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Optimizing The EOQ With Service Time For Product With Constant Demand Rate In The Inventory System When The Amount Received Is Uncertain.

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Abstract:
If the arrival of quantity of an item is not certain when some quantity is ordered and this quantity is needed to give some time for service to all arrived items in inventory before allowing it for sell and it spends time in the inventory besides the starting time of depleting it from the inventory up to the zero level of inventory. This service time is cost of holding on all received items in the inventory. The optimal order quantity depends only on the mean and standard deviation of the amount arrived and if we can reduce the service time by increasing service persons or facilities the total cost goes near to cost of EOQ model with uncertain arrival of quantity.

Key words Inventory system, Demand rate, Service rate, Replenishment rate, Lead time, Queuing system Service time, re-order point.

1. Introduction

The classical inventory management model also known as Wilson EOQ model is used to determine the optimal order quantity which minimizes total inventory cost by balancing ordering cost and holding cost. In Wilson's model it is assumed that the demand is constant, replenishment is instantaneous and no shortage occurs. Also, it is assumed that the received quantity matches the quantity requisitioned. However, in the real-world situations, replenishment is often uncertain due to variable lead time or fluctuating demand. Tis leads to the inventory model with uncertain replenishment of items, which extends the classical model by arrival of items in random with fixed order quantity.

It is obvious that when people are going to by any item, demand is there and as demand is there queue is there. If people arrive too frequently and because of finite capacity of server customer have to wait until they are served. Which makes a queue. Including all such terms we have a new term which is known as "Queuing system". If it is known, how many customers can be served at a time, we can specify the distribution of service time. In the most situation the service time is random variable and having the distribution for all arrivals. If the average number of arrivals exceeds the maximum average service rate of the system, as time goes customers will not be served any how at some time point and hence to get steady state condition we must have, $\lambda < \mu$ for one server in the system and $\lambda < c\mu$ for c servers in the system

2. Literature review

Edward A. Silver (1976) worked on the quantity received from a supplier may not match the quantity ordered—due to defects, shortages, shipping issues, etc. Silver's extended Economic Order Quantity (EOQ) model tackles this by considering that only the mean and standard deviation of the received amount matter in determining the optimal order quantity. Traditional models generally assume one-sided randomness but A. Hamid Noori and Gerald Keller ((1986)) extended the classic continuous-review (Q, r) inventory model to account for uncertainty in both demand during lead time and supply availability. Chirag Trivedi, Y.K. Shan, Nita H. Shah. (1994), present an EOQ model in which a temporary price discount is offered, but the supply received is random rather than equal to the order. Nita H. Shah, Chirag J. Trivedi (1996), extended the classic Economic Order Quantity (EOQ) model is by considering both random lead times and random demand instead of fixed values. Karush (1957) showed that inventory with random demand and lead times can be modelled as a queuing system. Berman O. Kaplan E.H., and Shimshak D.G. (1993) considered the inventory during the provision of service and inventory depleted according to the demand rate when there are no customers waiting in the queue and when customers are waiting in a queue it is depleted according to service rate. Ha (1997) worked on single-item make-to-stock production system and considered poison demand and exponential production times and used an M/M/1/S queuing system for modeling the system. Arda and Hennet (2006) analyzed inventory control of a multi-supplier strategy in a two-level supply chain with random arrival for customers and random delivery time for suppliers, the system was represented as a queuing network. Jung Woo Baek and Seung Ki Moon (2014), provides a queueing-theoretic model for production-inventory systems with lost sales, develops exact analytical results for performance evaluation, and offers managerial insights into balancing production, inventory levels, and customer service. Seyedhoseini et al. (2015) applied queuing theory to propose a mathematical model for inventory systems with substitute flexibility.

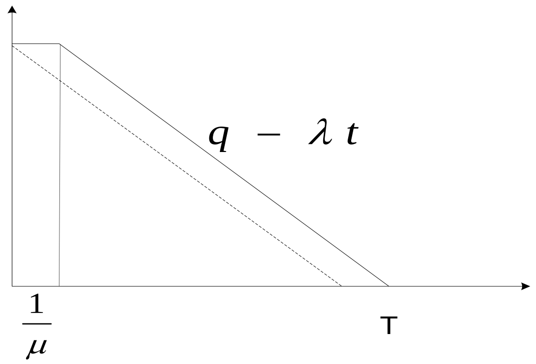
When the depletion of item does not start immediately and it reaches the service station for service, the waiting time of item increases and hence it increases the total cost. It is interesting to check the total cost when arrival of items is uncertain and service time of any number of items is less than the capacity of the Wherehouse before the items are sold. Therefore, we have assumed that in which the item is served before the customer arrives and the customer has to wait at the place, the queue gradually builds up. Thus, the model has been designed to study what could be the optimal quantity of the item when the queue is formed during uncertain quantity. And a hypothetical situation has also been considered to check its effectiveness and sensitivity of the model.

