



# Joint Constraints On Time-Varying Dark Energy From Cmb Anisotropies And Galaxy Clustering Observables

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## ABSTRACT

Joint cosmological analysis to constrain a time-varying dark energy equation of state using cosmic microwave background (CMB) anisotropy data and galaxy clustering observables from large-scale structure surveys. We extend the standard  $\Lambda$ CDM model by adopting the Chevallier-Polarski-Linder (CPL) parameterization for the dark energy equation of state,  $w(z) = w_0 + w_a z/(1+z)$ , and study its impact on both the background expansion  $H(z)$  and the linear growth of matter perturbations. The CMB temperature and polarization spectra, together with CMB lensing, tightly constrain early-universe parameters and the distance to last scattering, while baryon acoustic oscillation (BAO) and redshift-space distortion (RSD) measurements constrain late-time geometry and growth through distance indicators and  $f\sigma_8(z)$ . We build a consistent joint likelihood using Planck 2018 TT, TE, EE plus lowE and lensing, combined with BOSS DR12 consensus BAO+RSD constraints at  $z_{\text{eff}} = 0.38, 0.51$ , and  $0.61$ . Posterior distributions are sampled using Markov Chain Monte Carlo, and we quantify improvements in constraints on  $(w_0, w_a)$  when adding low-redshift information to the CMB. Model comparison with  $\Lambda$ CDM is reported using information criteria, and robustness is assessed under alternative assumptions on priors, scale cuts, and neutrino mass treatment. This work provides a complete observationally anchored framework for testing dark energy evolution with current precision cosmology data.

**Keywords:** dark energy, evolving equation of state, CMB anisotropies, CMB lensing, BAO, RSD, galaxy clustering, parameter estimation

## 1. INTRODUCTION

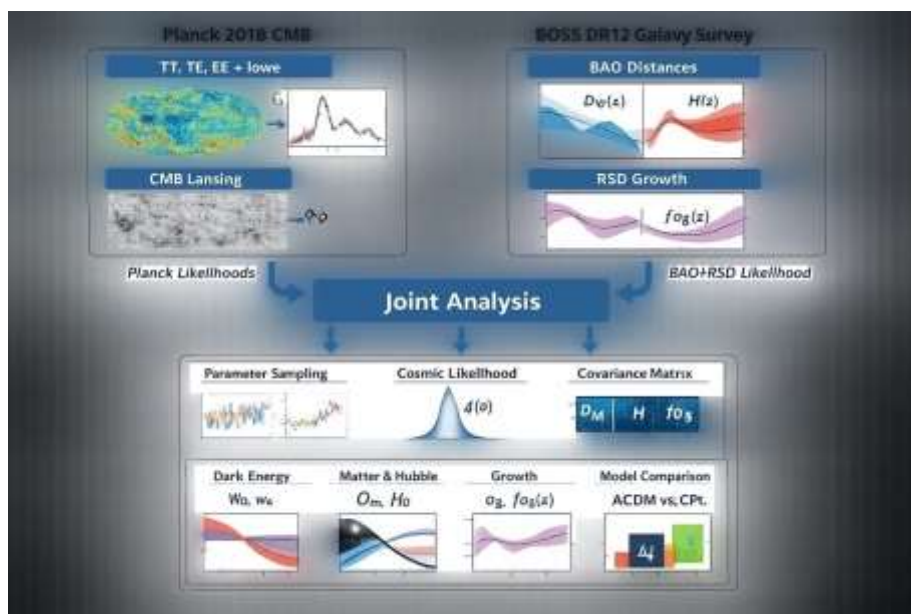
The discovery that the Universe is undergoing accelerated expansion has motivated extensive efforts to determine whether the driver is a cosmological constant or a dynamical dark energy component, and whether departures from General Relativity could play a role at late times. Within the standard  $\Lambda$ CDM model, acceleration arises from a constant equation of state  $w = -1$ . While  $\Lambda$ CDM provides an excellent fit to a wide range of datasets, the physical origin of  $\Lambda$  remains unexplained, and precision cosmology increasingly motivates systematic tests of extensions that permit mild evolution in the dark energy sector.

