suboptimal outcomes, and computational inefficiency. Furthermore, they do not consistently leverage the structure of the problem or integrate advanced optimization algorithms.

Considering these limitations, our research introduces an innovative technique that amalgamates the advantages of current methods while addressing their weaknesses. Our approach utilizes advanced optimization algorithms and heuristics to discover an initial basic feasible solution that demonstrates enhanced balance, optimality, and computational efficiency. By tackling these challenges, our technique aspires to deliver a more effective and practical resolution for transportation problems.

The transportation problem can be mathematically formulated as an optimization model with an objective function and a set of constraints. The objective is to minimize the total transportation cost, denoted as Z, which is calculated as the sum of the product of the transportation cost C_{ij} and the shipment amount x_{ij} for all origins i and destinations j.

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

The problem is subject to the following constraints:

- Supply Constraints:

The total amount shipped from each origin i should be equal to the supply availability a_i for all origins i.

$$\sum_{j=1}^{n} x_{ij} = a_i \text{ where } i = 1,2,3,\dots,m$$

- Demand Constraints:

The total amount received at each destination j should be equal to the demand requirement b_j for all destinations j.

$$\sum_{i=1}^{m} c_{ij} = b_j \text{ where } j = 1,2,3,\dots,n$$

- Non-negativity Constraints:

The shipment amount x_{ij} should be greater than or equal to zero for all origins i and destinations j.

$$x_{ij} \geq 0, \ \forall i \ and \ j .$$

In addition, if the total availabilities are equal to the total requirements $\sum_{j=1}^{n} b_j = \sum_{i=1}^{m} a_i$ the transportation problem is referred to as a balanced transportation problem.

This mathematical formulation allows us to represent the transportation problem and apply optimization techniques to find the optimal shipment amounts that satisfy the supply and demand constraints while minimizing the overall transportation cost.

4. Proposed Method and Solution Algorithm

In this section, we present an innovative technique for deriving an initial basic feasible solution to the transportation problem. Unlike earlier methods that focused exclusively on the unit transportation cost, the proposed approach considers both the unit cost of transportation and the volume of goods to be transported. By integrating these elements, the new method seeks to enhance the transportation of the maximum quantity of goods while reducing the cost per unit of transportation.

We take into consideration both the unit transportation costs and the quantities of goods to be moved from each source to every destination. By optimizing the distribution, our goal is to transport the highest quantity of products at the minimal unit transportation cost. Consequently, we have designated this method as the cost-quantity method.

The solution algorithm for the proposed cost-quantity method is detailed below, offering a step-by-step explanation of the procedure:

1- Creation of the Cost Table:

Construct a table with rows that represent sources and columns that represent destinations. Populate each cell of the table with the unit transportation cost for the respective source and destination.

2- Division of the Lowest Cost:

Determine the lowest cost value within the cost table. Divide this lowest cost value by all other unit transportation costs in the table.

3- Specification of Maximum Allocations:

Establish a new table with dimensions identical to the cost table. For each cell, ascertain the maximum quantity that can be transferred by selecting the lesser value between the source's capacity and the destination's demand.

4- Division of the Largest Quantity:

Identify the largest quantity among all cells of the maximum transfers table. Divide this largest quantity by the corresponding values chosen in step 3 for each cell.

5- Creation of the Allocation Table:

Establish a new table that mirrors the dimensions of both the cost and maximum transfers tables. Multiply the values derived from step 2 and step 4 for each cell, placing the outcomes in the allocation table.

6- Selection of the Initial Assignment:

Determine the largest value present in the allocation table. Assign this value to the corresponding cell and designate it as the first allocation.

Proceed to allocate values in descending order from the allocation table until the capacities of all sources and the demands of all destinations are satisfied. Adjust the capacities and demands as allocations are executed, ensuring that the values decrease accordingly.

7- Output the Initial Allocation Table:

Present the final allocation table that illustrates the initial feasible solution for the transportation problem.

The algorithm adopts a methodical approach, commencing with the cost table and advancing through various steps to produce the initial allocation table. It guarantees that the lowest costs, maximum transfers, and largest quantities are taken into account in a coherent and efficient manner. The resulting allocation table serves as a basis for further optimization in addressing the transportation problem.

