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An Improved Vogel's Approximation And Iterative Techniques For Transportation Optimization

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Abstract: This article presents an enhanced Vogel Approximation Method (VAM) for fixed-point issues in transportation systems. Two improved algorithms are suggested to produce better initial feasible solutions, as well as a third way for fixed-point formulation, since traditional VAM frequently produces poor results for unbalanced issues. Furthermore, a web-based model that provides more precise and economical solutions for single-supply and single-demand scenarios is developed using successive under-relaxation and Gauss-Seidel iteration approaches.

Keywords: Fixed-point problem; Transportation model; Vogel Approximation Method (VAM); Improved heuristic algorithms; Initial feasible solution; Unbalanced transportation problem

I. Introduction

In this chapter, we have used an upgraded version of the Vogel Approximation Method (VAM) to handle fixed-point issues in the transportation context based on their functions and requirements. We are aware that VAM usually allocates elements to the fake cells before the other cells in the table. For uneven challenges, this first option might not be the best. To address this issue, we have two suggested algorithms. This chapter contains three suggested approaches, two of which are fundamental ideas to improve the initial solutions to the transportation problem and one of which is to tackle the fixed-point problem.

There are numerous heuristic methods that can be used to obtain an initial simple potential solution. While some heuristics are quite good at coming up with a novel, workable solution, they are also not very great at cutting costs overall.

The web model is being prepared under consecutive relaxation, Gauss Seidel iterations. As the best approach, this offers a superior approximation solution for single supply and single demand in the fixed-point methodology.

One of the core models in operations research is the transportation problem (TP), which aims to meet supply and demand constraints while minimizing the overall cost of moving commodities from many sources to multiple destinations. It is an essential tool for resource allocation, supply chain management, and logistics. To find preliminary workable solutions for transportation issues, traditional techniques including Vogel's Approximation Method (VAM), the North-West Corner Rule, and the Least Cost Method have long been employed. Among these, VAM is highly regarded for striking a balance between solution quality and computational simplicity. However, it performs poorly in large-scale or unbalanced problems, frequently yielding solutions that are substantially different from the best (Bansal et al., 2021).

Before thoroughly examining the viable space, traditional VAM frequently assigns shipments to dummy cells, which could result in first allocations that are not ideal. This problem is especially noticeable in unbalanced transportation systems, including agricultural logistics and distribution networks, when overall supply and demand are different. To overcome these issues, recent research has suggested a number of improvements to the VAM architecture. For example, Prakash and Singh (2023) created a heuristic variation integrating fuzzy logic to better address uncertainty in transportation costs, while Kumar and Raj (2022) produced a modified penalty-based VAM that increases cost efficiency for big datasets. These developments highlight heuristic refinement's continued importance in transportation optimization.

Even with these advancements, achieving the global minimal cost frequently necessitates additional optimization of the first workable solution found by VAM. These heuristic solutions can be effectively refined by iterative techniques like consecutive under-relaxation and the Gauss–Seidel method. Because they enable progressive convergence toward optimality, iterative algorithms are especially useful for addressing fixed-point formulations and nonlinear cost structures. According to a recent study by Alam et al. (2024), supply chain models' computational correctness and stability are much improved when iterative techniques are incorporated into heuristic frameworks. In a similar vein, Das and Kaur (2022) investigated hybrid heuristic-iterative models that improved convergence rates and reduced transportation system costs.

By suggesting an improved version of VAM in combination with iterative strategies to enhance transportation optimization outcomes, this chapter adds to the expanding corpus of research. In order to produce better initial viable solutions, the method presents two improved algorithms that alter the conventional penalty and allocation processes of VAM. A third approach employs Gauss-Seidel iterations and consecutive under-relaxation to solve fixed-point problems. When combined, these techniques offer a solid framework for attaining computational efficiency and cost reduction in both balanced and unbalanced transportation models. Additionally, this study creates a web-based model that incorporates these enhanced techniques, allowing single-supply and single-demand systems to compute and visualize in real-time. According to Chen et al. (2023), who stressed the relevance of web-enabled optimization tools in improving decision-making and resource utilization, such digital implementations are becoming more and more important in contemporary logistics planning.