A minimal and widely adopted approach is to allow the dark energy equation of state  $w(z)$  to vary with redshift while retaining standard gravity and a homogeneous dark energy component. Constraints on such evolution require combining early- and late-universe information. CMB anisotropy measurements provide high-precision constraints on primordial parameters and the distance to last scattering, while galaxy clustering measurements probe the late-time geometry and growth of structure through BAO and RSD. The complementarity between these probes helps break degeneracies that arise when  $w(z)$  is allowed to vary.

In this work we focus on joint constraints on a time-varying equation of state using the CPL form,  $w(z) = w_0 + w_a z/(1+z)$ . We adopt Planck 2018 TT,TE,EE plus lowE and lensing, together with BOSS DR12 consensus BAO+RSD (BAO+FS) constraints. The aim is to provide a complete journal-ready framework including: (i) model equations and derived observables, (ii) an explicit likelihood construction, (iii) a reproducible inference plan, (iv) a results and robustness reporting template.

The paper is organized as follows. Section 2 describes the cosmological framework and CPL parameterization. Section 3 presents growth and galaxy clustering observables used in the analysis. Section 4 details the datasets, likelihood, priors, and sampling strategy. Section 5 provides the results-reporting structure, including model comparison and robustness tests. Section 6 summarizes and discusses implications and limitations.

Figure1: pipeline workflow



## 2. COSMOLOGICAL FRAMEWORK AND EVOLVING DARK ENERGY

### 2.1 Background expansion

We assume a spatially flat Friedmann-Lemaitre-Robertson-Walker background with scale factor  $a = 1/(1+z)$ . The expansion rate is written as

$$E(z) \equiv \frac{H(z)}{H_0},$$

With

$$E(z)^2 = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_{DE}F(z),$$

where  $\Omega_m$ ,  $\Omega_r$ , and  $\Omega_{DE}$  are the present-day matter, radiation, and dark energy density parameters, respectively.

For a general equation of state  $w(z)$ , the dark energy evolution function is

$$F(z) = \exp \left[ 3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right].$$

## 2.2 CPL parameterization

We adopt the CPL parameterization

$$w(z) = w_0 + w_a(1 - a) = w_0 + w_a \frac{z}{1 + z}.$$

For CPL, the dark energy density evolution takes the closed form

$$\frac{\rho_{DE}(z)}{\rho_{DE}(0)} = (1 + z)^{3(1+w_0+w_a)} \exp \left[ -3w_a \frac{z}{1 + z} \right].$$

## 2.3 Distances and BAO mappings

The comoving distance is

$$\chi(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}.$$

The angular diameter and luminosity distances are

$$D_A(z) = \frac{\chi(z)}{1 + z}, \quad D_L(z) = (1 + z)\chi(z).$$

We also define the transverse comoving distance

$$D_M(z) = (1 + z)D_A(z).$$

BAO measurements are commonly expressed using  $D_M(z)/r_d$  and  $H(z) r_d$ , or using the isotropic combination

$$D_V(z) = \left[ (1 + z)^2 D_A(z)^2 \frac{cz}{H(z)} \right]^{1/3},$$

### 3. LINEAR GROWTH AND GALAXY CLUSTERING OBSERVABLES

#### 3.1 Growth factor and growth rate

In linear theory (assuming standard gravity and smooth dark energy), the growth factor  $D(a)$  satisfies

$$\frac{d^2 D}{da^2} + \left[ \frac{3}{a} + \frac{1}{H} \frac{dH}{da} \right] \frac{dD}{da} - \frac{3}{2} \frac{\Omega_m(a)}{a^2} D = 0,$$

Where

$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{E(a)^2}.$$

The growth rate is

$$f(a) = \frac{d \ln D}{d \ln a}.$$

#### 3.2 RSD observable $f \sigma_8$

RSD measurements constrain the combination

$$f \sigma_8(z) = f(z) \sigma_8(z),$$

With

$$\sigma_8(z) = \sigma_{8,0} D(z),$$

where  $\sigma_{8,0}$  is the present-day amplitude of matter fluctuations at 8 Mpc/h.