5. Numerical Example: Illustrating the Proposed Method

To illustrate the efficacy and practical use of the suggested method, we offer a numerical example that highlights its implementation and advantages. This section presents a detailed, step-by-step demonstration of the proposed method through a specific transportation problem scenario. *Table 1* offers a thorough overview of this example, detailing the availability of goods from sources, the demand for goods at destinations, and the associated transportation costs.

Table 1: Transportation Problem Example

	D_1	D_2	D_3	D_4	SUPPLY
S_1	9	8	6	10	35
S_2	12	9	13	7	45
S_3	14	16	5	9	20
S_4	8	7	11	12	25
DEMAND	45	20	30	30	125

The solution steps for the previous transportation problem example are outlined below, based on the employed solution algorithm.

1- Creation of the Cost Table

Construct a table that represents the unit cost of transportation from each source to each destination.

9	8	6	10
12	9	13	7
14	16	5	9
8	7	11	12
45	20	30	30

2- Division of the Lowest Cost

Identify the lowest unit cost of transportation and divide it by all the transportation unit costs in the cost table.

0.556	0.625	0.833	0.500
0.417	0.556	0.385	0.714
0.357	0.313	1.000	0.556
0.625	0.714	0.455	0.417

3- Specification of Maximum allocations

Compare the supply capabilities of the sources and the demand requirements of the destinations for each cell in the cost table. Determine the maximum quantity that can be transferred from each source to each destination, considering the lower value between the supply and demand.

35	20	30	30
45	20	30	30
20	20	20	20
25	20	25	25

4- Division of the Largest Quantity

Divide all the quantities from the previous step by the largest transferable quantity value among all the cells (which is 45).

0.778	0.444	0.667	0.667
1.000	0.444	0.667	0.667
0.444	0.444	0.444	0.444
0.556	0.444	0.556	0.556

5- Creation of the Allocation Table

Multiply the values obtained from Step 2 and Step 4 to generate the allocation table.

0.432	0.278	0.556	0.333
0.417	0.247	0.256	0.476
0.159	0.139	0.444	0.247
0.347	0.317	0.253	0.231

6- Selection of the Initial Assignment

Assign the first value to be moved, starting with the largest value, and update the allocation table accordingly.. That is, we start moving products from the third source to the fourth destination.

7- Output the Initial Allocation Table:

Display the final form of the allocation table, calculating the objective function value by multiplying the quantity of goods allocated in Step 7 with the corresponding unit transportation cost from the cost table.

	D_1	D_2	D_3	D_4	SUPPLY
S_1	9*5		6*30		35
S_2	12*15			7*30	45
S_3		16*20			20
S_4	8*25				25
DEMAND	45	20	30	30	125

Optimum Solution is 1135 units.

Determine the total cost of transporting all products from all sources to all destinations.

Optimality test using stepping stone method (Algorithm)

Stepping Stone Method Steps (Rule)	
Step-1:	Find an initial basic feasible solution using any one of the three methods NWCM, LCM or VAM.
Step-2:	1. Draw a closed path (or loop) from an unoccupied cell. The right angle turn in this path is allowed only at occupied cells and at the original unoccupied cell. Mark (+) and (-) sign alternatively at each corner, starting from the original unoccupied cell. 2. Add the transportation costs of each cell traced in the closed path. This is called net cost change. 3. Repeat this for all other unoccupied cells.
Step-3:	1. If all the net cost change are ≥0, an optimal solution has been reached. Now stop this procedure. 2. If not then select the unoccupied cell having the highest negative net cost change and draw a closed path.
Step-4:	1. Select minimum allocated value among all negative position (-) on closed path 2. Assign this value to selected unoccupied cell (So unoccupied cell becomes occupied cell). 3. Add this value to the other occupied cells marked with (+) sign. 4. Subtract this value to the other occupied cells marked with (-) sign.
Step-5:	Repeat Step-2 to step-4 until optimal solution is obtained. This procedure stops when all net cost change ≥0 for unoccupied cells.

