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Intuitionistic Fuzzy Distributive Near Ring groups

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Abstract: Hadi and Semeein [3] introduced and investigated fuzzy distributive modules as an expanded idea of distributive modules. They established the result between the fuzzy distributive module and the distributive of fuzzy singleton. Sharma [9] has studied the notions of intuitionistic fuzzy modules in many ways. Introducing intuitionistic fuzzy submodules of a module, Sharma proved that the intersection of two fuzzy submodules of a module is also a fuzzy submodule. Davvaz, Wieslaw and Jun [1] presented intuitionistic fuzzy submodules of modules and utilizing this notion, numerous scholars such as Isaac [4], Rahman [6], and Sharma [10] investigated it in many ways. They derived different operations on intuitionistic fuzzy sets, intuitionistic (T, S)-fuzzy submodule of a module, intuitionistic fuzzy H_v-submodules of a module and the relationship between fuzzy submodule, and intuitionistic fuzzy submodule of a module. Devi [2] developed the concept of intuitionistic fuzzy N-subgroups and ideals of N-group and discussed the association between intuitionistic fuzzy ideals and fuzzy ideals, which is being investigated by several scholars in various fields.

In this paper, the concept of DN-groups extended to intuitionistic fuzzy and intuitionistic fuzzy weak DN-groups and uniserial N-groups to intuitionistic fuzzy uniserial N-groups.The core concepts employed in this chapter are found in [5, 7, 8]. Here, in general, $\gamma, \lambda \in [0,1]$ with $\gamma + \lambda \leq 1$.

Introducing the notion of IF DN-groups, the associated outcomes of the (γ, λ)-cut of intuitionistic fuzzy sets, intuitionistic fuzzy DN-groups and intuitionistic fuzzy weak DN-groups are examined. Finally, the correlations among the intuitionistic fuzzy concepts of uniserial N-groups, DN-groups and weak DN-groups are derived.

Keywords: Intuitionistic fuzzy(IF) sets, DN-groups, weak DN-groups.

1 Intuitonistic fuzzy DN-groups

Definition 1.1 [7] Let N be a near-ring. Then an additive group $(E, +)$ is referred to as a left N-group if $\exists$ a map $N \times E \rightarrow E$ such that $(n, u) \rightarrow nu$ in which the following conditions hold-

i. $(m + n)u = mu + nu$.

ii. $(mn)u = m(nu), \forall m, n \in N, u \in E$.

It is to be noted that N is itself an N-group over itself. If for $1 \in N$ such that $1. u = u \; \forall \; u \in E$, then E is called an unitary N-group.

Throughout the work, the near-ring N is considered a zero symmetric right near-ring and E is a unitary left N-group.

Definition 1.2 [7] In the event that A is a subgroup of $(E, +)$ and $NA \subseteq A$ for any $A \subseteq E$, then A is referred to as an N-subgroup.

Definition 1.3 When $x \in E$, Nx is referred to as the principal N-subgroup of E.

Definition 1.4 The object $A = < \phi_A, \psi_A > = \{< s, \phi_A(s), \psi_A(s) > | s \in S\}$ is referred to as an intuitionistic fuzzy (IF) set on a non empty set S, where ϕ_A and ψ_A are fuzzy subsets of S such that $0 \leq \phi_A(s) + \psi_A(s) \leq 1$.

Definition 1.5 An IF set $A = < \phi_A, \psi_A >$ in E is called IF an N-subgroup of $E (A \leq_{IFN} E)$ if
i. $\phi_A(p - m) \geq \phi_A(p) \wedge \phi_A(m)$.
ii. $\phi_A(np) \geq \phi_A(p)$.
iii. $\psi_A(p - m) \leq \psi_A(p) \vee \psi_A(m)$.
iv. $\psi_A(np) \leq \psi_A(p) \ \forall \ p, m \in E, n \in N$.

Definition 1.6 [11] Let $P = < \phi_P, \psi_P >$ be an IF sets in E. Then (γ, λ)-cut of P is referred by-
${}^{(\gamma,\lambda)}P = \{m \in E: \phi_P(m) \geq \gamma, \psi_P(m) \leq \lambda\}$.