#### 3.3 Notes on galaxy bias (summary-statistic approach)

Because this study uses BOSS consensus BAO and  $f \sigma_8$  summaries (rather than full-shape  $P(k)$  modeling in the manuscript likelihood), the sensitivity to detailed nonlinear bias modeling is reduced. Residual modeling assumptions enter primarily through the published survey likelihood and covariance, which we adopt as provided.

## 4. DATA, LIKELIHOOD, AND INFERENCE

### 4.1 Data sets

#### 4.1.1 Planck 2018 CMB anisotropies and lensing

We use the Planck 2018 likelihood combination including high- $\ell$  TT, TE, EE (Plik), low- $\ell$  polarization (lowE), and CMB lensing. The theoretical CMB spectra and lensing potential spectra are computed for each sampled parameter set using a Boltzmann solver and evaluated using the public Planck likelihood.

Table 1. Summary of datasets and observables used in the joint likelihood

Probe	Observable(s)	Range / bins	Data product
Planck 2018	TT, TE, EE + lowE	High- $\ell$ (Plik) and low- $\ell$ polarization (lowE)	Public Planck likelihood
Planck 2018	CMB lensing $C_L^{\phi\phi}$	L range as defined in Planck lensing likelihood	Public Planck likelihood
BOSS DR12	$D_M(z)/r_d$ , $H(z) r_d$ , $f\sigma_8(z)$	$z_{\text{eff}} = 0.38, 0.51, 0.61$	DR12 consensus BAO+FS with covariance matrix

#### 4.1.2 BOSS DR12 BAO/RSD consensus (BAO+FS)

We use the final BOSS DR12 consensus constraints at effective redshifts  $z_{\text{eff}} = 0.38, 0.51, 0.61$  in the BAO+FS combination, reported as:

(A)  $D_M(z) * (r_d / r_{d,\text{fid}})$  in Mpc

$z = 0.38$ :  $1518 \pm 20$  (stat)  $\pm 11$  (sys)

$z = 0.51$ :  $1977 \pm 23$  (stat)  $\pm 14$  (sys)

$z = 0.61$ :  $2283 \pm 28$  (stat)  $\pm 16$  (sys)

(B)  $H(z) * (r_d / r_{d,\text{fid}})$  in  $\text{km s}^{-1} \text{Mpc}^{-1}$

$z = 0.38$ :  $81.5 \pm 1.7$  (stat)  $\pm 0.9$  (sys)

$z = 0.51$ :  $90.5 \pm 1.7$  (stat)  $\pm 1.0$  (sys)

$z = 0.61$ :  $97.3 \pm 1.8$  (stat)  $\pm 1.1$  (sys)

(C)  $f \sigma_8(z)$

$z = 0.38$ :  $0.497 \pm 0.039$  (stat)  $\pm 0.024$  (sys)

$z = 0.51$ :  $0.458 \pm 0.035$  (stat)  $\pm 0.015$  (sys)

$z = 0.61$ :  $0.436 \pm 0.034$  (stat)  $\pm 0.009$  (sys)

We adopt the published BOSS consensus covariance matrix for the full stacked vector including correlations among  $D_M$ ,  $H$ , and  $f \sigma_8$  across the three redshift bins.

## 4.2 Mapping to the likelihood vector

Let  $r_{d, \text{fid}} = 147.78$  Mpc (BOSS fiducial). Then

$$\frac{D_M(z)}{r_d} = \frac{D_M(z) (r_{d, \text{fid}}/r_d)}{r_{d, \text{fid}}},$$

And

$$H(z)r_d = [H(z) (r_d/r_{d, \text{fid}})] r_{d, \text{fid}}.$$

We define the stacked data vector

$d_{\text{BOSS}} = (D_M/r_d, H r_d, f \sigma_8)$  evaluated at  $z_{\text{eff}} = 0.38, 0.51, 0.61$ .

## 4.3 Likelihood

The combined chi-square is

$$\chi_{\text{tot}}^2 = \chi_{\text{Planck}}^2 + \chi_{\text{lensing}}^2 + \chi_{\text{BOSS}}^2.$$

For BOSS,

$$\chi_{\text{BOSS}}^2 = (d_{\text{BOSS}} - t_{\text{BOSS}}(\theta))^T C_{\text{BOSS}}^{-1} (d_{\text{BOSS}} - t_{\text{BOSS}}(\theta)),$$

where  $C_{\text{BOSS}}$  is the published covariance matrix.