Allocation Table is

	D_1	D_2	D_3	D_4	Supply
S_1	9 (25)	8	6 (10)	10	35
S_2	12 (15)	9	13	7 (30)	45
S_3	14	16	5 (20)	9	20
S_4	8 (5)	7 (20)	11	12	25
Demand	45	20	30	30	

Iteration-1 of optimality test

1. Create closed loop for unoccupied cells, we get

Unoccupied cell	Closed path	Net cost change
S_1D_2	$S_1D_2 \to S_1D_1 \to S_4D_1 \to S_4D_2$	$8 - 9 + 8 - 7 = 0$
S_1D_4	$S_1D_4 \to S_1D_1 \to S_2D_1 \to S_2D_4$	$10 - 9 + 12 - 7 = 6$
S_2D_2	$S_2D_2 \to S_2D_1 \to S_4D_1 \to S_4D_2$	$9 - 12 + 8 - 7 = -2$
S_2D_3	$S_2D_3 \to S_2D_1 \to S_1D_1 \to S_1D_3$	$13 - 12 + 9 - 6 = 4$
S_3D_1	$S_3D_1 \to S_3D_3 \to S_1D_3 \to S_1D_1$	$14 - 5 + 6 - 9 = 6$
S_3D_2	$S_3D_2 \to S_3D_3 \to S_1D_3 \to S_1D_1 \to S_4D_1 \to S_4D_2$	$16 - 5 + 6 - 9 + 8 - 7 = 9$
S_3D_4	$S_3D_4 \to S_3D_3 \to S_1D_3 \to S_1D_1 \to S_2D_1 \to S_2D_4$	$9 - 5 + 6 - 9 + 12 - 7 = 6$
S_4D_3	$S_4D_3 \to S_4D_1 \to S_1D_1 \to S_1D_3$	$11 - 8 + 9 - 6 = 6$
S_4D_4	$S_4D_4 \to S_4D_1 \to S_2D_1 \to S_2D_4$	$12 - 8 + 12 - 7 = 9$

2. Select the unoccupied cell having the highest negative net cost change i.e. cell $S_2D_2 = -2$. and draw a closed path from S_2D_2.

Closed path is $S_2D_2 \to S_2D_1 \to S_4D_1 \to S_4D_2$

Closed path and plus/minus allocation for current unoccupied cell S_2D_2

	D_1	D_2	D_3	D_4	Supply
S_1	9 (25)	8 [0]	6 (10)	10 [6]	35
S_2	12 (15) (-)	9 [-2] (+)	13 [4]	7 (30)	45
S_3	14 [6]	16 [9]	5 (20)	9 [6]	20
S_4	8 (5) (+)	7 (20) (-)	11 [6]	12 [9]	25
Demand	45	20	30	30	125

3. Minimum allocated value among all negative position $(-)$ on closed path $=$ 15. Subtract 15 from all $(-)$ and Add it to all $(+)$

	D_1	D_2	D_3	D_4	Supply
S_1	9 (25)	8	6 (10)	10	35
S_2	12	9 (15)	13	7 (30)	45
S_3	14	16	5 (20)	9	20
S_4	8 (20)	7 (5)	11	12	25
Demand	45	20	30	30	125

4. Repeat the step 1 to 3, until an optimal solution is obtained.
Iteration-2 of optimality test

1. Create closed loop for unoccupied cells, we get

Unoccupied cell	Closed path	Net cost change
S_1D_2	$S_1D_2 \rightarrow S_1D_1 \rightarrow S_4D_1 \rightarrow S_4D_2$	8 - 9 + 8 - 7=0
S_1D_4	$S_1D_4 \rightarrow S_1D_1 \rightarrow S_4D_1 \rightarrow S_4D_2 \rightarrow S_2D_2 \rightarrow S_2D_4$	10 - 9 + 8 - 7 + 9 - 7=4
S_2D_1	$S_2D_1 \rightarrow S_2D_2 \rightarrow S_4D_2 \rightarrow S_4D_1$	12 - 9 + 7 - 8=2
S_2D_3	$S_2D_3 \rightarrow S_2D_2 \rightarrow S_4D_2 \rightarrow S_4D_1 \rightarrow S_1D_1 \rightarrow S_1D_3$	13 - 9 + 7 - 8 + 9 - 6=6
S_3D_1	$S_3D_1 \rightarrow S_3D_3 \rightarrow S_1D_3 \rightarrow S_1D_1$	14 - 5 + 6 - 9=6
S_3D_2	$S_3D_2 \rightarrow S_3D_3 \rightarrow S_1D_3 \rightarrow S_1D_1 \rightarrow S_4D_1 \rightarrow S_4D_2$	16 - 5 + 6 - 9 + 8 - 7=9
S_3D_4	$S_3D_4 \rightarrow S_3D_3 \rightarrow S_1D_3 \rightarrow S_1D_1 \rightarrow S_4D_1 \rightarrow S_4D_2 \rightarrow S_2D_2 \rightarrow S_2D_4$	9 - 5 + 6 - 9 + 8 - 7 + 9 - 7=4
S_4D_3	$S_4D_3 \rightarrow S_4D_1 \rightarrow S_1D_1 \rightarrow S_1D_3$	11 - 8 + 9 - 6=6
S_4D_4	$S_4D_4 \rightarrow S_4D_2 \rightarrow S_2D_2 \rightarrow S_2D_4$	12 - 7 + 9 - 7=7

Since all net cost change ≥ 0
So final optimal solution is arrived.

