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Pingala's Binary Method: A Comprehensive Study of Prastāra, Saṅkhyā, Naṣṭa, Uddiṣṭa

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Abstract: Binary representation and combinatorial analysis are fundamental to modern mathematics, computer science, and engineering. These concepts are generally regarded as products of relatively recent intellectual history. However, ancient Indian scholarship demonstrates that systematic binary thinking and combinatorial methods were developed much earlier. Pingala, an ancient Indian prosodist, introduced a structured framework for binary enumeration through the analysis of Sanskrit poetic metres. By classifying syllables into Laghu (short) and Guru (long), Pingala devised a complete binary system along with procedures for generation, counting, ranking, unranking, classification, and spatial estimation.This paper presents a comprehensive and detailed study of Pingala's binary methods—Prastāra, Saṅkhyā, Naṣṭa, and Uddiṣṭa—along with illustrative examples, conceptual interpretation, and applications in modern science and engineering, highlighting their significance as early foundations of discrete mathematics.

***Index Terms* -** Binary Numbers, Combinatorics, Sanskrit Prosody, Laghu–Guru, Varṇameru

Introduction

The history of mathematics often presents binary numbers, combinatorics, and algorithmic reasoning as developments of the modern era, closely associated with European mathematicians of the seventeenth century. However, a careful examination of ancient Indian mathematical literature reveals that these concepts were systematically explored more than two millennia earlier. One of the most remarkable contributions in this direction is found in the prosodic treatise Chandaḥśāstra authored by **Pingala** around 300 BCE.

Pingala's objective was not mathematical formalism in the modern sense but the exhaustive analysis of Sanskrit poetic metres (*chandas*). Nevertheless, the methods he developed for enumerating, counting, indexing, and classifying metrical patterns correspond precisely to binary mathematics and combinatorial algorithms. *Chandaḥśāstra* systematically addresses these problems using six well-defined procedures: Prastāra, Saṅkhyā, Naṣṭa, Uddiṣṭa, Lagakriyā, and Varṇameru.

This Paper establishes the historical context, outlines the motivation behind Pingala's work, and emphasizes the significance of recognizing ancient Indian contributions to discrete mathematics and computer-science–related thinking.

Laghu–Guru System and Binary Representation

Sanskrit prosody is fundamentally based on syllabic duration rather than stress or accent. Each syllable in a verse is classified into one of two categories:

- **Laghu (L)** – a short syllable
- **Guru (G)** – a long syllable

Any *varṇa-vṛtta* (syllabic metre) is therefore a finite sequence composed exclusively of Laghu and Guru syllables. Pingala recognized that the structural behavior of such sequences depends solely on the binary choice available at each syllabic position.

By mapping:

- Laghu → 1
- Guru → 0

each metrical pattern becomes equivalent to a binary sequence. For example, a four-syllable metre such as *L G L G* corresponds to the binary sequence *1 0 1 0*. This mapping is explicitly mentioned in the reference text and forms the conceptual foundation for all subsequent methods.

This abstraction demonstrates a profound understanding of symbolic representation and positional structure, anticipating the binary encoding used in modern computation.one example for this method is as showin below

कर्मण्येवाधिकारस्ते मा फलेषु कदाचन ।
मा कर्मफलहेतुर्भूर्मा ते सङ्गोऽस्त्वकर्मणि ॥
karmaṇyevādhikāraste mā phaleṣu kadācana |
mā karmaphalaheturbhūr mā te saṅgo’stvakarmaṇi ||

kar	**ma**	**ṇye**	**vā**	**dhi**	**kā**	**ra**	**ste**	**mā**	**pha**	**le**	**ṣu**	**ka**	**dā**	**ca**	**na**	
L	G	G	G	L	G	L	G	G	L	G	L	L	G	L	L	
mā	**kar**	**ma**	**pha**	**la**	**he**	**tur**	**bhūr**	**mā**	**te**	**saṅ**	**go**	**’s**	**tva**	**kar**	**ma**	**ṇi**
G	L	G	L	L	G	G	G	G	L	G	G	L	G	L	G	L