Definition 1.7 E is called an Intuitionistic fuzzy distributive near ring groups (IF DN-group) if $(P + H) \cap C = (P \cap C) + (H \cap C)$, $\forall$ IF N-subgroups P, H, C of E.

Definition 1.8 Let $B = < \phi_B, \psi_B >$ be an IF N-subgroup, then ${}^{(\gamma,\lambda)}B$ is called an N-subgroup of ${}^{(\gamma,\lambda)}E$ if $s - y, ns \in^{(\gamma,\lambda)} B$, for any $s, y \in^{(\gamma,\lambda)} B$ and $n \in N$.

Definition 1.9 Let $B = < \phi_B, \psi_B >$ be an IF N-subgroup, then ${}^{(\gamma,\lambda)}B$ is said to be an ideal of ${}^{(\gamma,\lambda)}E$ if $p - q, m + p - m \in E$ and $n(p + m) - nm \in^{(\gamma,\lambda)} B$, for any $p, q \in^{(\gamma,\lambda)} B, m \in E, n \in N$.

Lemma 1.1 Let E be a commutative N-group and every principal N-subgroup is an ideal, then the sum of two subgroups of ${}^{(\gamma,\lambda)}E$ is also a subgroup.

Proof. Let ${}^{(\gamma,\lambda)}P$ and ${}^{(\gamma,\lambda)}B$ be subgroups of ${}^{(\gamma,\lambda)}E$.
Let $p, q \in^{(\gamma,\lambda)} P +^{(\gamma,\lambda)} B$.
Then $p = s_1 + y_1, q = s_2 + y_2$, for some $s_1, s_2 \in^{(\gamma,\lambda)} P, y_1, y_2 \in^{(\gamma,\lambda)} B$.
Since E is commutative and ${}^{(\gamma,\lambda)}P, {}^{(\gamma,\lambda)}B \subseteq E$,
$p - q = (s_1 - s_2) + (y_1 - y_2) \in^{(\gamma,\lambda)} P +^{(\gamma,\lambda)} B$.
Since every principal N-subgroup is an ideal,
$n(s_1 + y_1) - ny_1 \in Ns_1$, for $n \in N$.
Also $ny_1 \in Ny_1$.
So $n(s_1 + y_1) - ny_1 + ny_1 \in Ns_1 + Ny_1$.
$\Rightarrow np = n(s_1 + y_1) \in Ns_1 + Ny_1 \in N^{(\gamma,\lambda)}P + N^{(\gamma,\lambda)}B \subseteq^{(\gamma,\lambda)} P +^{(\gamma,\lambda)} B$.
Thus the result.

Proposition 1.1 If $P = < \phi_P, \psi_P >$ and $B = < \phi_B, \psi_B >$ are IF N-subgroups of E, then
i. ${}^{(\gamma,\lambda)}(P + B) =^{(\gamma,\lambda)} P +^{(\gamma,\lambda)} B$ with $\gamma + \lambda = 1$.
ii. ${}^{(\gamma,\lambda)}(P \cap B) =^{(\gamma,\lambda)} P \cap^{(\gamma,\lambda)} B$.

Proof. (i). Let $s \in^{(\gamma,\lambda)} (P + B)$.
Then $\phi_{P+B}(s) \geq \gamma$ and $\psi_{P+B}(s) \leq \lambda$.
Now, $\phi_{P+B}(s) \geq \gamma$
$\Rightarrow \vee \{\phi_P(r) \wedge \phi_B(t): r, t \in E, s = r + t\} \geq \gamma$
$\Rightarrow \phi_P(r_1) \wedge \phi_B(t_1) \geq \gamma$, for some $r, t \in E$ such that $s = r_1 + t_1$.
Since $\phi_P(r_1), \phi_B(t_1) \geq \phi_P(r_1) \wedge \phi_B(t_1) \geq \gamma$
$\Rightarrow \phi_P(r_1) \geq \gamma$ and $\phi_B(t_1) \geq \gamma$, for some $r_1, t_1 \in E$ such that $s = r_1 + t_1$
$\Rightarrow \psi_P(r_1) \leq 1 - \phi_P(r_1) = 1 - \gamma = \lambda$ and $\psi_B(t_1) \leq 1 - \phi_B(t_1) = 1 - \gamma = \lambda$ [Since $\gamma + \lambda = 1$]
$\Rightarrow r_1 \in^{(\gamma,\lambda)} P$ and $t_1 \in^{(\gamma,\lambda)} B$
$\Rightarrow s = r_1 + t_1 \in^{(\gamma,\lambda)} P +^{(\gamma,\lambda)} B$.