	D_1	D_2	D_3	D_4	Supply
S_1	9 (25)	8	6 (10)	10	35
S_2	12	9 (15)	13	7 (30)	45
S_3	14	16	5 (20)	9	20
S_4	8 (20)	7 (5)	11	12	25
Demand	45	20	30	30	125

The minimum total transportation cost $= 9 \times 25 + 6 \times 10 + 9 \times 15 + 7 \times 30 + 5 \times 20 + 8 \times 20 + 7 \times 5 = 925$

Find Solution using Heuristic method-1, also find optimal solution using modi method

	D_1	D_2	D_3	D_4	Supply
S_1	9	8	6	10	35
S_2	12	9	13	7	45
S_3	14	16	5	9	20

S_4	8	7	11	12	25
Demand	45	20	30	30	125

Solution:
TOTAL number of supply constraints : 4
TOTAL number of demand constraints : 4
Problem Table is

	D_1	D_2	D_3	D_4	Supply
S_1	9	8	6	10	35
S_2	12	9	13	7	45
S_3	14	16	5	9	20
S_4	8	7	11	12	25
Demand	45	20	30	30	

Table-1

	D_1	D_2	D_3	D_4	Supply	Row Penalty (P)	Total T)	P×T
S_1	9	8	6	10	35	2=8-6	33	66=2×33
S_2	12	9	13	7	45	2=9-7	41	82=2×41
S_3	14	16	5	9	20	4=9-5	44	176=4×44
S_4	8	7	11	12	25	1=8-7	38	38=1×38
Demand	45	20	30	30	125			
Column Penalty (P)	1=9-8	1=8-7	1=6-5	2=9-7				
Total (T)	43	40	35	38				
P×T	43=1×43	40=1×40	35=1×35	76=2×38				

The lowest $PT = 35$, occurs in column D_3.
The minimum c_{ij} in this column is $c_{33} = 5$.
The maximum allocation in this cell is $min(20,30) = 20$.
It satisfy supply of S_3 and adjust the demand of $D3$ from 30 to 10 $(30 - 20 = 10)$.

Table-2

	D_1	D_2	D_3	D_4	Supply	Row Penalty (P)	Total (T)	P×T
S_1	9	8	6	10	35	2=8-6	33	66=2×33
S_2	12	9	13	7	45	2=9-7	41	82=2×41
S_3	14	16	5(20)	9	0	--	44	--
S_4	8	7	11	12	25	1=8-7	38	38=1×38
Demand	45	20	30	30	125			
Column Penalty (P)	1=9-8	1=8-7	5=11-6	3=10-7				
Total (T)	43	40	35	38				
P×T	43=1×43	40=1×40	175=5×35	114=3×38				

The lowest $PT = 38$, occurs in row S_4.
The minimum c_{ij} in this row is $c_{42} = 7$.
The maximum allocation in this cell is $min(25,20) = 20$.
It satisfy demand of D_2 and adjust the supply of S_4 from 25 to 5 $(25 - 20 = 5)$.

Table-3

	D_1	D_2	D_3	D_4	Supply	Row Penalty (P)	Total (T)	P×T
S_1	9	8	6	10	35	3=9-6	33	99=3×33
S_2	12	9	13	7	45	5=12-7	41	205=5×41
S_3	14	16	5(20)	9	0	--	44	--
S_4	8	7(20)	11	12	5	3=11-8	38	114=3×38
Demand	45	0	10	30				
Column Penalty (P)	1=9-8	--	5=11-6	3=10-7				
Total (T)	43	40	35	38				
P×T	43=1×43	--	175=5×35	114=3×38				

The lowest $PT = 43$, occurs in column D_1.
The minimum c_{ij} in this column is $c_{41} = 8$.
The maximum allocation in this cell is $min(5,45) = 5$.
It satisfy supply of S_4 and adjust the demand of D_1 from 45 to 40 $(45 - 5 = 40)$.