II. PRELIMINARIES AND KEY PROPERTIES

In this research, the transportation model is applied to solve the Chickpea produce in bulk transportation and storage problem. A linear programming (L.P.) formulation is utilized to optimize the collection of gram crops from tehsil-level centers and their transportation to designated warehouses. The optimization results are obtained using LINGO software within the framework of operations research (O.R.).

2.1. FORMULATE TRANSPORTATION PROBLEM INTO LINEAR PROGRAMMING PROBLEM

For *a* collection centers at the tehsil level and *b* destination warehouses, the transportation model is represented by the following L.P.P. structure.

$$Min = \sum_{i=1}^{a} \sum_{j=1}^{b} V_{ij} U_{ij}$$

Concerning the limitation:

$$\sum_{i=1}^{b} U_{ij} \leq M \quad , \; i = 1, 2, ..., b$$

$$\sum_{j=1}^{a} U_{ij} \leq N \quad , \; J = 1, 2, ..., a$$

where Vij is the cost of transportation from one Tehsil Labels centre beginning at i to warehouse j, and Uij is the number of trucks (units) of transportation from the Tehsil Labels centres to warehouse j. At the source, the quantity of supply is Mi.

Table 1: An issue with bulk shipping and storage of chickpea products from Tehsil label centres.

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764	1050	1575	2751	250
Nagod	987	525	840	1974	180
Maihar	2436	903	1260	2436	175
Rampur	1050	1050	987	2163	225
Demand	290	185	155	200	Total= 830

The transportation model is used in the current study to solve the bulk transportation and storage problem of chickpea produce. When bulk chickpea produce is collected from Tehsil Label Centres and transported to a warehouse, the L.P.P. is utilized to address the TP.
Given that Σ ai = Σbi = 830. As a result, transportation is equal, and there is a precise, ideal solution to the issues.

2.2. IMPLEMENTATION OF A PROBLEM USING THE PROPOSED APPROACH AND NWCR (NORTH WEST CORNER RULES)

It is an easy and effective way to get a first workable solution. We call it NWCR.

Step1: The first allocation was made in the northwest corner cell, i.e., in the cell (rewa, warehouse1) and allocated the minimum (250, 290) = 250. As a result, we allocated 250 units to these cells, fully satisfying the capacity of the plant tehsil labels center Reva and leaving the remaining (290-250) = 40 units of demand for warehouse 1.

Table 2:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 250	1050	1575	2751	250-250=0
Nagod	987	525	840	1974	180
Maihar	2436	903	1260	2436	175
Rampur	1050	1050	987	2163	225

Step2: The cell in the first column and second row, or (Nagod, warehouse1), should then be processed down non-horizontally. The allotment at this point is min. (40,180) = 40. This allocation of 40 units to the cell (Nagod, warehause1) fully meets warehause1's demand, leaving warehouse 1 Tehsil labelling Centre Nagod with a balance of (180-40) = 140 units of supply.

Table 3:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 **250**	1050	1575	2751	0
Nagod	987 **40**	525	840	1974	180-40=140
Maihar	2436	903	1260	2436	175
Rampur	1050	1050	987	2163	225
Demand	40-40=0	185	155	200	Total= 830

Step3: We now descend non-vertically to the following cell in the second row and column, which is (Nagod, warehouse2). The allocation at this point is minimum (185,140) = 140. This allocation of 140 units to the cell (Nagod, warehouse2) fully satisfies the centre Nagod's capacity, leaving warehouse 2's demand of (185-140) = 45 units.

Table 4:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 **250**	1050	1575	2751	0
Nagod	987 **40**	525 **140**	840	1974	140-140=0
Maihar	2436	903	1260	2436	175
Rampur	1050	1050	987	2163	225
Demand	0	185-140=45	155	200	Total= 830

Step4: Additionally, processing non-vertically to the cell in the third row and second column, i.e., (maihar, warehouse1). The allocation at this point is min. (175,45) = 45. This 45-unit allotment to the cell (Maihar, warehouse-2) This fully meets warehouse2's demand, leaving centre Maihar's capacity of (175-45) = 130 units.