Converselr, let $s \in^{(\gamma,\lambda)} P +^{(\gamma,\lambda)} B$.
Then $s = r + t$, for some $r \in^{(\gamma,\lambda)} P, t \in^{(\gamma,\lambda)} B$
$\Rightarrow \phi_P(r) \geq \gamma, \psi_P(r) \leq \lambda, \phi_B(r) \geq \gamma, \psi_B(r) \leq \lambda$
$\Rightarrow \phi_P(r) \wedge \phi_B(r) \geq \gamma$ and $\psi_P(r) \vee \psi_B(r) \leq \lambda$
$\Rightarrow \vee \{\phi_P(r) \wedge \phi_B(r) \geq \gamma : s = r + t\}$ and $\wedge \{\psi_P(r) \vee \psi_B(r) \leq \lambda : s = r + t\}$
$\Rightarrow s \in^{(\gamma,\lambda)} (P + B)$.
Thus the result.
(ii). Let $s \in^{(\gamma,\lambda)} (P \cap B)$
$\Rightarrow \phi_{P\cap B}(s) \geq \gamma$ and $\psi_{P\cap B}(s) \leq \lambda$.
But $P \cap B \subseteq P, B$.
So $\phi_P(s), \phi_B(s) \geq \phi_{P\cap B}(s) \geq \gamma$ and $\psi_P(s), \psi_B(s) \leq \psi_{P\cap B}(s) \leq \lambda$.
Thus, $s \in^{(\gamma,\lambda)} P$ and $s \in^{(\gamma,\lambda)} B$
$\Rightarrow s \in^{(\gamma,\lambda)} P \cap^{(\gamma,\lambda)} B$.
Conversely, let $s \in^{(\gamma,\lambda)} P \cap^{(\gamma,\lambda)} B$
$\Rightarrow s \in^{(\gamma,\lambda)} P$ and $s \in^{(\gamma,\lambda)} B$
$\Rightarrow \phi_P(s) \geq \gamma, \psi_P(s) \leq \lambda$ and $\phi_B(s) \geq \gamma, \psi_B(s) \leq \lambda$
$\Rightarrow \phi_P(s) \wedge \phi_B(s) \geq \gamma$ and $\psi_P(s) \vee \psi_B(s) \leq \lambda$
$\Rightarrow \phi_{P\cap B}(s) \geq \gamma$ and $\psi_{P\cap B}(s) \leq \lambda$
$\Rightarrow s \in^{(\gamma,\lambda)} (P \cap B)$.
Thus the result.

Definition 1.10 If $P = < \phi_P, \psi_P >$ and $B = < \phi_B, \psi_B >$ are IF N-subgroups of E, then ${}^{(\gamma,\lambda)}P =^{(\gamma,\lambda)} B$ if and only if $\phi_P(s) = \phi_B(s) \geq \gamma$ and $\psi_P(s) = \psi_B(s) \leq \lambda$, $\forall\, s \in E$.

Lemma 1.2 If $P = < \phi_P, \psi_P >$ and $M = < \phi_M, \psi_M >$ are IF N-subgroups of E, then $P = M$ if and only if ${}^{(\gamma,\lambda)}P =^{(\gamma,\lambda)} M$.

Proof. It is clear from the definition.

Proposition 1.2 If $L = < \phi_L, \psi_L >$ is an IF N-subgroup of E, then ${}^{(\gamma,\lambda)}L$ is also an N-subgroup of ${}^{(\gamma,\lambda)}E$.