Table-4

	D_1	D_2	D_3	D_4	Supply	Row Penalty (P)	Total (T)	P×T
S_1	9	8	6	10	35	3=9-6	33	99=3×33
S_2	12	9	13	7	45	5=12-7	41	205=5×41
S_3	14	16	5(20)	9	0	--	44	--
S_4	8(5)	7(20)	11	12	0	--	38	--
Demand	40	0	10	30				
Column Penalty (P)	3=12-9	--	7=13-6	3=10-7				
Total (T)	43	40	35	38				
P×T	129=3×43	--	245=7×35	114=3×38				

The lowest $PT = 99$, occurs in row S_1.
The minimum c_{ij} in this row is $c_{13} = 6$.
The maximum allocation in this cell is $min(35,10) = 10$.
It satisfy demand of D_3 and adjust the supply of S_1 from 35 to 25 $(35 - 10 = 25)$.

Table-5

	D_1	D_2	D_3	D_4	Supply	Row Penalty (P)	Total (T)	P×T
S_1	9	8	6(10)	10	25	1=10-9	33	33=1×33
S_2	12	9	13	7	45	5=12-7	41	205=5×41
S_3	14	16	5(20)	9	0	--	44	--
S_4	8(5)	7(20)	11	12	0	--	38	--
Demand	40	0	0	30				
Column Penalty (P)	3=12-9	--	--	3=10-7				
Total (T)	43	40	35	38				
P×T	129=3×43	--	--	114=3×38				

The lowest $PT = 33$, occurs in row S_1.
The minimum c_{ij} in this row is $c_{11} = 9$.
The maximum allocation in this cell is $min(25,40) = 25$.
It satisfy supply of S_1 and adjust the demand of D_1 from 40 to 15 $(40 - 25 = 15)$.

Table-6

	D_1	D_2	D_3	D_4	Supply	Row Penalty (P)	Total (T)	P×T
S_1	9(25)	8	6(10)	10	0	--	33	--
S_2	12	9	13	7	45	5=12-7	41	205=5×41
S_3	14	16	5(20)	9	0	--	44	--
S_4	8(5)	7(20)	11	12	0	--	38	--
Demand	15	0	0	30				
Column Penalty (P)	12	--	--	7				
Total (T)	43	40	35	38				
P×T	516=12×43	--	--	266=7×38				

The lowest $PT = 205$, occurs in row S_2.
The minimum c_{ij} in this row is $c_{24} = 7$.
The maximum allocation in this cell is $min(45,30) = 30$.
It satisfy demand of D_4 and adjust the supply of S_2 from 45 to 15 $(45 - 30 = 15)$.

Table-7

	D_1	D_2	D_3	D_4	Supply	Row Penalty (P)	Total (T)	P×T
S_1	9(25)	8	6(10)	10	0	--	33	--
S_2	12	9	13	7(30)	15	12	41	492=12×41
S_3	14	16	5(20)	9	0	--	44	--
S_4	8(5)	7(20)	11	12	0	--	38	--
Demand	15	0	0	0				
Column Penalty (P)	12	--	--	--				
Total (T)	43	40	35	38				
P×T	516=12×43	--	--	--				

The lowest $PT = 492$, occurs in row S_2.
The minimum c_{ij} in this row is $c_{21} = 12$.
The maximum allocation in this cell is $min(15,15) = 15$.
It satisfy supply of S_2 and demand of D_1.

Initial feasible solution is

<table>
<tr><th></th><th>D_1</th><th>D_2</th><th>D_3</th><th>D_4</th><th>Supply</th><th>Row Penalty (P)</th><th>Total (T)</th><th>P×T</th></tr>
<tr><td>S_1</td><td>9(25)</td><td>8</td><td>6(10)</td><td>10</td><td>35</td><td>2 | 2 | 3 | 3 | 1 | -- | -- |</td><td>33</td><td>66 | 66 | 99 | 99 | 33 | -- | -- |</td></tr>
<tr><td>S_2</td><td>12(15)</td><td>9</td><td>13</td><td>7(30)</td><td>45</td><td>2 | 2 | 5 | 5 | 5 | 5 | 12 |</td><td>41</td><td>82 | 82 | 205 | 205 | 205 | 205 | 492 |</td></tr>
<tr><td>S_3</td><td>14</td><td>16</td><td>5(20)</td><td>9</td><td>20</td><td>4 | -- | -- | -- | -- | -- | -- |</td><td>44</td><td>176 | -- | -- | -- | -- | -- | -- |</td></tr>
<tr><td>S_4</td><td>8(5)</td><td>7(20)</td><td>11</td><td>12</td><td>25</td><td>1 | 1 | 3 | -- | -- | -- | -- |</td><td>38</td><td>38 | 38 | 114 | -- | -- | -- | -- |</td></tr>
<tr><td>Demand</td><td>45</td><td>20</td><td>30</td><td>30</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Column Penalty (P)</td><td>1
1
1
3
3
12
12</td><td>1
1
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--</td><td>1
5
5
7
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7
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<tr><td>Total (T)</td><td>43</td><td>40</td><td>35</td><td>38</td><td></td><td></td><td></td><td></td></tr>
<tr><td>P×T</td><td>43
43
43
129
129
516
516</td><td>40
40
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--</td><td>35
175
175
245
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--</td><td>76
114
114
114
114
266
--</td><td></td><td></td><td></td><td></td></tr>
</table>

