IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Revealing The Nexus: A Philosophical Investigation Into The Mutually Beneficial Parallel Existence Of Set Theory And Relational Frameworks

Shakespeare Malita

Ex-Student Of Kalyani University

University of Kalyani

Abstract:-

Within modern mathematics and philosophy, the concept of a 'set' is soundly captured in the works of Moore and Russell; it affords a context in which we can organize collections of objects (real or abstract) in a systematic way. The term "set" was first used by Georg Cantor in his studies on trigonometric series. This changed completely the history of math and logic as set theory introduced an exact language that allowed us to model relations between particular elements. According to Cantor, a set is a collection that contains distinct objects as parts or members within one entity; on the other hand, Patrick Suppes defines a set as any kind of object composed of elements–in this case both simple and complex. This article differentiates between two aspects of sets, the mathematical and philosophical one. More specifically, it implies that sets play an important role in logic and relation . It starts by giving the definition of a set and its representation using Roster and Set-Builder methods. After this the types of sets such as finite set, infinite set, equal sets, subsets, proper subsets are described in depth. It also discusses concept of membership and the main set operations – union, intersection are examined. It also occupies the domain beyond mathematics. This paper scrutinizes the issue of relations from a philosophical point of view and their role in knitting individuals and objects as well as in creating the basis of meaning and existence in human context. Relations are categorized into eight primary classes depending on reflexivity, symmetry, and transitivity. They are such relationships that people explain through 'is a brother of' 'uses the same WhatsApp group'—real life examples are given through such relationships. Thus, this research demonstrates that the set theory both in the formal sense and in the human life experience is organized through relational frameworks. It concludes that set theory is not merely an abstract mathematical theory but it is also a philosophical paradigm that gives a great depth of understanding of the order, relationship and interdependence in the universe.

Keywords:

Set theory, Types of sets, Set operations, Membership, Relation, Reflexive, Symmetric, Transitive

Introduction:

The notion of set is one of those basic but widely applicable concepts that become the foundation both in math and philosophy. It is a building block in reading geometry, equations, statistics and logical reasoning. Georg Cantor himself invented it, and the field opened up new horizons in mathematics via a systematic treatment to countable and uncountable sets. In philosophy, relationships are everything that isn't quantity in the sense of mathematics: what it means for objects, thoughts or people to interact. A set can be anything, things, or ideas according to Patrick Suppes. The dynamics among the elements of set reveal how togetherness creates presence and meaning. Hence sets and relations, as the study of reflective mathematics and discerning philosophy combines in itself logical rigidness with mental confluence.

Objectives:

The main goal of this article is to stress the role of set theory and its implications in mathematics and in the philosophy. More precisely, the aims will be:

1)To describe and classify sets by finite , infinite, equal set, subset, and proper subset.

2)To be able to describe a variety of ways in which sets can be represented and to explain basic set operations, including union, intersection, and difference, Roster notation and Set Builder.

3)To question the philosophical realities of relations and consider how the relationships among elements, people,and things affect our being, living, and knowing.

4)To classify relations we build up from foundational properties—reflexive, symmetric and transitive—and so eight standard types of relations between sets.

1)CONCEPT OF SETS:

The concept of set is the part and parcel of both mathematics and philosophy in the modern era. It plays a vital role in these fields, as the idea and understanding of sets help us gain a clear perspective on various topics such as geometry, functions, sequences, probability, and relations. The German mathematician Georg Cantor was the inventor of set theory. He developed this concept while working on a particular topic known as the problem of trigonometric series. The practical applications of set theory are vast; because of this, it has played a crucial role not only in mathematics but also in logic and philosophy. It is universally acknowledged that set theory is an essential topic in mathematics, but it is equally important and significant for logicians and philosophers as well. According to Georg Cantor, "A set is a collection of definite, distinct objects of our perception or of our thought, which can be conceived as a whole." According to the famous logician Patrick Suppes, "By a set, we mean any kind of collection of any sort."

1.1Basic Definition of Sets:

In our physical world and in our daily life, we often go through sets without our comprehension. For example, when we go to purchase a piece of tea cu, the shopkeeper often mentions that you cannot take one cup, if you have to pick up this cup, then you have to take the whole set of cups, that is, A Set of Cups. Then we often call an assemblage of some objects a set. For example, A cricket team, A set of army, a set of players in the team. Since we call an assemblage of many things a set, some more examples can be given according to that form, such as:

The rivers of India i.e., Ganga, Jamuna, Padma, Hooghly, etc.

The vowels in the English alphabets, i.e., A, E, I, O, U.

