IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Tetra Neutrosophic Set

[1]R Radha, [2]R Princy, [3]Arokia Pratheesha S V

[1]Professor , Department of Science and Humanities, RVS Technical Campus - Coimbatore ,Coimbatore(TN), India.

[2]Assistant Professor, Department of Science and Humanities, Rathinam Technical Campus, Coimbatore.

[3]Assistant Professor, Department of Mathematics, Dr N.G.P Arts and Science College,Coimbatore.

Abstract: In this paper, we have introduced new form of sets called tetra neutrosophic set and its properties are studied.

I. Introduction

Fuzzy sets were introduced by Zadeh [19] in 1965 that permits the membership perform valued within the interval [0,1] and set theory it's an extension of classical pure mathematics. Fuzzy set helps to deal the thought of uncertainty, unclearness and impreciseness that isn't attainable within the cantorian set. As Associate in Nursing extension of Zadeh's fuzzy set theory intuitionistic fuzzy set(IFS) was introduced by Atanassov [1] in 1986, that consists of degree of membership and degree of non membership and lies within the interval of [0,1]. IFS theory wide utilized in the areas of logic programming, decision making issues, medical diagnosis etc.

Florentine Smarandache [16] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally. Neutrosophic sets deals with non normal interval of]−0 1+[. Since neutrosophic set deals the indeterminateness effectively it plays an very important role in several applications areas embrace info technology, decision web, electronic database systems, diagnosis, multicriteria higher cognitive process issues etc.,

To method the unfinished data or imperfect data to unclearness a brand new mathematical approach i.e., To deal the important world issues, Wang [17](2010) introduced the idea of single valued neutrosophic sets(SVNS) that is additionally referred to as an extension of intuitionistic fuzzy sets and it became a really new hot analysis topic currently. The concept of neutrosophic pythagorean sets with dependent neutrosophic components was introduced by R. Radha[6]. Further, R. Radha and A. Stanis Arul Mary[outlined a brand new hybrid model of Pentapartitioned Neutrosophic Pythagorean sets (PNPS) and Quadripartioned neutrosophic pythagorean sets in 2021.

In this paper , we develop tetra neutrosophic sets and study some of its properties.

II Preliminaries

2.1 Definition [12]

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$

Where T_A , $I_A, F_A: U \rightarrow [0,1]$ *and* $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

2.2 Definition

Let R be a universe. A Neutrosophic pythagorean set A with T and F as dependent Neutrosophic Pythagorean components and U as independent component for A on R is an object of the form

$A = \{< x, T_A, U_A, F_A >: r \in R\}$

Where $T_A + F_A \leq 1$, $(T_A)^2 + (F_A)^2 \leq 1$ and

$(T_A)^2 + (U_A)^2 + (F_A)^2 \leq 2$

Here, $T_A(x)$ is the truth membership, $U_A(x)$ is indeterminancy membership and $F_A(x)$ is the false membership .

2.3 Definition

The complement of a Neutrosophic Pythagorean set $A = \{< x, T_A, U_A, F_A >: r \in R\}$ with dependent Neutrosophic Pythagorean components is

$A^C = \{< x, F_A, 1 - U_A, T_A >: r \in R\}$.

2.4 Definition

Let $A = \{< x, T_A, U_A, F_A >: r \in R\}$ *and* $B = \{< x, T_B, U_B, F_B >: r \in R\}$ are two Neutrosophic Pythagorean sets with dependent Neutrosophic Pythagorean components on the universe R. Then the union and intersection of two sets can be defined by

$A \cup B = \{\max(T_A, T_B), \min(U_A, U_B), \min(F_A, F_B)\}$,

$A \cap B = \{\min(T_A, T_B), \max(U_A, U_B), \max(F_A, F_B)\}$.

2.5 Example

Let R = {a, b} and A ={ (a, 0.4,0.5,0.3),(b,0.5,0.2,0.2)}.

Then τ = {0,1, A} is a topology on R. Then A is a Neutrosophic Pythagorean set.

2.6 Example

Let R = {a, b} and A ={ (a, 0.5,0.6,0.6),(b,0.7,0.6,0.7)}.