3. Notations and Assumptions for the model:

The model is developed under the very stringent assumption and the notations used for the derivation are as under:

- q = the order quantity
- q* = Economic order quantity to be determined.
- C_3 = Replenishment cost per order which is known and constant
- C_1 = Holding cost per unit per unit time which is known and constant
- L = Lead time which is zero.
- The stockout cost is zero.
- b = the bias factor
- $\lambda =$ Demand rate is constant and known
- $\frac{1}{\mu}$ = Total Service time to serve q items.
- T = Total Cycle time.
- $EOQ = \sqrt{\frac{2\lambda C_3}{C_1}}$ = The economic quantity in units.
- $E(y|q) = bq$
- $ECPUT(q)$ = Expected costs per unit time if a requisition quantity q is used
- $ECWS(Q)$ = Expected cost without service
- The time horizon if infinite.
- $TC(q)$ = Average total cost per cycle
- The replenishment rate is infinite

Consider the following figure. Where inventory has taken time $\frac{1}{\mu}$ for the process to be done on all items before it starts for depletion. After the time $\frac{1}{\mu}$ items are depleted at the rate of λ and the inventory becomes zero at time T.



4. The Mathematical Model:

For developing the expected costs per unit time, we consider the time as being made up of cycles and the new cycle begins each time when the q quantity is ordered. As the lead time is zero the order will be placed when the inventory level drops to zero.

Suppose that the quantity y is received instead of q. That is y is a random variable and its maximum value is q. Because when the quantity y arrives, it stays for some time duration in the inventory for service before the selling. This stay is delayed by the service time $\frac{1}{\mu}$, which affects the inventory depletion rate. Therefore, the Inventory level at any time t, $0 \leq t \leq T$ will be,

$$I(t) = \begin{cases} y & if\ 0 \leq t \leq \frac{1}{\mu} \\ y - \lambda\left(t - \frac{1}{\mu}\right) & if\ \frac{1}{\mu} < t \leq T \end{cases}$$

The cycle time T depends on the quantity y and hence the time T is a function of. As it is assumed that up to maximum inventory level the service time is constant and does not depend up on the quantity y, the total cycle time is,

$$T(y) = \frac{1}{\mu} + \frac{y}{\lambda}$$

The expected value of T(y) is given by

$$E(T|q) = \frac{1}{\mu} + \frac{E(y|q)}{\lambda} = \frac{1}{\mu} + \frac{bq}{\lambda}$$

Let C(y) be the costs in the current cycle. And hence it will be,

$$C(y) = C_1 \int_0^T I(t)dt + C_3$$

$$= \int_0^{1/\mu} y\, dt + \int_{1/\mu}^{T} \left[y - \lambda\left(t - \frac{1}{\mu}\right)\right] dt + C_3$$

Letting $\left(t - \frac{1}{\mu}\right) = u$ we have

$$= \int_0^{1/\mu} y\, dt + \int_0^{y/\lambda} [q - \lambda u]\, du + C_3$$

Thus, we have,

$$C(y) = \frac{y}{\mu} + \frac{y^2}{2\lambda} + C_3$$

The expected value of C(y) is given by

$$E(C|q) = C_1 \cdot \left(\frac{E(y|q)}{\mu} + \frac{1}{2\lambda}E(y^2|q)\right) + C_3$$

As $E(y|q) = bq$ and $E(y^2|q) = {\sigma_{y|q}}^2 + [E(y|q)]^2$

$$E(C|q) = C_1 \cdot \left[\frac{bq}{\mu} + \frac{1}{2\lambda}\left({\sigma_{y|q}}^2 + (bq)^2\right)\right] + C_3$$