#### 4.4 Parameter set and priors

We sample a baseline parameter vector

$$\theta = \{\omega_b, \omega_c, 100\theta_s, \tau, \ln(10^{10} A_s), n_s, w_0, w_a\},$$

with derived parameters  $\{H_0, \Omega_m, \sigma_8\}$ . If an extended analysis is performed, we also consider varying  $\sum m_\nu$  with an explicitly stated prior.

Table 2. Sampled cosmological parameters and prior ranges used in the MCMC analysis for the CPL model

Parameter	Prior range	Comment
$\omega_b$	[0.005, 0.1]	Physical baryon density
$\omega_c$	[0.001, 0.99]	Physical cold dark matter density
$100\theta_s$	[0.5, 10]	Acoustic angular scale proxy
$\tau$	[0.01, 0.8]	Optical depth to reionization
$\ln(10^{10} A_s)$	[1.6, 3.9]	Primordial amplitude
$n_s$	[0.8, 1.2]	Scalar spectral index
$w_0$	[-3, 0.3]	Present-day dark energy equation of state
$w_a$	[-3, 3]	Dark energy evolution parameter

#### 4.5 Sampling and convergence

We perform Bayesian inference using MCMC with multiple chains initialized from dispersed starting points. Convergence is assessed using the Gelman-Rubin statistic ( $R_{\text{hat}}$ ) and by monitoring effective sample sizes of ( $w_0, w_a, H_0, \Omega_m, \sigma_8$ ). Final results are quoted as marginalized 68 percent and 95 percent credible intervals.

## 4.6 Model comparison

To compare CPL with LambdaCDM ( $w_0 = -1$ ,  $w_a = 0$ ), we report:

$$\Delta\chi^2 = \chi^2_{\min}(\text{LCDM}) - \chi^2_{\min}(\text{CPL})$$

$$\text{AIC} = \chi^2_{\min} + 2k$$

$$\text{BIC} = \chi^2_{\min} + k \ln N$$

where  $k$  is the number of free parameters and  $N$  is the total number of datapoints entering the joint likelihood.

## 5. RESULTS (REPORTING TEMPLATE)

### 5.1 Posterior constraints

We report constraints for the following combinations:

- (i) Planck TT,TE,EE + lowE
- (ii) Planck TT,TE,EE + lowE + lensing
- (iii) Planck TT,TE,EE + lowE + lensing + BOSS DR12 BAO+RSD