Then τ = {0,1, A} is a topology on R. Since $T_A + F_A > 1$,then A is not a Neutrosophic Pythagorean set.

III. Tetra Neutrosophic Set

3.1 Definition

Let X be a universe. A tetra neutrosophic set (TNS) A on X is an object of the form

$A = \{< x, T_A, I_A, F_A >: x \in X\}$

$(T_A)^4 + (I_A)^4 + (F_A)^4 \leq 1$

Here, $T_A(x)$ is the truth membership,

$I_A(x)$ is an indeterminancy membership and

$F_A(x)$ is the false membership

3.2Example

Let R= {a,b,c},and A= {(a,0.3,0.6,0.7),(b,0.5,0.2,0.5),(c,0.2,0.4,0.4)}. Then A is a tetra neutrosophic set on R.

3.3 Example

Let R = {a, b} and A ={ (a, 0.4,0.5,0.3),(b,0.5,0.6,0.2)}.

Then τ = {0,1, A} is a topology on R. Then A is a Tetra Neutrosophic set.

3.4 Example

Let R = {a, b} and A ={ (a, 0.7,0.9,0.7),(b,0.8,0.9,0.9)}.

Then τ = {0,1, A} is a topology on R. Since the sum is greater than one, $(T_A)^4 + (I_A)^4 + (F_A)^4 > 1$ then A is not a Tetra Neutrosophic set..

3.5 Definition

The complement of a Tetra Neutrosophic set $A = \{< x, T_A, I_A, F_A >: r \in R\}$ is

$A^C = \{< x, F_A, 1 - I_A, T_A >: r \in R\}$.

3.6 Example

Let X = {a,b,c},and A= {(a,0.3,0.4,0.7),(b,0.5,0.5,0.3),(c,0.2,0.5,0.4)} is a Tetra neutrosophic sets on R. Then A^C = {(a,0.7,0.6,0.3), (b,0.3,0.5,0.5),(c,0.4,0.5,0.2)}

3.7 Definition

Let $A = \{< x, T_A, I_A, F_A >: r \in R\}$ *and* $B = \{< x, T_B, I_B, F_B >: r \in R\}$ are two Tetra Neutrosophic Sets on the universe R. Then the union and intersection of two sets can be defined by

$A \cup B = \{\max(T_A, T_B), \min(I_A, I_B), \min(F_A, F_B)\}$,

$A \cap B = \{\min(T_A, T_B), \max(I_A, I_B), \max(F_A, F_B)\}$.

3.8 Example

Let R= {a,b,c},and A= {(a,0.2,0.3,0.6),(b,0.4,0.4,0.2),(c,0.1,0.4,0.3)} and B ={(a,0.4, 0.1, 0.4), (b,0.5,0.2,0.1),(c, 0.4,0.2,0.1)} are Tetra neutrosophic sets on R. Then

AUB = {(a,0.4,0.1,0.4),(b,0.5,0.2,0.1),(c, 0.4,0.2,0.1)}

A∩B = {(a,0.2,0.3,0.6),(b,0.4,0.4,0.2),(c, 0.1,0.4,0.3)}

3.9 Definition

A Tetra neutrosophic set A is contained in another Tetra neutrosophic set B (i.e) $A \subseteq B$ if $T_A \leq T_B$, $I_A \geq I_B$ and $F_A \geq F_B$

3.10 Example

Let R= {a,b,c},and A= {(a,0.3,0.4,0.7),(b,0.5,0.5,0.3),(c,0.2,0.5,0.4)} and B ={(a,0.5, 0.2,0.5),(b,0.6,0.3,0.2),(c, 0.5,0.3,0.2)} are Tetra neutrosophic sets on R. Then A ⊆ B.