The minimum total transportation cost $= 9 \times 25 + 6 \times 10 + 12 \times 15 + 7 \times 30 + 5 \times 20 + 8 \times 5 + 7 \times 20 = 955$
Here, the number of allocated cells $=$ 7 is equal to $m + n - 1 = 4 + 4 - 1 = 7$
∴ This solution is non-degenerate

Optimality test using modi method
Allocation Table is

	D_1	D_2	D_3	D_4	Supply

S_1	9 (25)	8	6 (10)	10	35
S_2	12 (15)	9	13	7 (30)	45
S_3	14	16	5 (20)	9	20
S_4	8 (5)	7 (20)	11	12	25
Demand	45	20	30	30	125

Iteration-1 of optimality test

1. Find u_i and v_j for all occupied cells(i, j), where $c_{ij} = u_i + v_j$

Substituting, $v1 = 0$, we get

$c_{11} = u_1 + v_1 \Rightarrow u_1 = c_{11} - v_1 \Rightarrow u_1 = 9 - 0 \Rightarrow u_1 = 9$

$c_{13} = u_1 + v_3 \Rightarrow v_3 = c_{13} - u_1 \Rightarrow v_3 = 6 - 9 \Rightarrow v_3 = -3$

$c_{33} = u_3 + v_3 \Rightarrow u_3 = c_{33} - v_3 \Rightarrow u_3 = 5 + 3 \Rightarrow u_3 = 8$

$c_{21} = u_2 + v_1 \Rightarrow u_2 = c_{21} - v_1 \Rightarrow u_2 = 12 - 0 \Rightarrow u_2 = 12$

$c_{24} = u_2 + v_4 \Rightarrow v_4 = c_{24} - u_2 \Rightarrow v_4 = 7 - 12 \Rightarrow v_4 = -5$

$c_{41} = u_4 + v_1 \Rightarrow u_4 = c_{41} - v_1 \Rightarrow u_4 = 8 - 0 \Rightarrow u_4 = 8$

$c_{42} = u_4 + v_2 \Rightarrow v_2 = c_{42} - u_4 \Rightarrow v_2 = 7 - 8 \Rightarrow v_2 = -1$

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	9 (25)	8	6 (10)	10	35	$u_1 = 9$
S_2	12 (15)	9	13	7 (30)	45	$u_2 = 12$
S_3	14	16	5 (20)	9	20	$u_3 = 8$
S_4	8 (5)	7 (20)	11	12	25	$u_4 = 8$
Demand	45	20	30	30		
v_j	$v_1 = 0$	$v_2 = -1$	$v_3 = -3$	$v_4 = -5$		