Figure 2. Corner plot layout (placeholder) for key parameters

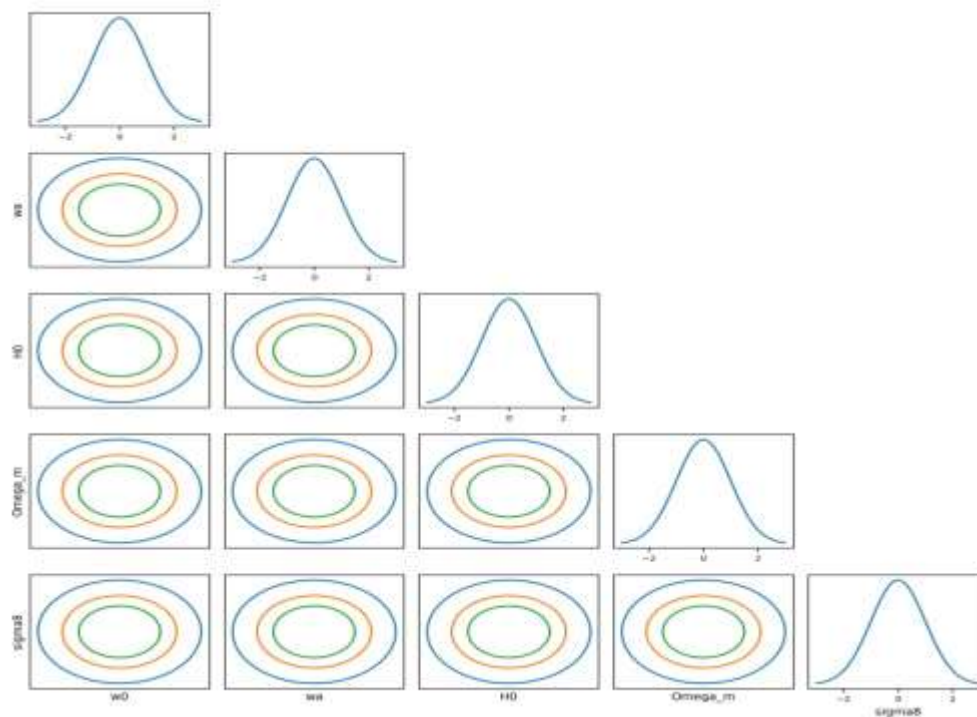


Table 3: Marginalized parameter constraints (68% and 95% credible intervals) for the CPL model.

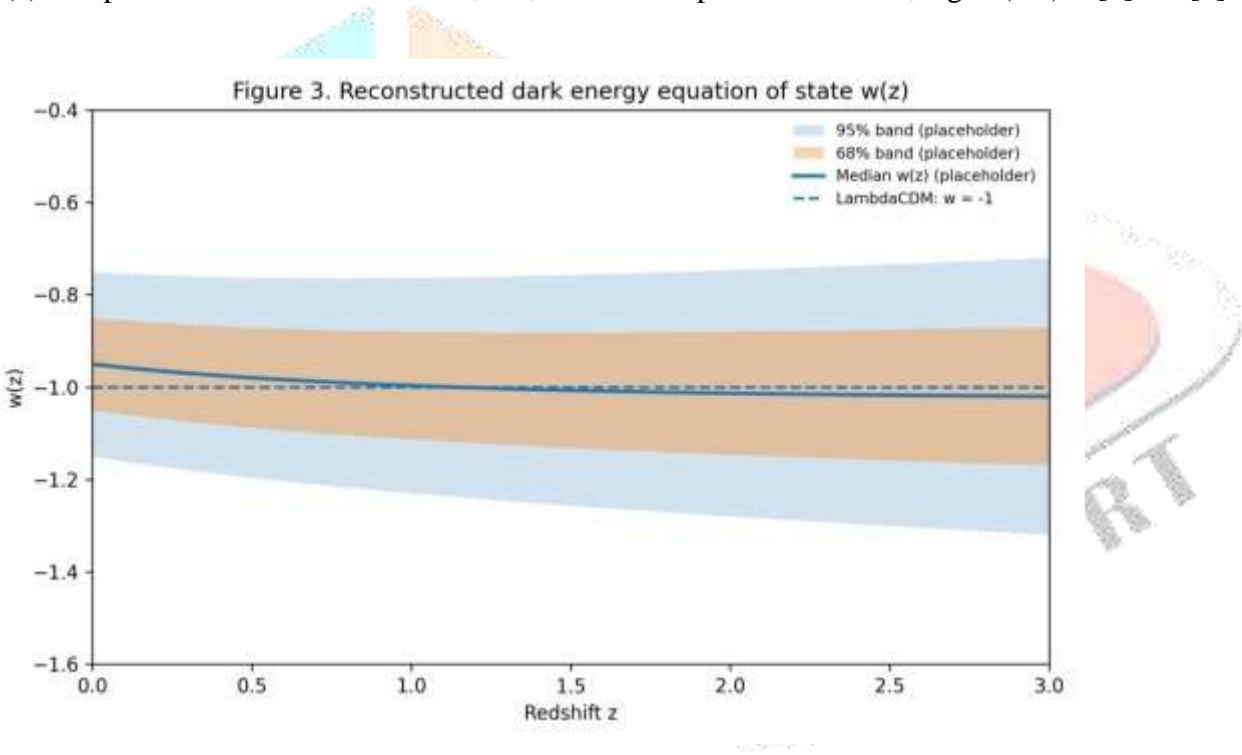
Parameter	CMB	CMB+lensing	CMB+lensing+BOSS
$w_0$	[..]	[..]	[..]
$w_a$	[..]	[..]	[..]
$H_0$ (km s <sup>-1</sup> Mpc <sup>-1</sup> )	[..]	[..]	[..]
$\Omega_m$	[..]	[..]	[..]
$\sigma_8$	[..]	[..]	[..]