Table 5:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 **250**	1050	1575	2751	0
Nagod	987 **40**	525 **140**	840	1974	0
Maihar	2436	903 **45**	1260	2436	175-45=130
Rampur	1050	1050	987	2163	225
Demand	0	45-45=0	155	200	Total= 830

Step5: We proceed horizontally to the next cell in the third row and column, which is (Maihar, warehouse3). The allocation at this point is min. (155,130) = 130. This distribution of 130 units to the cell (Maihar, warehouse 3) fully satisfies the centre Maihar's capacity, leaving the remaining (155-130) = 25 units of warehouse 3's need.

Table 6:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 **250**	1050	1575	2751	0
Nagod	987 **40**	525 **140**	840	1974	0
Maihar	2436	903 **45**	1260 **130**	2436	130-130=0
Rampur	1050	1050	987	2163	225
Demand	0	10	155-130=25	200	Total= 830

Step6: We descend vertically to the cell in the fourth row and third column, or (Nagod, warehouse1). The allotment at this point is min. (25,225) = 25. This distribution of 25 units to the cell (Rampur, warehause3) fully meets warehause3's demand, leaving warehouse 3 Tehsil labels Centre Rampur with a capacity of (225-25) = 200 units.

Table 7:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 **250**	1050	1575	2751	0
Nagod	987 **40**	525 **140**	840	1974	0
Maihar	2436	903 **45**	1260 **130**	2436	0
Rampur	1050	1050	987 **25**	2163	225-25=200

Step7: We relocate the cell to the fourth row and column, or (Rampur, warehouse4). The allotment at this point is min. (200,200) = 200. 200 units were assigned to the cell (Rampur, warehouse4). which exits the capacity Tehsil Labelling Centre Rampur and warehouse4 after fully meeting the demand and capacity.

Table 8:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 **250**	1050	1575	2751	0
Nagod	987 **40**	525 **140**	840	1974	0
Maihar	2436	903 **45**	1260 **130**	2436	0
Rampur	1050	1050	987 **25**	2163 **200**	200-200=0
Demand	0	0	0	200-200=0	Total= 830

Table 9:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	**250** 1764	1050	1575	2751	250
Nagod	**40** 987	**140** 525	840	1974	180
Maihar	2436	**45** 903	**130** 1260	2436	175
Rampur	1050	1050	**25** 987	**200** 2163	225
Demand	290	185	155	200	Total= 830

North West Corner Rules (NWCR) by unit cost of transportation.

=1764*250+987*40+525*140+903*45+1260*130+987*25+2163*200=Rs 12,15,690.

2.3 APPLICATION OF LCM TO A PROBLEM (PROPOSED APPROACH)

This approach reduces computation and the amount of time needed to arrive at an exact solution at the highest level by accounting for the least unit cost.

Step1: In this case, cell (Nagod, warehouse2) has the lowest cost (i.e., 525). We choose the cell (Nagod, warehouse2) at random. We allocate 180 units in this cell, which meets the Tehsil label Centre Nagod capacity. The transportation table is being deleted.

Table 10:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764	1050	1575	2751	250
Nagod	987	525 180	840	1974	180-180=0
Maihar	2436	903	1260	2436	175
Rampur	1050	1050	987	2163	225
Demand	290	185-180=5	155	200	Total= 830

Step2: Cell (Maihar, warehouse2) has the lowest cost (i.e., 903). We choose the cell (Maihar, warehouse2) at random. We allocate five units in this cell, meeting the requirements of Demand Warehouse 2. The transportation table is being deleted**.**

Table 11:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764	1050	1575	2751	250
Nagod	987	525 **180**	840	1974	0
Maihar	2436	903 **5**	1260	2436	175-5=170
Rampur	1050	1050	987	2163	225
Demand	290	5-5=0	155	200	Total= 830

Step3: Cell (Rampur, warehouse2) has the next lowest cost (i.e., 987). We choose the cell (Rampur, warehouse3) at random. We allocate 155 units in this cell, meeting Demand warehouse 2. The transportation table is being deleted.