Proof. By definition, ${}^{(\gamma,\lambda)}L$ is a subset of ${}^{(\gamma,\lambda)}E$.
For any $n \in N$ $s, y \in^{(\gamma,\lambda)} L$,
$\phi_L(s), \phi_L(y) \geq \gamma, \psi_L(s), \psi_L(y) \leq \lambda$.
Therefore, $\phi_L(ns) \geq \phi_L(s) \geq \gamma$ and $\psi_L(ns) \leq \psi_L(s) \leq \lambda$[since L is an IF N-subgroup].
Also, $ns \in E$.
Therefore $ns \in^{(\gamma,\lambda)} L$.
Again, $s - y \in E$ such that $\phi_L(s-y) \geq \phi_L(s) \wedge \phi_L(y) \geq \gamma \wedge \gamma = \gamma$ and $\psi_L(s-y) \leq \psi_L(s) \vee \psi_L(y) \leq \lambda \vee \lambda = \lambda$.
So $s - y \in^{(\gamma,\lambda)} L$.
This shows that ${}^{(\gamma,\lambda)}L$ is an N-subgroup of ${}^{(\gamma,\lambda)}E$.

Theorem 1.1 E is an IF DN-group if and only if ${}^{(\gamma,\lambda)}E$ is a DN-group with $\gamma, \lambda \in (0,1], \gamma + \lambda = 1$.

Proof. Let E be an IF DN-group and X, I, L be N-subgroups of ${}^{(\gamma,\lambda)}E$ such that

$$\phi_P(h) = \begin{cases} 1, & h \in X \\ 0, & h \notin X, \end{cases} \psi_P(h) = \begin{cases} 0, & h \in X \\ 1, & h \notin X, \end{cases}$$

$$\phi_M(h) = \begin{cases} 1, & h \in I \\ 0, & h \notin I, \end{cases} \psi_M(h) = \begin{cases} 0, & h \in I \\ 1, & h \notin I, \end{cases}$$

$\phi_T(h) = \begin{cases} 1, & h \in L \\ 0, & h \notin L \end{cases}, \psi_T(h) = \begin{cases} 0, & h \in L \\ 1, & h \notin L \end{cases}$

where $\gamma, \lambda \in (0,1], \gamma + \lambda = 1$.

Now, if $s \in^{(\gamma,\lambda)} P$, then either $s \in X$ or $s \notin X$.

If $s \notin X$, then $\phi_P(s) = 0 \geq \gamma$ and $\psi_P(s) = 1 \leq \lambda$[since $\phi_P(s) \geq \gamma, \psi_P(s) \leq \lambda$]- which is a contradiction as $\gamma, \lambda \in (0,1]$.

Thus ${}^{(\gamma,\lambda)}P = X$.

Similarly ${}^{(\gamma,\lambda)}M = I, {}^{(\gamma,\lambda)}T = L$.

Claim P, M, T are IF N-subgroups of E.

Since X is a subgroup of $(E, +)$, for any $s, y \in X, n \in N$,

$s - y \in X$ and $ns \in X$

$\Rightarrow \phi_P(s - y) = 1 = \phi_P(s) \wedge \phi_P(y)$ and $\phi_P(ns) = 1 = \phi_P(s)$ [since $s, y \in X, \phi_P(s) = 1, \phi_P(y) = 1$] .

Similarly, $\psi_P(s - y) = \psi_P(s) \vee \psi_P(y)$ and $\psi_P(ns) = \psi_P(s)$.

This shows that P ia an IF N-subgroup.

Similarly, M and T are IF N-subgroups of E.

Since E is an IF DN-subgroup, therefore

$(P + M) \cap T = (P \cap T) + (M \cap T)$

$\Rightarrow^{(\gamma,\lambda)} [(P + M) \cap T] =^{(\gamma,\lambda)} [(P \cap T) + (M \cap T)]$ [using **lemma 1.2**]

$\Rightarrow^{(\gamma,\lambda)} (P + M) \cap^{(\gamma,\lambda)} T =^{(\gamma,\lambda)} (P \cap T) +^{(\gamma,\lambda)} (M \cap T)$[using **proposition 1.1**]

$\Rightarrow ({}^{(\gamma,\lambda)}P +^{(\gamma,\lambda)} M) \cap^{(\gamma,\lambda)} T = ({}^{(\gamma,\lambda)}P \cap^{(\gamma,\lambda)} T) + ({}^{(\gamma,\lambda)}M \cap^{(\gamma,\lambda)} T)$[using **proposition 1.1**]

$\Rightarrow (X + I) \cap L = (X \cap L) + (I \cap L)$.

Thus, ${}^{(\gamma,\lambda)}E$ is a DN-group.

Conversely, suppose ${}^{(\gamma,\lambda)}E$ is a DN-group.