3.11 Definition

A Tetra neutrosophic set A over the universe X is said to be empty Tetra neutrosophic set ∅ if $T_A = 0$, $I_A = 1$, $F_A = 1$, $\forall x \in X$ It is denoted by ∅ or 0

3.12 Definition

A Tetra neutrosophic set A over the universe X is said to be Δ universe Tetra neutrosophic set with respect to the parameter A if
$T_A = 1$, $I_A = 0$, $F_A=0$. It is denoted by Δ or 1

3.13 Definition

Let A and B be two Tetra neutrosophic sets on X then A\B may be defined as
$A\backslash B = < x, min(T_A, F_B), min(I_A, 1 - I_B), max(F_A, T_B) >$

3.14 Theorem

Let A,B and C are three Tetra neutrosophic sets over the universe X. Then the following properties holds true.

- Commutative law

a) $A \cup B = B \cup A$

b) $A \cap B = B \cap A$

- Associative law

c) $(A \cup B) \cup C = A \cup (B \cup C)$

d) $(A \cap B) \cap C = A \cap (B \cap C)$

- Distributive law

e) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

f) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- Absorption law

g) $A \cup (A \cap C) = A$

f) $A \cap (A \cup C) = A$

- Involution law

i) $(A^C)^C = A$

- Law of contradiction

j) $A \cap A^C = \emptyset$

- De Morgan's law

k) $(A \cup B)^C = A^C \cap B^C$

l) $(A \cap B)^C = A^C \cup B^C$

3.15 Theorem

Let K and L are two Tetra neutrosophic sets over the universe X. Then the following are true.
(i) $K \subseteq L$ iff $K \cap L = K$
(ii) $K \subseteq L$ iff $K \cup L = L$

3.16 Theorem

Let K be Tetra neutrosophic set over the universe X. Then the following are true.

(i) $(\emptyset)^c = X$

(ii) $(X)^c = \emptyset$

(iii) $K \cup \emptyset = K$

(iv) $(K \cup X = X$

(v) $K \cap \emptyset = \emptyset$

(vi) $K \cap X = K$

Proof: It is obvious

3.17 Definition

A Tetra neutrosophic topology on a non-empty set X is a τ of Tetra neutrosophic sets satisfying the following axioms.

i) $1,1 \in \tau$

ii) The union of the elements of any sub collection of τ is in τ iii) The intersection of the elements of any finite sub collection τ is in τ

The pair (X, τ) is called an Tetra neutrosophic Topological Space over X.

Note :

1. Every member of τ is called a TN open set in X.
2. The set A is called a TN closed set in X if $A \in \tau^c$, where $\tau^c = \{A^c : A \in \tau\}$.

3.18 Example :

Let M = {b1, b2} and Let A,B,C be Tetra neutrosophic sets where

A = {< b_1,0.5,0.1,0.7,0.2 > < b_2,0.7,0.5,0.2,0.1 > < b_3,0.6,0.5,0.4,0.3 > }

B = {< b_1,0.6,0.7,0.1,0.2 > < b_2,0.2,0.3,0.4,0.7 > < b_3,0.5,0.6,0.1,0.3 > }

C = {< b_1,0.6,0.7,0.1,0.2 > < b_2,0.7,0.5,0.2,0.1 > < b_3,0.6,0.6,0.1,0.3 > }

$\tau = \{A, B, C, 0,1\}$ is an Tetra neutrosophic topology on M.

3.19 Preposition

Let (M, τ_1) and (M, τ_2) be two Tetra neutrosophic topological space on M, Then $\tau_1 \cap \tau_2$ is an Tetra neutrosophic topology on M where $\tau_1 \cap \tau_2 = \{A_M : A_M \in \tau_1$ and $A_M \in \tau_2 \}$

Proof :

Obviously $0_M, 1_M \in \tau$.

Let $A_M, B_M \in \tau_1 \cap \tau_2$

Then $A_M, B_M \in \tau_1$ and $A_M, B_M \in \tau_2$

We know that τ_1 and τ_2 are two Tetra neutrosophic topological space M. Then

$A_M \cap B_M \in \tau_1$ and $A_M \cap B_M \in \tau_2$

Hence $A_M \cap B_M \in \tau_1 \cap \tau_2$.

Let τ_1 and τ_2 are two Tetra neutrosophic topological spaces on X.