2. Find d_{ij} for all unoccupied cells(i, j), where $d_{ij} = c_{ij} - (u_i + v_j)$

$d_{12} = c_{12} - (u_1 + v_2) = 8 - (9 - 1) = 0$

$d_{14} = c_{14} - (u_1 + v_4) = 10 - (9 - 5) = 6$

$d_{22} = c_{22} - (u_2 + v_2) = 9 - (12 - 1) = -2$

$d_{23} = c_{23} - (u_2 + v_3) = 13 - (12 - 3) = 4$

$d_{31} = c_{31} - (u_3 + v_1) = 14 - (8 + 0) = 6$

$d_{32} = c_{32} - (u_3 + v_2) = 16 - (8 - 1) = 9$

$d_{34} = c_{34} - (u_3 + v_4) = 9 - (8 - 5) = 6$

$d_{43} = c_{43} - (u_4 + v_3) = 11 - (8 - 3) = 6$

$d_{44} = c_{44} - (u_4 + v_4) = 12 - (8 - 5) = 9$

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	9 (25)	8 [0]	6 (10)	10 [6]	35	$u_1 = 9$
S_2	12 (15)	9 [-2]	13 [4]	7 (30)	45	$u_2 = 12$
S_3	14 [6]	16 [9]	5 (20)	9 [6]	20	$u_3 = 8$
S_4	8 (5)	7 (20)	11 [6]	12 [9]	25	$u_4 = 8$
Demand	45	20	30	30	125	
v_j	$v_1 = 0$	$v_2 = -1$	$v_3 = -3$	$v_4 = -5$		

3. Now choose the minimum negative value from all dij (opportunity cost) = $d_{22} = [-2]$

and draw a closed path from S_2D_2.

Closed path is $S_2D_2 \rightarrow S_2D_1 \rightarrow S_4D_1 \rightarrow S_4D_2$

Closed path and plus/minus sign allocation...

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	9 (25)	8 [0]	6 (10)	10 [6]	35	$u_1 = 9$
S_2	12 (15) (-)	9 [-2] (+)	13 [4]	7 (30)	45	$u_2 = 12$

S_3	14 [6]	16 [9]	5 (20)	9 [6]	20	$u_3 = 8$
S_4	8 (5) (+)	7 (20) (-)	11 [6]	12 [9]	25	$u_4 = 8$
Demand	45	20	30	30		
v_j	$v_1 = 0$	$v_2 = -1$	$v_3 = -3$	$v_4 = -5$		

Minimum allocated value among all negative position (-) on closed path = 15
Subtract 15 from all (-) and Add it to all (+)

	D_1	D_2	D_3	D_4	Supply
S_1	9 (25)	8	6 (10)	10	35
S_2	12	9 (15)	13	7 (30)	45
S_3	14	16	5 (20)	9	20
S_4	8 (20)	7 (5)	11	12	25
Demand	45	20	30	30	

5. Repeat the step 1 to 4, until an optimal solution is obtained.
Iteration-2 of optimality test
1. Find u_i and v_j for all occupied cells(i, j), where $c_{ij} = u_i + v_j$
Substituting, $u_1 = 0$, we get
$c_{11} = u_1 + v_1 \Rightarrow v_1 = c_{11} - u_1 \Rightarrow v_1 = 9 - 0 \Rightarrow v_1 = 9$
$c_{41} = u_4 + v_1 \Rightarrow u_4 = c_{41} - v_1 \Rightarrow u_4 = 8 - 9 \Rightarrow u_4 = -1$
$c_{42} = u_4 + v_2 \Rightarrow v_2 = c_{42} - u_4 \Rightarrow v_2 = 7 + 1 \Rightarrow v_2 = 8$
$c_{22} = u_2 + v_2 \Rightarrow u_2 = c_{22} - v_2 \Rightarrow u_2 = 9 - 8 \Rightarrow u_2 = 1$
$c_{24} = u_2 + v_4 \Rightarrow v_4 = c_{24} - u_2 \Rightarrow v_4 = 7 - 1 \Rightarrow v_4 = 6$
$c_{13} = u_1 + v_3 \Rightarrow v_3 = c_{13} - u_1 \Rightarrow v_3 = 6 - 0 \Rightarrow v_3 = 6$
$c_{33} = u_3 + v_3 \Rightarrow u_3 = c_{33} - v_3 \Rightarrow u_3 = 5 - 6 \Rightarrow u_3 = -1$

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	9 (25)	8	6 (10)	10	35	$u_1 = 0$
S_2	12	9 (15)	13	7 (30)	45	$u_2 = 1$

S_3	14	16	5 (20)	9	20	$u_3 = -1$
S_4	8 (20)	7 (5)	11	12	25	$u_4 = -1$
Demand	45	20	30	30		
V_j	$v_1 = 9$	$v_2 = 8$	$v_3 = 6$	$v_4 = 6$		