By the result of Renewal Reward Process, if a cycle is completed every time a renewal occurs and the long-run average reward per unit time is equal to the expected reward earned during a cycle divided by the expected length of a cycle. Thus, we have, the expected cots per unit time are

$$ECPUT(q) = \frac{E(C|q)}{E(T|q)}$$

Substituting values of $E(C|q)$ and $E(T|q)$ in the above equation we have,

$$ECPUT(q) = \frac{C_1 \cdot \left[\frac{bq}{\mu} + \frac{1}{2\lambda}\left({\sigma_{y|q}}^2 + (bq)^2\right)\right] + C_3}{\frac{1}{\mu} + \frac{bq}{\lambda}}$$

$$ECPUT(q) = \frac{2\lambda C_1 bq + \mu C_1 {\sigma_{y|q}}^2 + \mu C_1 b^2 q^2 + 2\lambda\mu C_3}{2(\lambda + \mu bq)}$$

$ECPUT(q)$ will be minimum if $\frac{d(ECPUT(q))}{dq} = 0$ and its solution $\frac{d^2(ECPUT(q))}{dq^2} > 0$

Let $\frac{d(ECPUT(q))}{dq} = 0$, then we have,

$$\frac{(2\lambda + 2\mu bq)[2\lambda C_1 b + 2\mu C_1 b^2 q] - 2\mu b\left(2\lambda C_1 bq + \mu C_1 {\sigma_{y|q}}^2 + \mu C_1 b^2 q^2 + 2\lambda\mu C_3\right)}{4(\lambda + \mu bq)^2} = 0$$

$$(2\lambda + 2\mu bq)[2\lambda C_1 b + 2\mu C_1 b^2 q] = 2\mu b\left(2\lambda C_1 bq + \mu C_1 {\sigma_{y|q}}^2 + \mu C_1 b^2 q^2 + 2\lambda\mu C_3\right)$$

$$(\lambda + \mu bq)[2\lambda C_1 b + 2\mu C_1 b^2 q] = \mu b\left(2\lambda C_1 bq + \mu C_1 {\sigma_{y|q}}^2 + \mu C_1 b^2 q^2 + 2\lambda\mu C_3\right)$$

$$\mu^2 C_1 b^3 q^2 + 2\lambda\mu C_1 b^2 q + b\left(2\lambda^2 C_1 - \mu^2 C_1 {\sigma_{y|q}}^2 - 2\lambda\mu^2 C_3\right) = 0$$

It is the quadratic equation in q. That is, $Aq^2 + Bq + C = 0$

$$A = \mu^2 C_1 b^3$$

$$B = 2\lambda\mu C_1 b^2$$

$$C = b\left(2\lambda^2 C_1 - \mu^2 C_1 {\sigma_{y|q}}^2 - 2\lambda\mu^2 C_3\right)$$

Solving this quadratic equation we have,

$$\Delta = 4b^4\mu^4\left[\frac{\lambda^2 {C_1}^2}{\mu^2} - C_1\left(\frac{2\lambda^2 C_1}{\mu^2} - C_1 {\sigma_{y|q}}^2 - 2\lambda C_3\right)\right]$$

Let $\Delta_0 = \frac{\lambda^2 {C_1}^2}{\mu^2} - C_1\left(\frac{2\lambda^2 C_1}{\mu^2} - C_1 {\sigma_{y|q}}^2 - 2\lambda C_3\right)$ and hence we have $\Delta = 4b^4\mu^4\Delta_0$

Thus, we have

$$q = \frac{-2\lambda\mu C_1 b^2 + \sqrt{\Delta}}{2\mu^2 C_1 b^3}$$

$$q = \frac{-2\lambda\mu C_1 b^2 + 2b^2\mu^2\sqrt{\Delta_0}}{2\mu^2 C_1 b^3}$$

$$q = \frac{2b^2\mu^2\left(-\frac{\lambda C_1}{\mu} + \sqrt{\Delta_0}\right)}{2b^2\mu^2 C_1 b}$$

$$q = \frac{1}{b}\left(\frac{-\frac{\lambda C_1}{\mu} + \sqrt{\Delta_0}}{C_1}\right)$$

Provided $b \neq 0, C_1 \neq 0$.

Which is the cost minimum value of q because it can be shown that $\frac{d^2(ECPUT(q))}{dq^2} > 0$

$$q^* = \frac{1}{b}\left(\frac{-\frac{\lambda C_1}{\mu} + \sqrt{\Delta_0}}{C_1}\right)$$

Provided $b \neq 0, C_1 \neq 0$.