Table 12:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764	1050	1575	2751	250
Nagod	987	525 **180**	840	1974	0
Maihar	2436	903 **5**	1260	2436	170
Rampur	1050	1050	987 **155**	2163	225-155=70
Demand	290	0	155-155=0	200	Total= 830

Step4: Cell (Rampur, warehouse1) has the next lowest cost (i.e., 1050). We choose cells at random (Rampur, warehouse1). We allocate 70 units in this cell, which meets the Tehsil labels Centre Rampur's capacity. The transportation table is being deleted.

Table 13:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764	1050	1575	2751	250
Nagod	987	525 180	840	1974	0
Maihar	2436	903 **5**	1260	2436	170
Rampur	1050 **70**	1050	987 **155**	2163	70-70=0
Demand	290-70=220	0	0	200	Total= 830

Step5: Cell (Rewa, warehouse1) has the next lowest cost (i.e., 1764). We choose cell (Rewa, warehouse1) at random. We allocate 220 units in this cell to meet the demand warehouse1. The transportation table is being deleted.

Table 14:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 220	1050	1575	2751	250-220=30
Nagod	987	525 180	840	1974	0
Maihar	2436	903 **5**	1260	2436	170
Rampur	1050 **70**	1050	987 **155**	2163	0
Demand	220-220=0	0	0	200	Total= 830

Step6: Cell (Reva, warehouse 4) has the next lowest cost (i.e., 2751). We choose the cell (Rampur, warehouse1) at random. We put 30 units in this cell, which meets the Tehsil labels Centre Rewa capacity. The transportation table is being deleted.

Table 15:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 220	1050	1575	2751 **30**	30-30=0
Nagod	987	525 **180**	840	1974	0
Maihar	2436	903 **5**	1260	2436	170
Rampur	1050 **70**	1050	987 **155**	2163	0
Demand	0	0	0	200-30=170	Total= 830

Step7: Cell (Rampur, warehouse1) has the next lowest cost (i.e., 1050). We choose cells at random (Rampur, warehouse1). We allocate 70 units in this cell, which meets the Tehsil labels Centre Rampur's capacity. The transportation table is being deleted.

Table 16:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 **220**	1050	1575	2751 **30**	30-30=0
Nagod	987	525 **180**	840	1974	0
Maihar	2436	903 **5**	1260	2436 **170**	170-170=0
Rampur	1050 **70**	1050	987 **155**	2163	0
Demand	0	0	0	170-170=0	Total= 830

Table 17:

Tehsil labels Centre	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Capacity
Rewa	1764 **220**	1050	1575	2751 **30**	250
Nagod	987	525 **180**	840	1974	180
Maihar	2436	903 **5**	1260	2436 **170**	175
Rampur	1050 **70**	1050	987 **155**	2163	225
Demand	290	185	155	200	Total= 830

LCM (Least Cost Method) by unit cost of transportation.

= 1764*220+2751*30+525*180+903*5+2436*170+1050*70+987*155=Rs 12, 10,230.

2.4 APPLICATION OF VAM TO A PROBLEM (PROPOSED APPROACH)

Step1: In each row and column, trace the penalty cost (within the implied meaning of cost over the standard cost as it should be), which is different between the smallest and next smallest cost ($C_1–C_{1-n}$) [where C = Cost of one stage and C_1 = next additional stage of cost identified as marginal cost] and place it in parenthesis against the corresponding row and column. The entry in the TP table that corresponds to the Tehsil labelling centre Rewa has the biggest penalty (525). Due to the Row Tehsil labels centre's minimum cost of 1050 in the cell (Rewa, warehouse2). This allocation is set at min (250,185) = 185.

Table 18:

Origin Tehsil labels Centre	**Target**				**Supply**	**Penalty of Row**
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4		
Rewa	1764	1050 **185**	1575	2751	250-185=65	**525***
Nagod	987	525	840	1974	180	315
Maihar	2436	903	1260	2436	175	357
Rampur	1050	1050	987	2163	225	63
Demand	290	185-185=0	155	200		

Penalty of Column	63	375	147	189	

Rewa requirement, leaving 65 units available at Rewa. As a result, warehouse 2's demand is exhausted, and the warehouse 2 column is deleted.

Step2: Once more, determine each row's and column's penalty cost for reduced table 2 and put it in parenthesis. The Row Tehsil labels Centre Maihar has the biggest penalty, which is (1176). Since the minimum cost in row Maihar is (1260), 155 units are allocated to this minimum cost cell, which exhausts Maihar's requirements and levels its supply. removing the table's column warehouse3.