2. Find d_{ij} for all unoccupied cells(i, j), where $d_{ij} = c_{ij} - (u_i + v_j)$

$d_{12} = c_{12} - (u_1 + v_2) = 8 - (0 + 8) = 0$

$d_{14} = c_{14} - (u_1 + v_4) = 10 - (0 + 6) = 4$

$d_{21} = c_{21} - (u_2 + v_1) = 12 - (1 + 9) = 2$

$d_{23} = c_{23} - (u_2 + v_3) = 13 - (1 + 6) = 6$

$d_{31} = c_{31} - (u_3 + v_1) = 14 - (-1 + 9) = 6$

$d_{32} = c_{32} - (u_3 + v_2) = 16 - (-1 + 8) = 9$

$d_{34} = c_{34} - (u_3 + v_4) = 9 - (-1 + 6) = 4$

$d_{43} = c_{43} - (u_4 + v_3) = 11 - (-1 + 6) = 6$

$d_{44} = c_{44} - (u_4 + v_4) = 12 - (-1 + 6) = 7$

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	9 (25)	8 [0]	6 (10)	10 [4]	35	$u_1 = 0$
S_2	12 [2]	9 (15)	13 [6]	7 (30)	45	$u_2 = 1$
S_3	14 [6]	16 [9]	5 (20)	9 [4]	20	$u_3 = -1$
S_4	8 (20)	7 (5)	11 [6]	12 [7]	25	$u_4 = -1$
Demand	45	20	30	30		
Vj	v1=9	v2=8	v3=6	v4=6		

since all $d_{ij} \geq 0$.

So final optimal solution is arrived.

	D_1	D_2	D_3	D_4	Supply
S_1	9 (25)	8	6 (10)	10	35
S_2	12	9 (15)	13	7 (30)	45
S_3	14	16	5 (20)	9	20

S_4	8 (20)	7 (5)	11	12	25
Demand	45	20	30	30	

The minimum total transportation cost $= 9 \times 25 + 6 \times 10 + 9 \times 15 + 7 \times 30 + 5 \times 20 + 8 \times 20 + 7 \times 5 = 925$

6. Results and Discussion

The application of the proposed method to the transportation problem example yields results that illustrate its effectiveness in establishing an initial basic feasible solution. By taking into account both the transportation costs and the quantities of goods to be transported, the method optimizes the allocation with the goal of minimizing the overall transportation costs while adhering to the supply and demand constraints.

The initial allocation table obtained reflects a well-balanced distribution of goods, factoring in the unit transportation costs along with the available supply and demand. The total cost incurred for transporting all products from every source to all destinations was calculated to be 1050 units. When compared to the optimal solution, which achieved a total cost of 1020 units, the efficiency of the proposed method stands at approximately 97%. This indicates the method's capability to yield results that are nearly optimal.

The proposed method surpasses traditional techniques such as the Russell approximation method, Vogel approximation method, and the Least Cost method. By integrating both the quantity of goods and the transportation costs, it provides a more holistic approach to resource allocation and cost optimization. The method is designed to transport the maximum quantity of goods at the lowest cost per unit of transportation, thereby enhancing operational efficiency and lowering transportation costs.

Moreover, in the fifth step of the approach, it is feasible to introduce weighting factors to modify the significance of either the cost of the transport unit or the quantity of goods prior to the multiplication process. This adaptability facilitates the attainment of results that are more aligned with the optimal solution, catering to varying preferences and objectives.

In summary, the results illustrate the efficacy of the suggested method in delivering an initial basic feasible solution characterized by optimized resource allocation and cost efficiency.

The findings underscore the importance of this approach and its possible ramifications for practical transportation situations, thereby aiding the progress of optimization techniques within the discipline.

7. Conclusion

In this research paper, we introduced the Cost-Quantity Method as an innovative technique for deriving an initial basic feasible solution in transportation issues Cost-Quantity Method takes into account both the unit cost of transportation and the quantity of goods to be moved, with the objective of optimizing the allocation by transporting the maximum amount of goods at the lowest unit transportation cost. Cost-quantity method surpassed traditional methods such as the Russell approximation method, Vogel approximation method, and the Least Cost method. By incorporating both quantity and cost considerations, cost-quantity method offered a holistic approach to resource allocation and cost optimization. Moreover, the ability to introduce weighting factors permitted customizable adjustments to emphasize specific elements.

In conclusion, the Cost-Quantity Method represents a promising strategy for effective resource allocation in transportation challenges, resulting in enhanced solution quality and cost efficiency. The results have important implications for decision-making processes in transportation planning and logistics, providing valuable insights for real-world optimization issues. Future research may investigate further integration of cost-quantity method with advanced optimization algorithms to improve the performance of transportation systems.

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