If the service time is taken zero, we have the model of EOQ model with uncertain arrival y our to ordered quantity Q and hence we have,

$$T(y) = \frac{1}{\mu} + \frac{y}{\lambda} \text{ and } E(C|q) = \frac{C_1}{2\lambda}\cdot\left({\sigma_{y|Q}}^2 + b^2q^2\right) + C_3$$

And expected cost without service will be,

$$ECWS(Q) = \frac{E(C|Q)}{E(T|Q)} = \frac{C_1{\sigma_{y|Q}}^2}{2bq} + \frac{C_1 bq}{2} + \frac{\lambda C_3}{bq}$$

$ECWS(q)$ will be minimum if $\frac{d(ECWS(q))}{dq} = 0$ and its solution $\frac{d^2(ECWS(Q))}{dq^2} > 0$

Let $\frac{d(ECWS(Q))}{dq} = 0$, then we have,

$$Q^* = \frac{1}{b}\sqrt{\frac{C_1{\sigma_{y|Q}}^2 + 2\lambda C_3}{C_1}} = \frac{1}{b}\sqrt{{\sigma_{y|Q}}^2 + \frac{2\lambda C_3}{C_1}} = \frac{1}{b}\sqrt{{\sigma_{y|Q}}^2 + EOQ^2}$$

Which is the cost minimum value of q because it can be shown that $\frac{d^2(ECWS(Q))}{dq^2} > 0$

It should be noted that the quantity q is same as quantity Q but for uncertain arrival with service and for uncertain arrival without service the optimal quantities are different and they are q* and Q* respectively. But the standard deviation $\sigma_{y|Q} = \sigma_{y|Q}$.

5. Sensitive Analysis

To check the effectiveness and utility of the current model along with the theoretical theory when we take the ratio of the quantity obtained by the current model to the quantity obtained by EOQ/b it tends to 1 as the service time tends to zero. That is, if we ignore the service time, it becomes the EOQ mode with uncertain arrival of y quantity when order of Q quantity placed.

when $\frac{1}{\mu} \longrightarrow 0$, $\Delta_0 \longrightarrow \sqrt{{C_1}^2{\sigma_{y|Q}}^2 + 2\lambda C_1 C_3}$ and $q^* = \frac{1}{b}\sqrt{{\sigma_{y|Q}}^2 + EOQ^2}$

$$\lim_{\frac{1}{\mu}\to 0}\frac{q^*}{Q^*} = 1$$

Inventory with service will have mor total variable cost than the total variable cost in classical inventory because of service time and hence holding cost. If we reduce the service time by increasing service persons or facilities, we will have no difference between our model and classical inventory model.

Case 1: Let $\sigma_{y|q} = \sigma$: If the standard deviation of the quantity received is independent of the quantity requisitioned:

Thus, we have

$$ECPUT(q) = \frac{2\lambda C_1 bq + \mu C_1 \sigma^2 + \mu C_1 b^2 q^2 + 2\lambda\mu C_3}{2(\lambda + \mu bq)}$$

If $\sigma = 0$, received quantity will be certain, $E(y|q) = bq = mean\ value$ and we have

That is the quantity actually received will be the EOQ.

And

$$q^* = \frac{1}{b}\left(\frac{-\frac{\lambda C_1}{\mu} + \sqrt{\Delta_0}}{C_1}\right)$$

With $\Delta_0 = \frac{\lambda^2 C_1{}^2}{\mu^2} - C_1\left(\frac{2\lambda^2 C_1}{\mu^2} - 2\lambda C_3\right)$

when $\frac{1}{\mu} \longrightarrow 0$,

$$\Delta_0 = 2\lambda C_1 C_3$$

$$q^* = \frac{1}{b}\sqrt{\frac{2\lambda C_3}{C_1}} = \frac{EOQ}{b}$$

Which is the optimal quantity when arrival is random and standard deviation of quantity arrived is independent of the quantity Q requisitioned. That is $\sigma_{y|q} = \sigma$.

$$Q^* = \frac{EOQ}{b}$$

Case 2: Let $\sigma_{y|q} = \sigma_1 q$: If the standard deviation of the quantity received is proportional to the quantity requisitioned:

Thus, we have

$$ECPUT(q) = \frac{2\lambda C_1 bq + \mu C_1 \sigma_1{}^2 q^2 + \mu C_1 b^2 q^2 + 2\lambda\mu C_3}{2(\lambda + \mu bq)}$$