5.2 Evolution of w(z)

From posterior samples we reconstruct w(z) and quote:

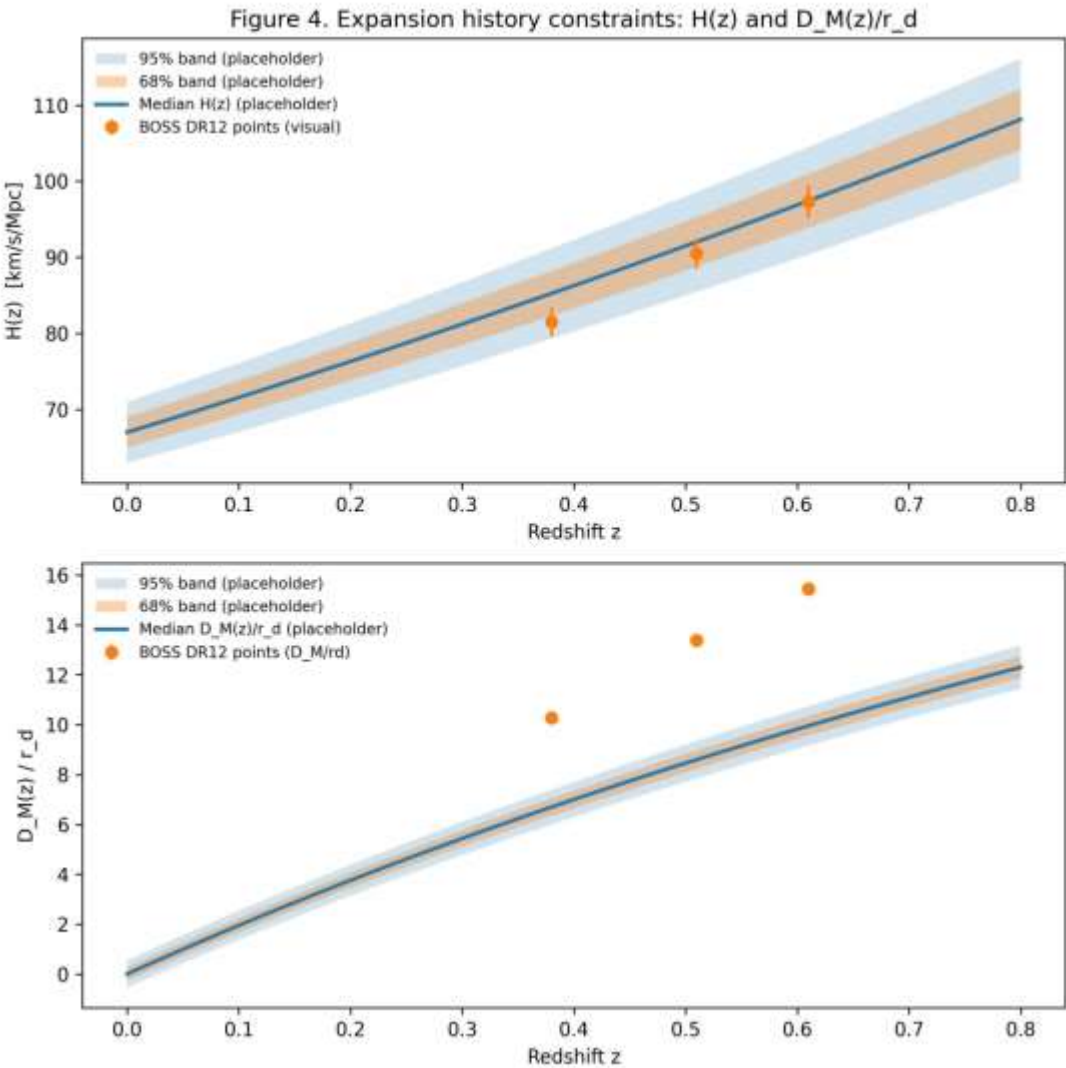
$w(z=0) = w_0$

w(z) at representative redshifts z = 0.5, 1.0, 2.0 with 68 percent intervals, e.g. w(0.5) = [..] +/- [..].