Table 1: Laghu Guru Matra

So from the above table if we denote L and G by 1 and 0 respectively binary sequence will be form as below

Line 1- 1 0 0 0 1 0 1 0 0 1 0 1 1 0 1 1

Line 2- 0 1 0 1 1 0 0 0 0 1 0 0 1 0 1 0 1

Mnemonic for the Eight Gaṇas of Piṅgala

Piṅgala introduced a systematic method for analyzing Sanskrit metres by grouping syllables into **units of three**, known as **gaṇas**. These gaṇas are constructed using the two fundamental prosodic units: **laghu (short syllable)** and **guru (long syllable)**. The motivation behind this grouping is to simplify the identification and classification of metres by reducing long syllabic sequences into smaller, manageable blocks.

Since each syllable can assume only two possible states—laghu or guru—a group of three syllables yields exactly **2^3 = 8** distinct combinations. Each of these combinations is assigned a specific **gaṇa name**, which acts as a mnemonic device, allowing scholars to remember and recognize metrical patterns efficiently.

The Eight Gaṇas and Their Binary Representation

यमाता–राज–भान–सलगम्
yamātā–rāja–bhāna–salagam

Each syllable of this mnemonic corresponds sequentially to one gaṇa, allowing the metrical structure of an entire verse to be reconstructed mentally without writing down individual laghu–guru patterns.

Sl. No.	Gaṇa Name	Laghu–Guru Pattern	Binary Word
1	**Ya-gaṇa**	L G G	100
2	**Ma-gaṇa**	G G G	000
3	**Ta-gaṇa**	G G L	001
4	**Ra-gaṇa**	G L G	010
5	**Ja-gaṇa**	L G L	101
6	**Bha-gaṇa**	G L L	011
7	**Na-gaṇa**	L L L	111
8	**Sa-gaṇa**	L L G	110

Table 2: Eight Ganas V/s Binary Representation

Prastāra Method – Systematic Enumeration

Concept

The **Prastāra** method is the procedure for generating all possible metrical patterns of a given length. In modern terminology, it corresponds to the exhaustive generation of all binary strings of length n.

Pingala's approach is algorithmic and recursive. Starting with shorter sequences, the existing array is duplicated, and a new column is appended. In this column, the first half of the rows is filled with one symbol (0 or Guru), and the second half with the other symbol (1 or Laghu). This process is repeated iteratively as the length increases.

For n syllables, the prastāra contains exactly 2^n rows, each representing a unique metrical configuration. The ordered arrangement of rows ensures that no pattern is repeated or omitted.

This chapter explains the generation process step by step, illustrates the structure of the prastāra table, and highlights its equivalence to binary counting from 0 to $2^n - 1$.

Procedure

1. Begin with sequences of length 1.
2. Replicate the existing array.
3. Add a new column.
4. Fill the first half of the rows with 0 and the second half with 1.
5. Repeat the process for increasing lengths.