$ECPUT(q)$ will be minimum if $\frac{d(ECPUT(q))}{dq} = 0$ and its solution $\frac{d^2(ECPUT(q))}{dq^2} > 0$

Let $\frac{d(ECPUT(q))}{dq} = 0$, then we have,

$$\frac{(2\lambda + 2\mu bq)[2\lambda C_1 b + 2\mu C_1 \sigma_1{}^2 q + 2\mu C_1 b^2 q] - 2\mu b(2\lambda C_1 bq + \mu C_1 \sigma_1{}^2 q^2 + \mu C_1 b^2 q^2 + 2\lambda\mu C_3)}{4(\lambda + \mu bq)^2} = 0$$

$$4\lambda^2 C_1 b + 4\lambda\mu C_1 \sigma_1{}^2 q + 4\lambda\mu C_1 b^2 q + 2\mu^2 C_1 \sigma_1{}^2 bq^2 + 2\mu^2 C_1 b^3 q^2 - 4\lambda\mu^2 C_3 b = 0$$

$$2\mu^2 C_1 b(\sigma_1{}^2 + b^2)q^2 + 4\lambda\mu C_1(\sigma_1{}^2 + b^2)q + 4\lambda b(\lambda C_1 - \mu^2 C_3) = 0$$

It is the quadratic equation in q. Then we have,

$$\Delta = 16\mu^4 b^2(\sigma_1{}^2 + b^2)^2\left[\frac{\lambda^2 C_1{}^2}{\mu^2 b^2} - \frac{2\lambda C_1}{(\sigma_1{}^2 + b^2)}\left(\frac{\lambda C_1}{\mu^2} - C_3\right)\right]$$

Let $\Delta_0 = \frac{\lambda^2 C_1{}^2}{\mu^2 b^2} - \frac{2\lambda C_1}{(\sigma_1{}^2+b^2)}\left(\frac{\lambda C_1}{\mu^2} - C_3\right)$ then, $\Delta = 16b^2\mu^4(\sigma_1{}^2+b^2)^2\,\Delta_0$

Thus, we have

$$q = \frac{-4\lambda\mu C_1(\sigma_1{}^2+b^2) + 4\mu^2 b(\sigma_1{}^2+b^2)\sqrt{\Delta_0}}{4\mu^2 C_1 b(\sigma_1{}^2+b^2)}$$

$$q = \frac{-\frac{\lambda C_1}{\mu b} + \sqrt{\Delta_0}}{C_1}$$

Provided $b \neq 0, C_1 \neq 0$.

Which is the cost minimum value of q because it can be shown that $\frac{d^2(ECPUT(q))}{dq^2} > 0$

$$q^* = \frac{-\frac{\lambda C_1}{\mu b} + \sqrt{\Delta_0}}{C_1}$$

when $\frac{1}{\mu} \longrightarrow 0$,

$$\Delta_0 = \frac{2\lambda C_1 C_3}{\sigma_1{}^2 + b^2}$$

And hence,

$$q^* = \sqrt{\frac{2\lambda C_3}{C_1} \times \frac{1}{\sigma_1{}^2 + b^2}}$$

$$\frac{q^*}{EOQ} = \sqrt{\frac{1}{\sigma_1{}^2 + b^2}}$$

Which is the optimal quantity when arrival is random and the standard deviation is proportional to the Q amount requisitioned. That is $\sigma_{y|q} = \sigma_1 Q$.

$$\frac{Q^*}{EOQ} = \sqrt{\frac{1}{\sigma_1{}^2 + b^2}}$$

6. Hypothetical Numerical Example:

Consider the following example:

Annual demand D = 120000
Ordering Cost per order = 1000
Holding cost per item per unit time =1000
The demand rate per month will be R= 10000

$$EOQ = \sqrt{\frac{2RC_3}{C_1}} = 141.421\ units\ per\ order$$

$$Q^* = \frac{EOQ}{b} = \frac{141.421}{b}$$

Case 1: Let $\sigma_{y|q} = \sigma = 0$ and for b =0.6, 0.8, 1, 1.2, 1.4, 1.6

For b= 0.6, $Q^* = \frac{EOQ}{b} = \frac{141.421}{0.6} = 235.702$

For b= 0.8, $Q^* = \frac{EOQ}{b} = \frac{141.421}{0.8} = 176.777$

μ	λ/μ	q*	q*/Q*
10000	1	232.9186	0.98819
20000	0.5	234.3119	0.994101
30000	0.333333	234.7757	0.996069
40000	0.25	235.0074	0.997052
50000	0.2	235.1465	0.997642
60000	0.166667	235.2391	0.998035
70000	0.142857	235.3053	0.998316
80000	0.125	235.3549	0.998526
90000	0.111111	235.3935	0.99869
100000	0.1	235.4244	0.998821