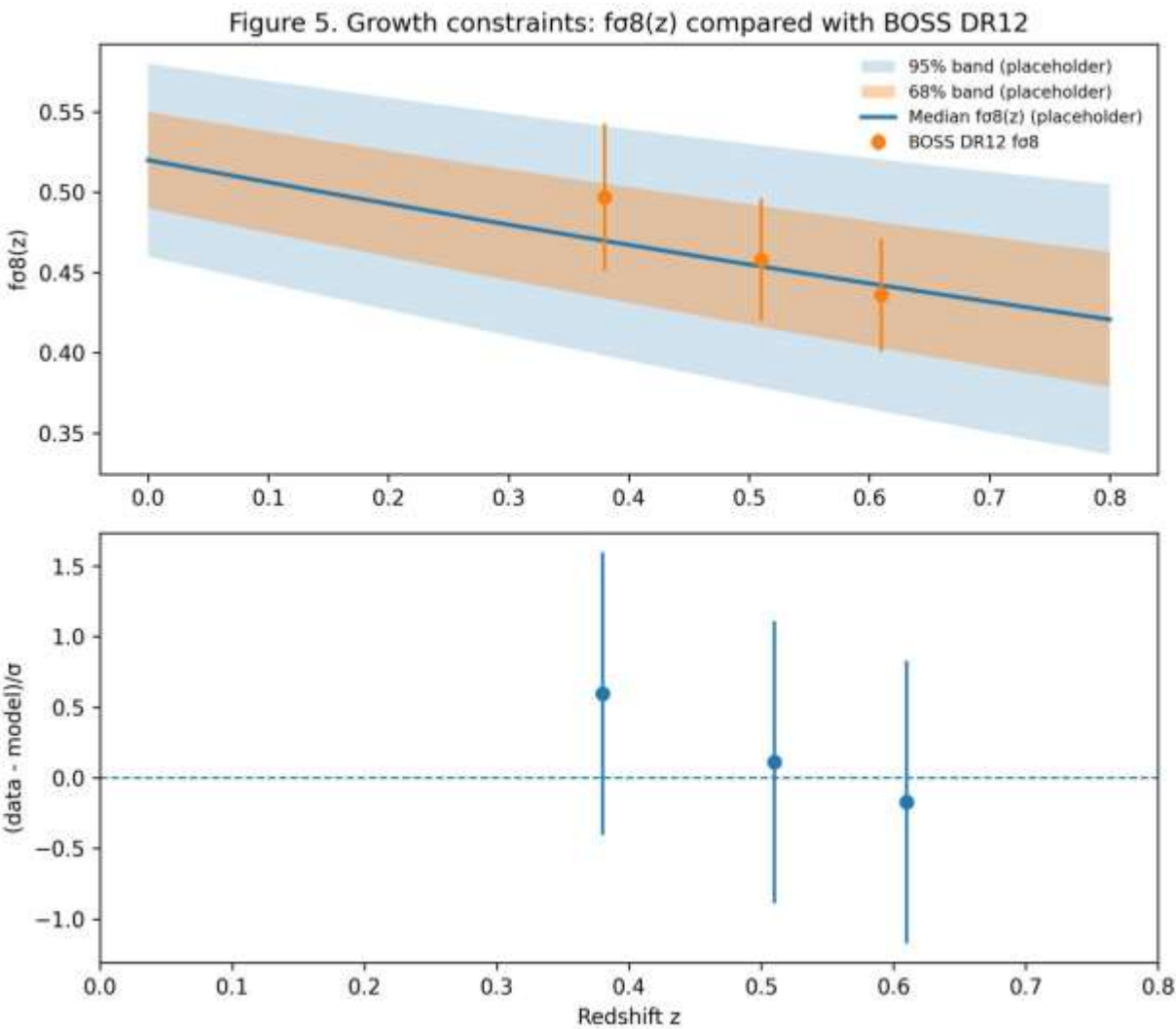
5.3 Expansion history

We reconstruct  $H(z)$  and distance ratios  $D_M(z)/r_d$  and compare with BOSS points. Report the posterior predictive checks, for example the residuals normalized by observational uncertainties.