Note: In all the above exemplars, the sets are systematized in an organized manner, but where they are not systematized in this way, we cannot say that. For example: The river of India, but the Amazon River will not be incorporated in this set. To illustrate a set, we entail to keep some points to be remembered in mind, such as:

1. The words objects, members, elements are synonyms.
2. We will identify the set with capital letters A, B, C, D, E, F, etc.
3. We will represent the members of the set with small letters a, b, c, d, e, f.

We can illustrate these sets mainly in two ways

1) Roster Form

2) Set Builder Form

1)**Roster Form:**

In this form, we place the members of the set in a brace and use a comma to segregate the members, and all the members are in a consistent and persistent manner. One thing to retain is that the members of the set or the members of one set don't come twice in the same set. Example:

A set of all outdoor games

G = {cricket, football, tennis, badminton, hockey}

2)**Set Builder Form:**

In this method, we represent the members of the set with a symbol and write qualities of that member of the set which is estranged by a colon. This total characteristic is encased within the middle braces. Example:

C = {x : x is a Consonant in the English alphabet}

1.2Types of Sets:

➢**Empty Set:**-A type of set in which no member or member subsists in reality is called Empty set. There is no member in this set. So it can also be called Null set or Void set.Example:

X = { x : x is a teacher who is currently studying in class xii in the same school }

Explanation:

This will be an Empty set, as no such teacher will be formulated who is a teacher of that school and also a student of a class of that same school.

➢**Finite Set:**-A set in which there is an Definite number of members is called finite set. Example:

A set of months in a year.

M = { January, February, March, April, May, June, July, August, September, October, November, December } This is a finite set because it has an indisputable number of members, and it can be enumerated.

➢**Infinite Set:-**It does not encompass a fixed number of members, i.e. it does not contain a fixed number of members but an indefinite number of members which cannot be tabulated.Examples:

1. E = {2, 4, 6, 8, 10 …} is the set of even numbers.

2. The set of natural numbers N is an Infinite Set. N = {1, 2, 3, 4…}

➢**Equal Sets:** If the members of two sets are equal, i.e. if the number of members of set A is even and the number of members of set B is equal, then it will be an equal set.Example:

Set A = {H, O, N, E, S, T} and set B = {H, O, N, T, Y, E, S} are equal sets.

Note: Order does not matter.

➢**Subset:-**A set will be a subset only when all the members of set A are present in set B. And if the number of set B is a little more, there is no problem. Subset is suggested by the symbol and read as 'is a subset of' in set theory. Subset is represented and advocated by the symbol ⊆.

Example:

X = {1, 3, 5, 7}, Y = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14…..}

The set elements that X's elements relate to set Y. Hence X is a subset of Y (X ⊆ Y).

➢**Proper Subset:-**Set A is a proper subset of set B only when all the members of set A are equal in number to all the members of set B and there is at least one more member which was not in the subset. Example:

X = {p, q, r}

Y = {p, q, r, s, t}

Here X is a proper subset of Y (X ⊂ Y) as all the elements of set X are in set Y and also X ≠ Y.

That means:

SUBSET → (A = B)

PROPER SUBSET → (A ≠ B)

1.3Basic Concepts of Set:

A) The Concept of Membership:-

The symbol "∈" read as "is an element of" or "is in," indicates membership in a set. The symbol "∉" read as "is not an element of" or "is not in," indicates lack of membership in a set.

Example:

If y is an element of set B, we write y ∈ B, which means "y is a member of B" or "y belongs to B."

If y is not an element of set B, we write y ∉ B, meaning "y is not a member of B."

1.4Operations on Sets:

In propositional logic, we are familiar with truth-functional operators like AND (·), OR (V), IF-THEN (→), IF AND ONLY IF (↔), etc. These are truth-functional operators, meaning they are used to form compound statements. Here In Sets we deal with some primary types of operators:

1. Intersection
2. Union

1) **Intersection**:- The set formed by the common members of the members of the sets A and B or the symbol by which we express it is called intersection. A and B be any two sets. The intersection of A and B is the set of all those elements which belong to both A and B. It is denoted by A ∩ B and read as “A intersection B.” The symbol “∩” is used to denote the intersection.

Example:

Let X = {1, 2, 3, 4} and Y = {3, 4, 5, 6}.

The intersection X ∩ Y = {3, 4}, since 3 and 4 are the elements common to both sets.

2) **Union:-** The union of two sets A and B, denoted as A ∪ B. The union A ∪ B means that all the members of the set A and the members of the set B are combined and then it is called the union A ∪ B.

Example:

Set P = {6, 7, 8}

Set Q = {8, 9, 10}

The union P ∪ Q = {6, 7, 8, 9, 10}, as it includes all elements from both sets.