Table 19:

Origin Tehsil labels Centre	**Target**				**Supply**	**Penalty of Row**
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4		
Rewa	1764	1050 **185**	1575	2751	65	189
Nagod	987	525	840	1974	180	147
Maihar	2436	903	1260 **155**	2436	175-155=20	**1176***
Rampur	1050	1050	987	2163	225	63
Demand	290	185	155-155=0	200		
Penalty of Column	63	-	147	189		

Step3: Once more, the biggest penalty is (1113), which is located in row Tehsil labels Centre Rampur because row Rampur has a minimum cost of 1050. We give this minimum cost cell a minimum of (225,290) = 225, which exhausts warehouse 2's demand and leaves Tehsil labels Centre Rampur's supply. removing the warehouse 2 column from the table

Table 20:

Origin Tehsil labels Centre	**Target**				**Supply**	**Penalty of Row**
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4		
Rewa	1764	1050 **185**	1575	2751	65	987
Nagod	987	525	840	1974	180	987
Maihar	2436	903	1260 **155**	2436	20	**0**
Rampur	1050 **225**	1050	987	2163	225-225=0	**1113***
Demand	290-225=65	185	155	200		
Penalty of Column	63	-	-	189		

Step4: Again, since the minimum cost in row Rewa is 1764, the biggest penalty is (987), which is located in row Tehsil labels Centre Rewa. We remove the row Tehsil labels Centre Rewa and column warehouse 1 from the table and assign min (65, 65) = 65 to this minimum cost cell.

Table 21:

Origin Tehsil labels Centre	**Target**				**Supply**	**Penalty of Row**
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4		
Rewa	1764 **65**	1050 **185**	1575	2751	65-65=0	**987***
Nagod	987	525	840	1974	180	987
Maihar	2436	903	1260 **155**	2436	20	**0**
Rampur	1050 **225**	1050	987	2163	225	**-**
Demand	65-65=0	185	155	200		
Penalty of Column	777	-	-	462		

Step5: Once more, the biggest penalty is (2436), which is located at Tehsil Labels Centre Maihar because row Maihar's minimum cost is 2436. This minimum cost cell is given min (20, 200) = 20. As a result, warehouse 4's supply of Tehsil labels is exhausted, leaving Centre Rampur. removing the Tehsil -labeled Centre Maihar row from Table 5.

Table 22:

Origin Tehsil labels Centre	**Target**				**Supply**	**Penalty of Row**
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4		
Rewa	1764 **65**	1050 **185**	1575	2751	250	**-**
Nagod	987	525	840	1974	180	1974
Maihar	2436	903	1260 **155**	2436 **20**	20-20=0	**2436***
Rampur	1050 **225**	1050	987	2163	225	**-**
Demand	290	185	155	200-20=180		
Penalty of Column	-	-	-	462		

Step6: Once more, the biggest penalty is (1974), which is in the Tehsil labels Centre Nagod because row Nagod's minimum cost is 1974. This minimum cost cell receives min (180, 180) = 180. removing the row Tehsil labels Centre Nagod and column warehouse 4 from the table.

Table 23:

Origin Tehsil labels Centre	**Target**				**Supply**	**Penalty of Row**
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4		
Rewa	1764 **65**	1050 **185**	1575	2751	250	-
Nagod	987	525	840	1974 **180**	180-180=0	**1974***
Maihar	2436	903	1260 **155**	2436 **20**	175	-
Rampur	1050 **225**	1050	987	2163	225	-
Demand	290	185	155	180-180=0		
Penalty of Column	-	-	-	462		

In this case, Σ Mi = 830 and Σ Nj = 30. Since Σ Mi = Σ Nj, the balanced transportation problem is displayed in the above table: -The first workable answer obtained using the VAM approach is U11 = 65, U12 = 185, U24 = 180, U33 = 155, U34 = 20, and U41 = 225. The value of the associative objective function (transportation cost) is Rs. 1144500.