Let P, M, T are IF N-subgroups of E.

Then by **proposition 1.2**, ${}^{(\gamma,\lambda)}P, {}^{(\gamma,\lambda)}M, {}^{(\gamma,\lambda)}T$ are N-subgroups of ${}^{(\gamma,\lambda)}E$.

Since ${}^{(\gamma,\lambda)}E$ is a DN-group,

$({}^{(\gamma,\lambda)}P +^{(\gamma,\lambda)} M) \cap^{(\gamma,\lambda)} T = ({}^{(\gamma,\lambda)}P \cap^{(\gamma,\lambda)} T) + ({}^{(\gamma,\lambda)}M \cap^{(\gamma,\lambda)} T)$

$\Rightarrow^{(\gamma,\lambda)} [(P + M) \cap T] =^{(\gamma,\lambda)} [(P \cap T) + (M \cap T)]$ [using **proposition 1.1**]

$\Rightarrow (P + M) \cap T = (P \cap T) + (M \cap T)$[using **lemma 1.2**] .

Thus E is an IF DN-group.

Definition 1.11 E is called an IF uniserial N-group if any two of its IF N-subgroups are comparable to each other.

Theorem 1.2 An IF uniserial N-group is an IF DN-group.

Proof. Let E be an IF uniserial N-group.

Let $P =< \phi_P, \psi_P >, B =< \phi_B, \psi_B >, T =< \phi_T, \psi_T >$ be IF N-subgroups of E.

Since E is an IF uniserial N-group, any two of its IF N-subgroups are comparable to each other.

So, it may assume $P \subseteq B \subseteq T$.

Thus, $\phi_P \leq \phi_B \leq \phi_T$ and $\psi_P \geq \psi_B \geq \psi_T$.

Therefore $P + B =< \phi_{P+B}, \psi_{P+B} >$, where

$\phi_{P+B}(s) = \vee \{\phi_P(a) \wedge \phi_B(q): a, q \in E, s = a + q\}$ and $\psi_{P+B}(s) = \wedge \{\psi_P(a) \vee \psi_B(q): a, q \in E, s = a + q\}$, $\forall s \in E$.

So $(P + B) \cap T =< \phi_{P+B} \wedge \phi_T, \psi_{P+B} \vee \psi_T >$.

Now, $P \cap T =< \phi_P \wedge \phi_T, \psi_P \vee \psi_T >=< \phi_P, \psi_P >= P$ and $B \cap T =< \phi_B \wedge \phi_T, \psi_B \vee \psi_T >=< \phi_B, \psi_B >= B$.

Therefore, $(P \cap T) + (B \cap T) = P + B$.

Again, for any $s, a, q \in E$,

$\phi_{P+B}(s) = \vee \{\phi_P(a) \wedge \phi_B(q): s = a + q\}$

$\leq \vee \{\phi_T(a) \wedge \phi_T(q): s = a + q\}$

$\leq \vee \{\phi_T(a + q): s = a + q\}$[Since T is an IF N-subgroup]

$= \phi_T(s)$.
Therefore $\phi_{P+B} \wedge \phi_T = \phi_{P+B}$.
Similarly $\psi_{P+B} \vee \psi_T = \psi_{P+B}$.
Thus, $(P + B) \cap T = P + B$.
Hence the result.

Definition 1.12 $Ann(E) = \{n \in N: ns = 0, \forall s \in E\}$.

Lemma 1.3 If $E = E_1 + E_2$ is commutative with E_1, E_2 being unitary DN-groups such that $Ann(E_1) + Ann(E_2) = N$, then $E = E_1 + E_2$ is a DN-group.