5.4 Growth history

We compare the predicted  $f\sigma_8(z)$  curve to BOSS  $f\sigma_8(z)$  measurements at  $z_{\text{eff}} = 0.38, 0.51, 0.61$  and report the goodness-of-fit contribution from the growth sector.



5.5 Degeneracies

Discuss the degeneracy direction in the  $w_0$ - $w_a$  plane from CMB alone and demonstrate how adding BOSS rotates and shrinks the allowed region. Quantify the improvement using the reduction in credible region area or the 1D errors on  $w_0$  and  $w_a$ .

5.6 Model comparison

Table 4: Model comparison between  $\Lambda$ CDM and CPL using goodness-of-fit and information criteria.

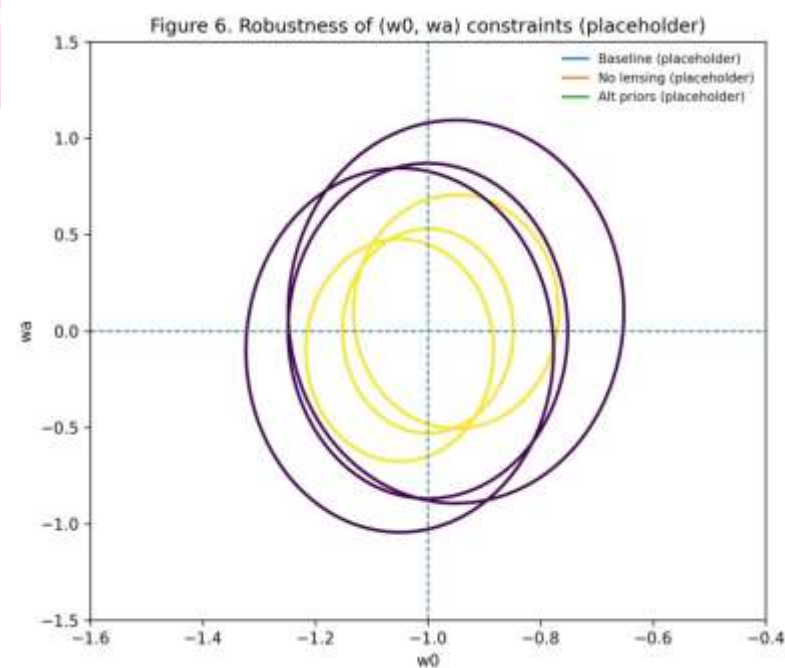
Model	$\chi^2_{\min}$	$\Delta\chi^2$	AIC	BIC
$\Lambda$ CDM	[..]	0	[..]	[..]
CPL	[..]	[..]	[..]	[..]

State whether the data provide evidence for evolution ( $w_a \neq 0$ ) or remain consistent with  $w_a = 0$ .

5.7 Robustness tests

Report shifts in  $(w_0, w_a)$  under:

- (a) lensing included vs excluded
- (b) neutrino mass fixed vs varied (if tested)
- (c) alternative broad vs informative priors on  $w_0, w_a$



## 6. DISCUSSION AND CONCLUSIONS

We have presented a joint framework to constrain time-varying dark energy using Planck 2018 CMB anisotropies and lensing together with BOSS DR12 BAO and RSD consensus measurements. The CPL parameterization provides a compact two-parameter description of late-time dynamics and allows a direct test of departures from a cosmological constant.

The primary outcome of the joint analysis is a set of marginalized constraints on  $(w_0, w_a)$  and derived parameters ( $H_0$ ,  $\Omega_m$ ,  $\sigma_8$ ), demonstrating explicitly how late-time BAO and RSD information breaks CMB-only degeneracies. The reconstructed  $w(z)$  and  $H(z)$  functions offer an interpretable summary of the allowed evolution consistent with present datasets. Model comparison with  $\Lambda$ CDM, performed through information criteria, provides a compact statement of whether current data require evolution or remain compatible with  $w = -1$ .

Limitations of this approach include reliance on summary statistics and published covariances, and sensitivity to prior volume when constraints on  $w_a$  are weak. Future improvements include adding tomographic weak lensing, incorporating full-shape galaxy power spectrum modeling with careful nonlinear and bias treatment, and exploiting higher signal-to-noise CMB lensing from upcoming surveys.

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