MIN=1764*65 + 1050*185 + 1974*180 + 1260*155 + 2436*20 + 1050*225 = Rs1144500

2.5 LINGO SOFTWARE FOR O.R

MODEL L.P.P Formulation of Transportation Problem[1],

We are now utilizing a table to modify the transportation issue in L.P.P.

$$MIN==1764*U_{11}+1050*U_{12}+1575*U_{13}+2751*U_{14}+987*U_{21}+525*U_{22}+840*U_{23}+1974*U_{24}+2436*U_{31}+903*U_{32}+1260*U_{33}+2436*U_{34}+1050*U_{41}+1050*U_{42}+987*U_{43}+2163*U_{44};$$

$$U_{11} + U_{12} + U_{13} + U_{14} <=250;$$

$$U_{21} + U_{22} + U_{23} + U_{24} <=180;$$

$$U_{31} + U_{32} + U_{33} + U_{34} <=175;$$

$$U_{41} + U_{42} + U_{43} + U_{44} <=225;$$

$$U_{11} + U_{21} + U_{31} + U_{41} >=290;$$

$$U_{12} + U_{22} + U_{32} + U_{42} >=185;$$

$$U_{13} + U_{23} + U_{33} + U_{43} >=155;$$

$$U_{14} + U_{24} + U_{34} + U_{44} >=200;$$

$$U_{11} >= 0; U_{12} >= 0; ; U_{13} >=0; U_{14} >=0;$$

$$U_{21} >=0; U_{22} >=0; U_{23} >=0; U_{24} >=0;$$

$$U_{31} >=0; U_{32} >=0; U_{33} >=0; U_{34} >=0;$$

$$U_{41} >=0; U_{42} >=0; U_{43} >=0; U_{44} >=0;$$

Transportation costs have an associative objective function value of Rs. 1144500. According to the aforementioned strategy, the cost of shipping one Quintal of chickpeas in bulk from each source to each K.M. (Goal) varies based on the distance and Lorry's methodology. First, we use the VAM approach to solve our transportation problem for chickpea produce in bulk. We next use LINGO Software on Integer Programming Formulation of Transportation Problem to achieve the best potential answer. For Tehsil Label Centers to reduce the cost of transportation

Rs1144500 at U_{11} =65, U_{12} =185, U_{24} = 180, U_{33}= 155, U_{34} = 20, U_{41}= 225.

III. FINDINGS AND TALKS

In the form of a numerical example, the Transportation techniques (2.2), (2.3), and (2.4) are applied to various types of transportation techniques; the resulting transportation expenses are Rs 1215690 by NWCR, Rs 1210230 by LCM, and Rs 1144500 by VAM.

Depending on the distance and Lorry's method, the cost of shipping one Quintal of chickpeas in bulk from each source to each K.M. (target) varies. First, we address our transportation problem for chickpea produce in bulk utilizing the NWCR, LCM, and VAM methods. Then, we use LINGO Software on Integer Programming Formulation of Transportation Problem to achieve the best potential answer. Tesil Label Centers will save Rs. 1144500 on transportation at U_{11} = 65, U_{12} = 185, U_{24} = 180, U_{33} = 155, U_{34} = 20, and U_{41} = 225.

Therefore, VAM is the best and most profitable option. Additionally, rank-wise outcomes are