In the below example as I prepare a sequence from 1 bit binary to 4 bit binary

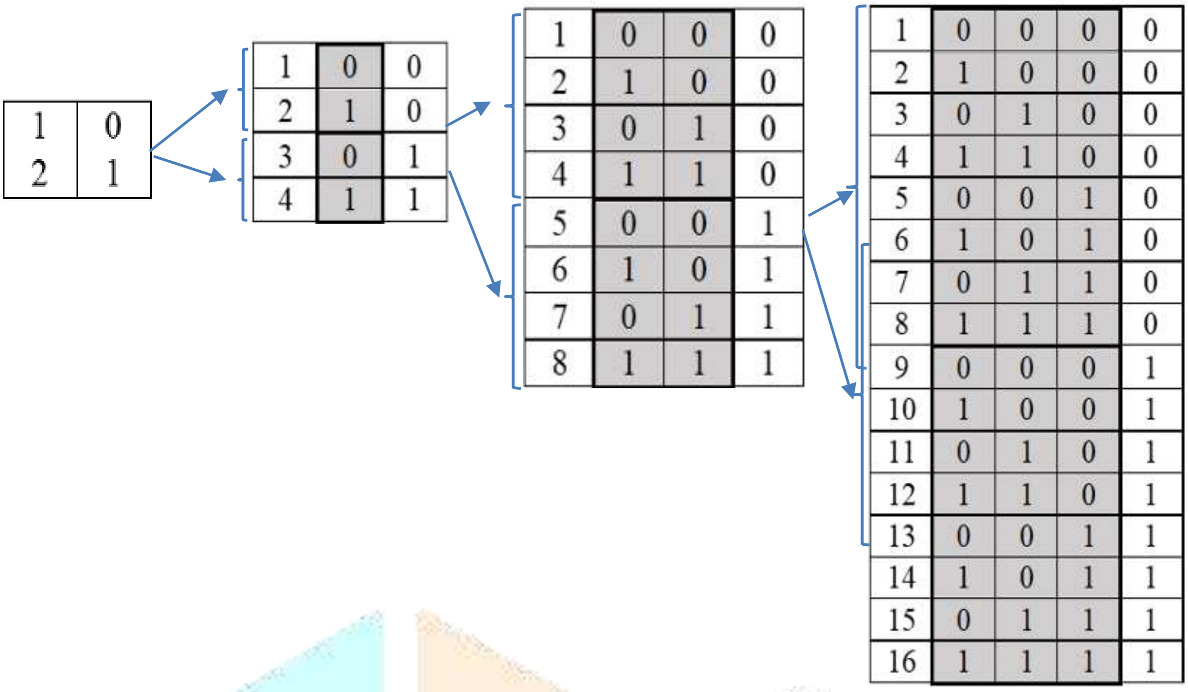

Figure 1: 1 binary to 4 bit binary Prastāra method

In modern science and technology this method is used for **Binary tree traversal** ,**State-space generation in AI**, **Test-case generation in software engineering**, **Gray code–like sequence design**, **Exhaustive search algorithms.**

Saṅkhyā Method: Counting the Number of Patterns

Concept

Saṅkhyā moves from construction to quantification. Instead of listing patterns, it determines how many patterns must exist in principle.

The recognition that the number of combinations doubles with each additional syllable reflects an understanding of exponential growth, a concept central to:

Algorithm complexity

Network scalability

Cryptographic strength

Mathematical Representation of method is 2^n so that if we prepare for the 4bit or for the 4 syllable of Laghu and Guru combination according to sankhya method put n=4 in above equation we get $2^4 = 16$, so that we can able to create 16 combination of 4bit.

Naṣṭa Method: Finding the Pattern from a Row Number

Concept

Naṣṭa solves the inverse problem: retrieving structure from position. This is a crucial computational operation because storage and retrieval often depend on indexing rather than generation. Pingala's division-based algorithm is remarkable because it Avoids generating the full Prastāra, Uses local operations only, Works for arbitrary lengths. This makes Naṣṭa highly efficient and scalable.

Following is the algorithm to find out the binary sequence according to row number using Nasta method.

Algorithm

1. Start with the given row number.
2. Divide the number by 2.
3. If divisible, write 1.
4. If not divisible, write 0, add 1, and divide by 2.
5. Repeat until the required length is obtained.

6. Reverse the obtained sequence if necessary.

Example

Find the pattern corresponding to row number 13 (length = 4):

- 13 → not divisible → write **0** → (13+1)/2 = 7
- 7 → not divisible → write **0** → (7+1)/2 = 4
- 4 → divisible → write **1** → 4/2 = 2
- 2 → divisible → write **1**

Binary sequence: **0011** , Metre: **G G L L**

Uddiṣṭa Method: Finding the Row Number from a Pattern

Concept

Uddiṣṭa is the reverse of Naṣṭa. It determines the row number of a given metrical pattern in the Prastāra array.

Algorithm

1. Start with value 1.
2. Scan the binary sequence from right to left.
3. For each digit:
   - If 1 → multiply the value by 2.
   - If 0 → multiply by 2 and subtract 1.
4. The final value is the row number.