2Understanding Relation in the Context of Set Theory: A Philosophical Perspective:

The concept of relation is an important concept in logic. Without relation, human life is limited. Human life is based on relation. All objects in this world are related to other objects. The significance of an object in the world is determined based on the relationship of that object with the another object. Suddenly, the relevance of the object is revealed through the relationship, just as the value of the life of an individual in society is determined through such relationships. Well, it can be said that human life is nothing but relationships. By establishing relationships, they find the meaning of life. People make their existence significant when they establish relationships with their surroundings, and the breakdown of relationships causes suffering in life. Some relationships are eternal and some relationships are temporary, such as the relationship between parents. The relationship between parents is created through the birth of a child. We will call such relationships eternal truths. It can also be said that the relationship between one’s own siblings. This is also a kind of eternal truth. Even if someone denies their parents or denies their brother or sister, this relationship may be destroyed socially but biologically this relationship is eternal. Again, some relationships are temporary, which are created within some time and then end within some time. The relationship of husband and wife. It is created and developed after the birth of a person and it is regulated by social law. But we cannot call this relationship an eternal truth because this relationship is not eternal, this relationship can break at any time and once this relationship is broken, we cannot call it an eternal truth anymore, it is a temporary relationship. So it can be easily said that no person or object in the world is unnecessary, each of them has a relationship with another person or object, which can be known or unknown. Nothing in the world appears without necessity, this is a very necessary truth. But what is a relationship or what is a relationship, how can the concept of relationship be defined?

We know enough about this relationship, we see relationships in the real world, but no one has given a specific definition of this relationship. Nor did Patrick Suppes give any definition because it is difficult to define. Since human life is nothing but relationships, it is not difficult to understand the concept of relationship, but it is difficult to define it. But by looking at its practical application and analyzing various relationships, a definition can be given that a relationship is some kind of interconnection established between multiple people or objects.

2.1TYPES OF RELATION

Suppes classifies relation as binary (involving 2 entities) and ternary, when it involves more than two. In set theory, we shall stick strictly to binary relations. Examples are: “b is the father of a”, “a is an offspring of b” or “2 is bigger than 5”. These relations are person to person connections, the persons they connect for which they get connected to these connected pairs we call elements of connection. By relating elements from one set to another with sets, a relation can be defined. There is a relationship only if there are sets.

The primary types of relations include reflexive, symmetric, and transitive. These three relations are explained below. For example, reflexive relation ensures that every element is related to itself, while symmetric relation implies that if one element is related to another, then the reverse is also true. Transitive relations establish a chain of relationships.

1) **Reflexive, Irreflexive, and Non-reflexive Relations**:

Relations are classified by several key properties. The first is whether an element of the set is related to itself or not. There are three cases:

Reflexive: Every element is related to itself. Like height relation, love relation. For example, everyone’s height is equal to their own height. My love relation can also be included in reflexive relation; for example, Ram loves Ram himself.

Irreflexive: No element is related to itself. For example, the concept of the relation of being a mother is irreflexive because no one can ever be their own mother.

Non-reflexive: Some elements are related to themselves, but some aren’t. For example, the relation of being the head of the family, because the head of the family can be the father from my side and the mother from my sister’s side. This is a non-reflexive relation.

2) **Symmetric and Asymmetric Relations:**

Symmetric Relation: “Symmetric relation is a type of binary relation in discrete mathematics defined on sets, which satisfies that if aRb exists, then bRa also exists for all pairs a, b that belong to set S. This means that if (a, b) belongs to R, then (b, a) also belongs to R.”

Or: “A relation R is symmetric when, for two elements p and q of set A, if p is related to q, then q is also related to p.”

Example: Marriage relationship. If Saurabh is married to Moumita, then Moumita is married to Saurabh. This is a symmetric relation.

Asymmetric Relation: “A relation R is asymmetric when, for two elements p and q of set A, if p is related to q, then q is not related to p.”

Example: Hierarchy at the workplace. If Ram is Shyam’s boss, then Shyam will never be Ram’s boss.

3) Transitive and Intransitive Relations:

Transitive Relation: “Transitive relations or transitive relationships are all about maintaining a clear chain of connections among their elements. If element A is connected with element B and element B is connected with element C, then it logically follows that element A must also be connected with element C.”

Example: Brother relationship. If X is Y’s brother and Y is Z’s brother, then X is Z’s brother.

Intransitive Relation: “Intransitive relations don’t follow the clear chain rule. If A is connected with B and B is connected with C, it doesn’t guarantee that A is connected with C. Such relations often appear in complex real-world situations.”

Example: If Ram is Shyam’s neighbor and Shyam is Yadu’s neighbor, then Ram cannot be Yadu’s neighbor.

2.2Set Relations: A Study of the Relations of Sets:

Relations indicate the way in which the elements of one set are related to the elements of another (some times to the elements of the same set). These relations are based on whether they are reflexive, symmetric, and transitive. There are, in all, eight fundamental types of set relations based on these properties.