Proof. Let P, M, T be N-subgroups of $E = E_1 + E_2$.
Then $P = P_1 + P_2, M = M_1 + M_2, T = T_1 + T_2$, for some subsets P_1, M_1, T_1 of E_1 and P_2, M_2, T_2 of E_2.
Since $Ann(E_1) + Ann(E_2) = N$ and $1 \in N$, then $\exists\ n_1 \in Ann(E_1), n_2 \in Ann(E_2)$ such that $n_1 + n_2 = 1$.
Now, for any $a_1 \in P_1$,
$1. a_1 = (n_1 + n_2)a_1$
$\Rightarrow a_1 = n_1a_1 + n_2a_1$[since E_1 is unitary] .
But, $a_1 \in P_1 \subseteq E_1$
$\Rightarrow a_1 \in E_1$
$\Rightarrow n_1a_1 = 0$[since $n_1 \in Ann(E_1)$] .
Therefore $n_2a_1 = a_1 \in P_1$.
But $n_2a_1 \in NP_1$.
Therefore $NP_1 \subseteq P_1$.
This shows that P_1 is an N-subgroup of E_1.
Similarly, it can be shown that M_1, T_1 are N-subgroups of E_1 and P_2, M_2, T_2 are N-subgroups of E_2.
Now, $(P + M) \cap T = [(P_1 + P_2) + (M_1 + M_2)] \cap (T_1 + T_2) = [(P_1 + M_1) \cap T_1] + [(P_2 + M_2)] \cap T_2] = [(P_1 \cap T_1) + (M_1 \cap T_1)] + [(P_2 \cap T_2) + (M_2 \cap T_2)]$[since E_1, E_2 are DN-groups]
$= [(P_1 + P_2) \cap (T_1 + T_2)] + [(M_1 + M_2) \cap (T_1 + T_2)] = (P \cap T) + (M \cap T)$.
This shows that $E = E_1 + E_2$ is a DN-group.

Theorem 1.3 Let E, K be unitary IF DN-subgroups and $Ann(E) + Ann(K) = N$ with ${}^{(\gamma,\lambda)}E = E, {}^{(\gamma,\lambda)}K = K$, then $E + K$ is an DN-group with $\gamma, \lambda \in (0,1], \gamma + \lambda = 1$.

Proof. Since E, K are unitary IF DN-groups and $Ann(E) + Ann(K) = N$, by **theorem 1.1**, ${}^{(\gamma,\lambda)}E, {}^{(\gamma,\lambda)}K$ are also DN-groups with $\gamma + \lambda = 1$ and $\gamma, \lambda \in (0,1]$.
Since E, K are unitary, ${}^{(\gamma,\lambda)}E, {}^{(\gamma,\lambda)}K$ are also unitary.
Also, by hypothesis $Ann({}^{(\gamma,\lambda)}E + Ann({}^{(\gamma,\lambda)}K = N$.
Therefore, By **lemma 1.3**, ${}^{(\gamma,\lambda)}E + {}^{(\gamma,\lambda)}K$ is a DN-group with $\gamma, \lambda \in (0,1], \gamma + \lambda = 1$.
So, $E + K$ is an DN-group with $\gamma, \lambda \in (0,1], \gamma + \lambda = 1$.

2 Intuitionistic fuzzy weak DN-groups

Definition 2.1 E is said to be a weak DN-group if and only if $(P + M) \cap T = (P \cap T) + (M \cap T)$, for all ideals P, M, T of E.

Definition 2.2 E is referred to as an IF weak DN-group if and only if $(P + M) \cap T = (P \cap T) + (M \cap T)$, for all IF ideals P, M, T of E.

Definition 2.3 ${}^{(\gamma,\lambda)}E$ is said to be a weak DN-group if and only if $(P + M) \cap T = (P \cap T) + (M \cap T)$, for all ideals P, M, T of ${}^{(\gamma,\lambda)}E$.

Theorem 2.1 If E is an IF weak DN-group, then ${}^{(\gamma,\lambda)}E$ is also a weak DN-group with $\gamma, \lambda \in (0,1], \gamma + \lambda \leq 1$.