Example

Binary sequence: **1011**

- Start: 1
- 1 → 2
- 1 → 4
- 0 → 7
- 1 → 14

Row number = **14**

Conclusion

This study establishes that Piṅgala's *Chandaḥśāstra* presents a systematic and rigorous framework for binary mathematics through the analysis of Sanskrit prosody. By representing syllables as *laghu* and *guru*, Piṅgala effectively introduced binary encoding and developed complete methods for enumeration, counting, ranking, unranking, and combinatorial classification. Techniques such as Prastāra, Saṅkhyā, Naṣṭa, Uddiṣṭa, Lagakriyā, Varṇameru, and the gaṇa system together form an integrated algorithmic structure comparable to modern discrete mathematics.

The conversion of poetic metres into binary sequences demonstrates that sophisticated computational thinking can emerge from linguistic traditions. Piṅgala's work anticipates key ideas used today in computer science, information theory, and engineering, including state-space generation, binary representation, and combinatorial analysis. Recognizing these contributions not only enriches the history of mathematics but also highlights the enduring relevance of ancient Indian knowledge systems to modern scientific thought.

References

1. *Piṅgala, Chandaḥśāstra, Ancient Sanskrit Treatise on Prosody.*
2. *Halāyudha, Mṛtasañjīvanī (Commentary on Chandaḥśāstra).*
3. *Kedāra Bhaṭṭa, Vṛttaratnākara.*
4. *Datta, B., & Singh, A. N., History of Hindu Mathematics, Vol. I, Motilal Banarsidass, 1962.*
5. *Joseph, G. G., The Crest of the Peacock: Non-European Roots of Mathematics, Princeton University Press, 2011.*
6. *Plofker, K., Mathematics in India, Princeton University Press, 2009.*
7. *Ifrah, G., The Universal History of Numbers, Wiley, 2000.*
8. *Kak, S. C., "Binary Numbers and Indian Prosody," Annals of the History of Computing, IEEE, 1986.*
9. *Kak, S. C., "The Algorithms of Pingala," Indian Journal of History of Science, 1994.*
10. *Sarasvati Amma, T. A., Geometry in Ancient and Medieval India, Motilal Banarsidass, 1979.*

11. Sen, S. N., & Bag, A. K., *The Śulbasūtras*, Indian National Science Academy, 1983.
12. Bag, A. K., *Mathematics in Ancient India*, Chaukhamba Orientalia, 1979.
13. Ramasubramanian, K., Srinivas, M. D., & Sriram, M. S., *Mathematics in India*, Hindustan Book Agency, 2010.
14. Boyer, C. B., & Merzbach, U. C., *A History of Mathematics*, Wiley, 2011.
15. Van der Waerden, B. L., *Science Awakening*, Oxford University Press, 1961.
16. Knuth, D. E., *The Art of Computer Programming*, Vol. 4, Addison-Wesley, 2011.
17. Raju, C. K., *Cultural Foundations of Mathematics*, Pearson, 2007.
18. Halbfass, W., *India and Europe: An Essay in Understanding*, SUNY Press, 1988.
19. Chatterjee, A., "Prosody and Mathematics in Ancient India," *Indian Philosophical Quarterly*, 2002.
20. Singh, P., *Indian Mathematics: Concepts and Development*, Pearson, 2015.
21. Sharma, S., *Ancient Indian Mathematics and Vedic Traditions*, Aryan Books, 2016.
22. Needham, J., *Science and Civilisation in China*, Vol. 3, Cambridge University Press, 1959.
23. Pingree, D., *Census of the Exact Sciences in Sanskrit*, American Philosophical Society, 1970.
24. Rao, T. S., *Mathematics in Sanskrit Literature*, Rashtriya Sanskrit Sansthan, 1995.
25. Datta, B., "The Mathematics of Prosody," *Bulletin of the Calcutta Mathematical Society*, 1933.
26. Kak, S. C., "Information Theory and Indian Prosody," *Cybernetics and Systems*, 1990.
27. Staal, F., *Discovering the Vedas*, Penguin, 2008.
28. Narasimhan, R., "Mathematical Structures in Sanskrit Prosody," *Journal of the Asiatic Society*, 1987.
29. Srinivas, M. D., "Combinatorics in Ancient India," *Current Science*, 2005.
30. Government of India, *Indian Knowledge Systems: Concepts and Applications*, AICTE/UGC Publication.