1. VAM (4.2.4) - Rs 1144500.
2. LCM (4.2.3) – Rs 1210230.
3. NWCR (4.2.2) – Rs 1215690.

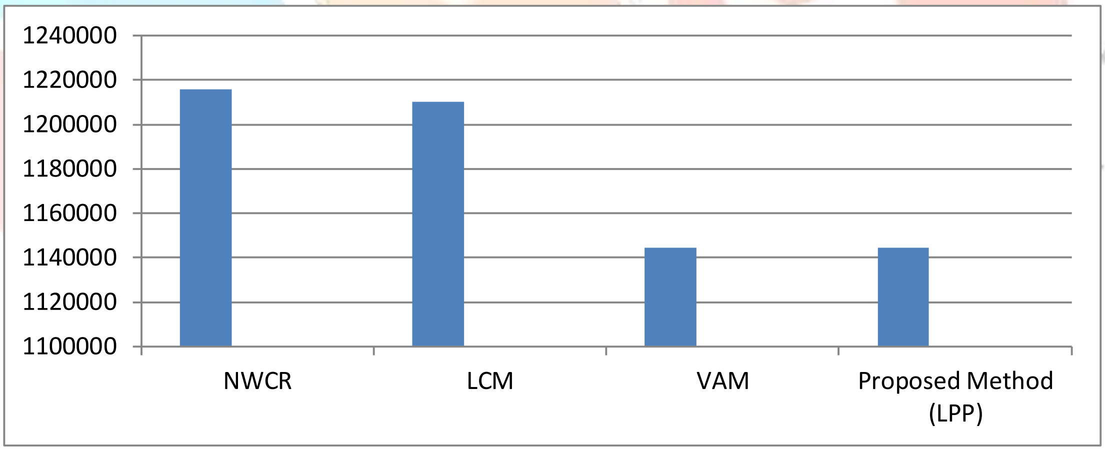

Figure 1: NWCR, LCM, and VAM & LPM Comparison Graph

According to the combined study, the LPP is displaying the outcome that is equal to VAM.

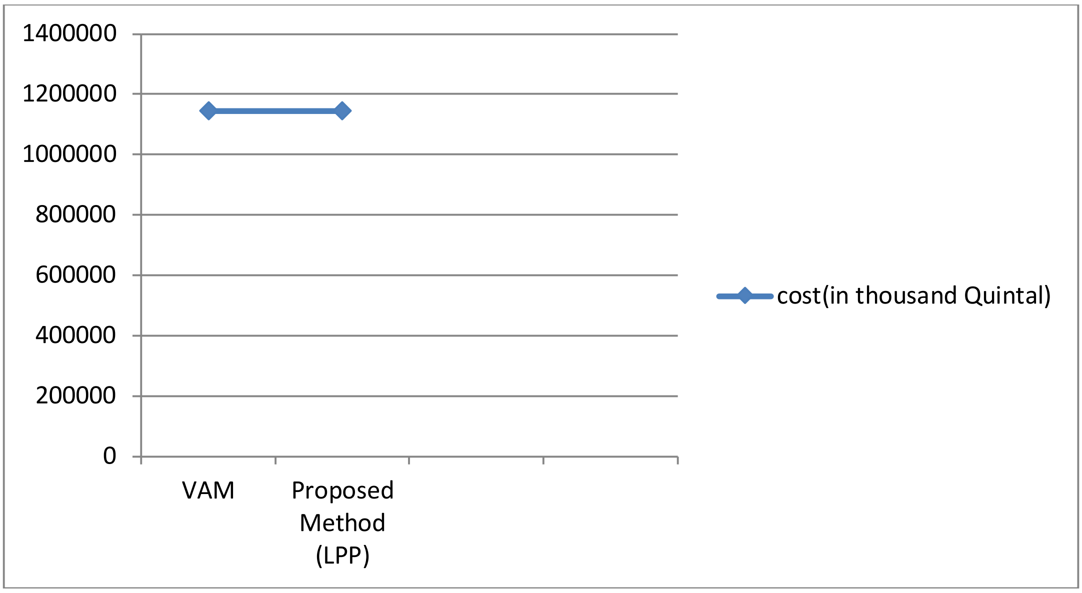

Figure 2: VAM and Proposed Method (LPP) Comparison Graph

IV. CONCLUSION

By combining an improved Vogel's Approximation Method (VAM) with iterative computing approaches including the Gauss-Seidel method and successive under-relaxation, this study offered an improved framework for tackling transportation optimization problems. When dealing with unbalanced transportation problems, where traditional allocation to dummy cells frequently yields inferior initial solutions, the improved VAM successfully overcomes the shortcomings of the conventional technique. By altering the allocation and punishment mechanisms, the suggested algorithms produce superior initial viable solutions that speed up convergence and lower overall transportation costs.

Additionally, the accuracy and stability of solutions are greatly enhanced by the use of iterative refining techniques. The under-relaxation technique improves numerical convergence, particularly for fixed-point formulations in single-supply and single-demand systems, while the Gauss–Seidel iteration guarantees progressive cost minimization through repeated changes. For both balanced and unbalanced transportation models, the combination of heuristic and iterative techniques offers a reliable and effective method.

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