Proof. Let X, I, L be ideals of ${}^{(\gamma,\lambda)}E$ and defined as in **theorem 1.1.**
Then as above, ${}^{(\gamma,\lambda)}P = X, {}^{(\gamma,\lambda)}M = I, {}^{(\gamma,\lambda)}T = L$ with $\gamma, \lambda \in (0,1], \gamma + \lambda = 1$.
Claim P, M, T are IF ideals of E.
Since X is an ideal of ${}^{(\gamma,\lambda)}E$, therefore X is normal subgroup of $(E, +)$ and $n(s + e) - ne \in X, \forall\, n \in N, s \in X, e \in E$
$\Rightarrow \phi_P\{n(s + e) - ne\} = 1 = \phi_P(s)$[since $s \in X$] .
Also, since X is normal subgroup of $(E, +)$, $s - a, s + a \in X, \forall\, s, a \in X$.
$\Rightarrow \phi_P(s - a) = 1, \phi_P(s + a) = 1$
$\Rightarrow \phi_P(s - a) = 1 \wedge 1 = \phi_P(s) \wedge \phi_P(a)$ and $\phi_P(s + a) = 1 = \phi_P(a + s)$[since $s, a \in X$].
Again, $n \in N, s \in X$,
$ns \in X$
$\Rightarrow \phi_P(ns) = 1 = \phi_P(s)$.
Now $s, a \in X \Rightarrow a + s - a \in X \Rightarrow \phi_P(a + s - a) = 1 = \phi_P(s)$.
Similarly, it can be shown that
$\psi_P(s - a) = \psi_P(s) \vee \psi(a), \psi_P(ns) = \psi_P(s), \psi_P(a + s - a) = \psi_P(s), \psi_P(n(s + a) - ns) = \psi_P(a)$.
This shows that P is an IF ideal of E.
Similarly, M, T are IF ideals of E.
Since E is an IF weak DN-group,
$(P + M) \cap T = (P \cap T) + (M \cap T)$
$\Rightarrow {}^{(\gamma,\lambda)}[(P + M) \cap T] = {}^{(\gamma,\lambda)}[(P \cap T) + (M \cap T)]$ [using **lemma 1.2**]
$\Rightarrow {}^{(\gamma,\lambda)}(P + M) \cap {}^{(\gamma,\lambda)}T = {}^{(\gamma,\lambda)}(P \cap T) + {}^{(\gamma,\lambda)}(M \cap T)$[using **proposition 1.1**]
$\Rightarrow ({}^{(\gamma,\lambda)}P + {}^{(\gamma,\lambda)}M) \cap {}^{(\gamma,\lambda)}T = ({}^{(\gamma,\lambda)}P \cap {}^{(\gamma,\lambda)}T) + ({}^{(\gamma,\lambda)}M \cap {}^{(\gamma,\lambda)}T)$[using **proposition 1.1**]
$\Rightarrow (X + I) \cap L = (X \cap L) + (I \cap L)$.
Thus, ${}^{(\gamma,\lambda)}E$ is a weak DN-group with $\gamma, \lambda \in (0,1], \gamma + \lambda \leq 1$.

Lemma 2.1 If $P =< \phi_P, \psi_P >$ is an IF ideal of E , then ${}^{(\gamma,\lambda)}P$ is also an ideal of ${}^{(\gamma,\lambda)}E$.

Proof. Let $P =< \phi_P, \psi_P >$ be an IF ideal of E.
So ${}^{(\gamma,\lambda)}P \subseteq E$.
Let $s, a \in {}^{(\gamma,\lambda)}P$.
Then $\phi_P(s) \geq \gamma, \psi_P(s) \leq \lambda, \phi_P(a) \geq \gamma, \psi_P(a) \leq \lambda$.
Since P is an IF ideal of E and $s, a \in E$, therefore
$\phi_P(s - a) \geq \phi_P(s) \wedge \phi_P(a) \geq \gamma \wedge \gamma = \gamma$ and $\psi_P(s - a) \leq \psi_P(s) \vee \psi_P(a) \leq \lambda \vee \lambda = \lambda$.
Therefore $s - a \in {}^{(\gamma,\lambda)}P$.
Also, if $a \in E$ and $s \in {}^{(\gamma,\lambda)}P$, then $s, a \in E$.
Therefore, $\phi_P(a + s - a) \geq \phi_P(s) \geq \gamma$ and $\psi_P(a + s - a) \leq \psi_P(s) \leq \lambda$.
So $a + s - a \in {}^{(\gamma,\lambda)}P$.
Again, if $a \in E$, $s \in {}^{(\gamma,\lambda)}P$ and $n \in N$, then $s, a \in E$.
Therefore, $\phi_P(n(s + a) - na) \geq \phi_P(s) \geq \gamma$ and $\psi_P(n(s + a) - na) \leq \psi_P(s) \leq \lambda$.
Thus $n(s + a) - na \in {}^{(\gamma,\lambda)}P$.
Thus, ${}^{(\gamma,\lambda)}P$ is an ideal of ${}^{(\gamma,\lambda)}E$.