μ	λ/μ	q*	q*/Q*
10000	1	175.2098	0.991136
20000	0.5	175.9943	0.995574
30000	0.333333	176.2554	0.997051
40000	0.25	176.3858	0.997789
50000	0.2	176.464	0.998231
60000	0.166667	176.5162	0.998526
70000	0.142857	176.5534	0.998737
80000	0.125	176.5813	0.998895
90000	0.111111	176.603	0.999018
100000	0.1	176.6204	0.999116

For b= 1, $Q^* = \frac{EOQ}{b} = \frac{141.421}{1} = 141.421$

For b= 1.2, $Q^* = \frac{EOQ}{b} = \frac{141.421}{1.2} = 117.851$

μ	λ/μ	q*	q*/Q*
10000	1	140.4178	0.992904
20000	0.5	140.9205	0.996458
30000	0.333333	141.0876	0.99764
40000	0.25	141.1711	0.998231
50000	0.2	141.2212	0.998585
60000	0.166667	141.2546	0.998821
70000	0.142857	141.2784	0.998989
80000	0.125	141.2963	0.999116
90000	0.111111	141.3102	0.999214
100000	0.1	141.3213	0.999293

μ	λ/μ	q*	q*/Q*
10000	1	117.1537	0.994082
20000	0.5	117.5032	0.997047
30000	0.333333	117.6193	0.998033
40000	0.25	117.6773	0.998525
50000	0.2	117.7121	0.99882
60000	0.166667	117.7353	0.999017
70000	0.142857	117.7519	0.999158
80000	0.125	117.7643	0.999263
90000	0.111111	117.7739	0.999345
100000	0.1	117.7817	0.99941

For b= 1.4, $Q^* = \frac{EOQ}{b} = \frac{141.421}{1.4} = 101.015$

μ	λ/μ	q*	q*/Q*
10000	1	100.5025	0.994924
20000	0.5	100.7595	0.997468
30000	0.333333	100.8449	0.998314
40000	0.25	100.8875	0.998736
50000	0.2	100.9131	0.998989
60000	0.166667	100.9302	0.999158
70000	0.142857	100.9423	0.999278
80000	0.125	100.9514	0.999368
90000	0.111111	100.9585	0.999438
100000	0.1	100.9642	0.999495

For b= 1.6, $Q^* = \frac{EOQ}{b} = \frac{141.421}{1.6} = 88.388$

μ	λ/μ	q*	q*/Q*
10000	1	87.99551	0.995556
20000	0.5	88.19248	0.997784
30000	0.333333	88.25789	0.998524
40000	0.25	88.29055	0.998894
50000	0.2	88.31013	0.999115
60000	0.166667	88.32318	0.999263
70000	0.142857	88.3325	0.999368
80000	0.125	88.33948	0.999447
90000	0.111111	88.34492	0.999509
100000	0.1	88.34926	0.999558

From the above table it can be seen that the optimal quantity q* with service time tends to the optimal quantity Q* without service time when arrival is uncertain and standard deviation of quantity arrived is independent of the quantity Q requisitioned when the service time tends to zero.

Case 2: Let $\sigma_{y|q} = \sigma_1 \cdot EOQ$ and for b =0.6, 0.8, 1, 1.2, 1.4, 1.6

Assume that y follows uniform distribution with q=EOQ and the y is in the range from 0.9EOQ to 1.1EOQ. That is

$y \sim U(0.9EOQ, 1.1EOQ)$ and $\sigma_{y|q} = \frac{1.1EOQ - 0.9EOQ}{\sqrt{12}} = 0.0577 \times EOQ$ and hence $\sigma_1 = 0.0577$

For above example,

$$\sigma_{y|q} = 0.0577 \times 141.421 = 8.1600$$

$$Q^* = \frac{EOQ}{\sqrt{\sigma_1{}^2 + b^2}} = \frac{141.421}{\sqrt{0.00333 + b^2}}$$