(1) Reflexive, Symmetric and Transitive Relation.

Example: Use the same WhatsApp group as

Explanation:

Reflexive: All are in the same group as themselves.

Symmetric: If A uses a group with B, then B uses it with A.

Transitive: If A uses the same group with B, and B with C, then A uses it with C.

(2) Symmetric and Transitive and not Reflexive.

Example: Is a brother of

Explanation:

Not Reflexive: Nobody is a brother to himself.

Symmetric: If A is a brother of B, then B is a brother of A.

Transitive: If A is brother of B, and B of C, then A is also brother of C.

(3) Reflexive and Transitive but Not Symmetric.

Example: Is a student of.

Analysis: Reflexive: One can be a student of oneself.

Not Symmetric: If A is a student of B, B is not a student of A.

Transitive: If A is a student of B and B is a student of C, A is a student of C.

(4) Reflexive and Symmetric but Not Transitive.

Example: Rides with the same automobile as.

Analysis: Reflexive: One rides with oneself.

Symmetric: If A rides with B, B rides with A.

Not Transitive: If A rides with B and B rides with C, A may not ride with C.

(5) Transitive but Not Reflexive and not Symmetric.

Example: Is a parent of.

Analysis: Not Reflexive: No one is their own parent.

Not Symmetric: If A is parent of B, B is not parent of A.

Transitive: If A is parent of B and B of C, A is Grand parent of C.

(6) Reflexive but not Symmetric and not Transitive.

Example: Is owner of.

Explanation: Reflexive: He is owner of his own property.

Not Symmetric: If A is owner of B, B is not owner of A.

Not Transitive: If A is owner of B, and B owner of C, A may not be owner of C.

(7) Symmetric but not Reflexive and not Transitive.

Example: Is colleague of.

Explanation: Not Reflexive: No one is their own colleague.

Symmetric: If A is colleague of B, B is colleague of A.

Not Transitive: If A is colleague of B, and B of C, A may not be colleague of C.

(8) Neither Reflexive, Symmetric, nor Transitive.

Example: Is boss of.

Explanation: Not Reflexive: No one is his own boss.

Not Symmetric: If A is boss of B, B is not boss of A.

Not Transitive: If A is boss of B, and B of C, A may not be boss of C.

Findings:-

1) Collections are found in human experience, in the metaphysical realm, and in mathematics.

2) Membership and set operations (union, intersection) are among the best methods for considering relationships between objects.

3)Set-defined relationships illustrate fundamental facets of human existence, including enduring ties like family ties and transient interactions like professional networks. Based on these attributes, we classify relationships into eight main categories that are predefined and whose examples, like "is a brother of" or "shares the same WhatsApp group as," can be effectively monitored in day-to-day interactions.

4) The findings show that set theory is a philosophical tool that can be used to comprehend social hierarchies.

References:-

1. Cantor, G. (1895). Beiträge zur Begründung der transfiniten Mengenlehre [Contributions to the founding of the theory of transfinite numbers]. Mathematische Annalen, 46, 481–512.

2. Cantor, G. (1955). Contributions to the founding of the theory of transfinite numbers (P. E. B. Jourdain, Trans.). Dover Publications. (Original work published 1895–1897)

3. Halmos, P. R. (1960). Naïve set theory. Van Nostrand.

4. Fraenkel, A. A., Bar-Hillel, Y., & Levy, A. (1973). Foundations of set theory (2nd ed.). North-Holland Publishing Company.

5. Jech, T. (2003). Set theory: The third millennium edition. Springer.

6. Russell, B. (1919). Introduction to mathematical philosophy. George Allen & Unwin.

7. Whitehead, A. N., & Russell, B. (1910–1913). Principia mathematica (Vols. I–III). Cambridge University Press.

8. Quine, W. V. O. (1969). Set theory and its logic (Rev. ed.). Harvard University Press.

9. Suppes, P. (1972). Axiomatic set theory. Dover Publications.

10. Copi, I. M., Cohen, C., & Flage, D. E. (2014). Introduction to logic (14th ed.). Routledge.

11. Chakraborty, S., & Guha, A. (2002). Mathematical logic. University of Calcutta.

12. National Council of Educational Research and Training. (2021). Mathematics: Set theory (Class 12). Author.

13. Rosen, K. H. (2012). Discrete mathematics and its applications (7th ed.). McGraw–Hill Education.

(Referenced chapters: Sets, relations and functions, sequences, sums, and cardinality of sets.)

14. Ashlock, D. (2010). Set theory: Foundations of mathematics. University of Guelph, Department of Mathematics and Statistics.

(Explores fundamental concepts of set theory, including membership, subsets, operations, and relations.)