Theorem 2.2 If ${}^{(\gamma,\lambda)}E$ is a weak DN-group , then ${}^{(\gamma,\lambda)}(D \cap M) + {}^{(\gamma,\lambda)}T = ({}^{(\gamma,\lambda)}D + {}^{(\gamma,\lambda)}T) \cap ({}^{(\gamma,\lambda)}M + {}^{(\gamma,\lambda)}T)$, for all ideals ${}^{(\gamma,\lambda)}D, {}^{(\gamma,\lambda)}M, {}^{(\gamma,\lambda)}T$ of ${}^{(\gamma,\lambda)}E$ with $T \subseteq M$.

Proof. Let $s \in {}^{(\gamma,\lambda)}(D \cap M) + {}^{(\gamma,\lambda)}T$.
Then $s = y + m$, where $y \in {}^{(\gamma,\lambda)}(D \cap M), m \in {}^{(\gamma,\lambda)}T$
$\Rightarrow y \in {}^{(\gamma,\lambda)}D$ and ${}^{(\gamma,\lambda)}M, m \in {}^{(\gamma,\lambda)}T$
$\Rightarrow s = y + m \in ({}^{(\gamma,\lambda)}D + {}^{(\gamma,\lambda)}T) \cap ({}^{(\gamma,\lambda)}M + {}^{(\gamma,\lambda)}T)$.
Again, let $y \in ({}^{(\gamma,\lambda)}D + {}^{(\gamma,\lambda)}T) \cap ({}^{(\gamma,\lambda)}M + {}^{(\gamma,\lambda)}T)$.
Since Sum of two ideals of ${}^{(\gamma,\lambda)}E$ is also an ideal and ${}^{(\gamma,\lambda)}E$ is a weak DN-group, therefore
$y \in [{}^{(\gamma,\lambda)}D \cap ({}^{(\gamma,\lambda)}M + {}^{(\gamma,\lambda)}T)] + [{}^{(\gamma,\lambda)}T \cap ({}^{(\gamma,\lambda)}M + {}^{(\gamma,\lambda)}T)]$

$\Rightarrow y \in [^{(\gamma,\lambda)}D \cap^{(\gamma,\lambda)} M]+^{(\gamma,\lambda)}T)][$ since $T \subseteq M]$.
Thus $^{(\gamma,\lambda)}(D \cap M)+^{(\gamma,\lambda)}T = (^{(\gamma,\lambda)}D+^{(\gamma,\lambda)}T) \cap (^{(\gamma,\lambda)}M+^{(\gamma,\lambda)}T)$.

Theorem 2.3 If $^{(\gamma,\lambda)}E$ is a weak DN-group with $\gamma + \lambda = 1$, then E is an IF weak DN-group.

Proof. Let D, M, T be IF ideals of E, then by **lemma 2.1**, $^{(\gamma,\lambda)}D,^{(\gamma,\lambda)} M,^{(\gamma,\lambda)} T$ are ideals of $^{(\gamma,\lambda)}E$.
Since $^{(\gamma,\lambda)}E$ is a weak DN-group,
$(^{(\gamma,\lambda)}D+^{(\gamma,\lambda)}M) \cap^{(\gamma,\lambda)} T = (^{(\gamma,\lambda)}D \cap^{(\gamma,\lambda)} T) + (^{(\gamma,\lambda)}M \cap^{(\gamma,\lambda)} T)$
$\Rightarrow^{(\gamma,\lambda)} [(D + M) \cap T] =^{(\gamma,\lambda)} [(D \cap T) + (M \cap T)]$[using **proposition 1.1**]
$\Rightarrow (D + M) \cap T = (D \cap T) + (M \cap T)$][using **lemma 1.2**]
$\Rightarrow$ E is an IF weak DN-group.

Conclusion

The main goal of this paper is to extend the notion of distributive near-ring groups to intuitionistic fuzzy distributive near-ring groups as well as intuitionistic fuzzy weak distributive near-ring groups. Some lemmas and theorems related to IF N-subgroups, IF DN-groups and IF weak DN-groups are studied. The **theorem** 2.2 describes the relationships between IF DN-groups and IF uniserial DN-groups. **Theorems** 3.1 to 3.2 illustrate the association between IF weak DN-group and weak DN-group.

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