For b= 0.6, $Q^* = \frac{EOQ}{\sqrt{\sigma_1{}^2+b^2}} = 234.619$

μ	λ/μ	q*	q*/Q*
10000	1	232.9474	0.992872
20000	0.5	233.7851	0.996442
30000	0.333333	234.0637	0.997629
40000	0.25	234.2028	0.998223
50000	0.2	234.2863	0.998578
60000	0.166667	234.3419	0.998815
70000	0.142857	234.3817	0.998985
80000	0.125	234.4114	0.999112
90000	0.111111	234.4346	0.99921
100000	0.1	234.4531	0.999289

For b= 0.8, $Q^* = \frac{EOQ}{\sqrt{\sigma_1{}^2+b^2}} = 176.318$

μ	λ/μ	q*	q*/Q*
10000	1	175.0643	0.992886
20000	0.5	175.6926	0.996449
30000	0.333333	175.9015	0.997634
40000	0.25	176.0059	0.998226
50000	0.2	176.0685	0.998581
60000	0.166667	176.1102	0.998818
70000	0.142857	176.14	0.998987
80000	0.125	176.1624	0.999113
90000	0.111111	176.1797	0.999212
100000	0.1	176.1936	0.999291

For b= 1, $Q^* = \frac{EOQ}{\sqrt{\sigma_1{}^2+b^2}} = 141.186$

For b= 1.2, $Q^* = \frac{EOQ}{\sqrt{\sigma_1{}^2+b^2}} = 117.715$

μ	λ/μ	q*	q*/Q*
10000	1	140.183	0.992892
20000	0.5	140.6856	0.996452
30000	0.333333	140.8528	0.997636
40000	0.25	140.9363	0.998228
50000	0.2	140.9864	0.998582
60000	0.166667	141.0198	0.998819
70000	0.142857	141.0436	0.998988
80000	0.125	141.0615	0.999114
90000	0.111111	141.0754	0.999213
100000	0.1	141.3213	0.999293

μ	λ/μ	q*	q*/Q*
10000	1	116.8789	0.992896
20000	0.5	117.2977	0.996454
30000	0.333333	117.437	0.997637
40000	0.25	117.5066	0.998229
50000	0.2	117.5483	0.998583
60000	0.166667	117.5762	0.998819
70000	0.142857	117.596	0.998988
80000	0.125	117.6109	0.999115
90000	0.111111	117.6225	0.999213
100000	0.1	117.6318	0.999292

For b= 1.4, $Q^* = \frac{EOQ}{\sqrt{{\sigma_1}^2+b^2}} = 100.929$

μ	λ/μ	q*	q*/Q*
10000	1	100.2128	0.992898
20000	0.5	100.5718	0.996455
30000	0.333333	100.6912	0.997638
40000	0.25	100.7508	0.998229
50000	0.2	100.7866	0.998584
60000	0.166667	100.8105	0.99882
70000	0.142857	100.8275	0.998988
80000	0.125	100.8402	0.999115
90000	0.111111	100.8502	0.999213
100000	0.1	100.8581	0.999292

For b= 1.6, $Q^* = \frac{EOQ}{\sqrt{{\sigma_1}^2+b^2}} = 88.3309$

μ	λ/μ	q*	q*/Q*
10000	1	87.70372	0.992899
20000	0.5	88.01788	0.996456
30000	0.333333	88.12235	0.997639
40000	0.25	88.17454	0.99823
50000	0.2	88.20584	0.998584
60000	0.166667	88.2267	0.99882
70000	0.142857	88.2416	0.998989
80000	0.125	88.25277	0.999115
90000	0.111111	88.26146	0.999214
100000	0.1	88.26841	0.999292

From the above table it can be seen that the optimal quantity q* with service time tends to the optimal quantity Q* without service time when arrival is uncertain and the standard deviation is proportional to the Q amount requisitioned when the service time tends to zero.

7. Conclusion

When the arrival of quantities is uncertain, the best order quantity depends on the mean and standard deviation of the amount actually received. In the situation, when the replenished items cannot be sold immediately, and they must undergo a processing stage the processing time increases the average customer waiting time and therefore raises the holding cost of all items in the system. Two cases of supply uncertainty are considered: (i) when the standard deviation of the received quantity is independent of the ordered amount, and (ii) when the standard deviation is proportional to the ordered amount. If we can reduce the process time on the product — that is, decrease the waiting time as much as possible — the new model simplifies to the classical case where there is no service time and the arrival of quantity is uncertain.

8. References

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