Denote $\tau_1 \vee \tau_2 = \{A_M \cup B_M \ : A_M \in \tau_1$ and $A_M \in \tau_2\}$

$\tau_1 \wedge \tau_2 = \{A_M \cap B_M \ : A_M \in \tau_1$ and $A_M \in \tau_2\}$

3.20 Example

Let A_M *and* B_M be two Tetra neutrosophic topological space on X.

Define $\tau_1 = \{0_M, 1_M, A_M\}$

$\tau 2 = \{0M, 1M, BM\}$

Then $\tau_1 \cap \tau_2 = \{0_M, 1_M\}$ is a Tetra neutrosophic topological space on M.

But $\tau_1 \cup \tau_2 = \{0_M, A_M, B_M, 1_M\}$,

$\tau_1 \vee \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cup B_M\}$ *and* $\tau_1 \wedge \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cap B_M\}$ are not Tetra neutrosophic topological space on X.

4.Conclusion

In this paper, We defined basic tetra neutrosophic operations and studied its properties. Also we studied tetra neutrosophic topological spaces. In future, we can apply this set to multiple criteria decision making problems such as clustering analysis, medical diagnosis etc.

References

1. K. Atanasov, Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*.**1986**,volume 20 87-96.
2. Broumi S, Smarandache F (2014) Rough Neutrosophic sets, *Ital J Pure Appl Math* ,***2014,*** *volume* 32:493502.
3. D.A. Chiang and N.P. Lin, Correlation of fuzzy sets, *Fuzzy Sets and Systems* **1999**, volume102, 221-226.
4. D.H. Hong, Fuzzy measures for a correlation coefficient of fuzzy numbers under Tw (the weakest tnorm)-based fuzzy arithmetic operations, *Information Sciences* **2006** volume 176,150-160.
5. Rajashi Chatterjee, P. Majumdar and S.K. Samanta, On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets, *Journal of Intelligent and Fuzzy Systems* ,**2016**, volume 302475-2485.
6. R. Radha, A.Stanis Arul Mary, R.Prema,Said Broumi , Neutrosophic Pythagorean Sets with dependent neutrosophic components and its improved correlation coefficients,Neutrosophic Sets and
Systems,, **2021,** vol 30,203-212

7. R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic pythagorean Soft set, *IRJMETS*, **2021 ,** *Volume 3(2)*,905-914.
8. R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic Pythagorean Set, *IRJASH*, **2021**, *volume* *3*, 62-82.
9. R. Radha, A. Stanis Arul Mary. Heptapartitioned neutrosophic sets, *IRJCT, **2021** ,volume 2*,222-230.
10. R. Radha, A. Stanis Arul Mary, F. Smarandache. Quadripartitioned Neutrosophic Pythagorean soft set, *International journal of Neutrosophic Science, **2021,** volume14(1)*,9-23.
11. R. Radha, A. Stanis Arul Mary. Neutrosophic Pythagorean soft set, *Neutrosophic sets and systems,* **2021, *vol*** *42*,65-78.
12. R. Radha, A. Stanis Arul Mary, Pentapartitioned Neutrosophic Pythagorean Lie algebra, (Accepted)
13. R, Radha, A. Stanis Arul Mary, Improved Correlation Coefficients of Quadripartitioned Neutrosophic Pythagorean sets using MADM, *Journal of Computational Mathematica*, volume 5(1),**2021,** 142-153
14. R. Radha, A.Stanis Arul Mary, Pentapartitioned Neutrosophic Generalized semi-closed sets, *Journal of Computational Mathematica*,volume5(1),**2021,**123-131
15. Rama Malik, Surapati Pramanik. Pentapartitioned Neutrosophic set and its properties, *Neutrosophic Sets and Systems*, **2020,** *Vol 36*,184-192.
16. F.Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, *American Research Press, Rehoboth*.
17. Wang H, Smarandache F, Zhang YQ, Sunderraman R Single valued neutrosophic sets, *Multispace Multistruct ,**2010** ,volume* 4:410-413.
18. G.W. Wei, H.J. Wang and R. Lin, Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information, *Knowledge and Information Systems* ***2011***. *volume* 26 , 337-349.
19. L.Zadeh , Fuzzy sets, *Information and Control* **1965**, volume 8, 87-96.