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Optimizing The EOQ With Service Time For Product In The Inventory System With Constant Demand Rate.

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Abstract:
In some situations, it is needed to give some time to items in inventory before allowing it for sell and it spends time in the inventory besides the starting time of depleting it from the inventory up to the zero level of inventory. This service time is cost of holding on all received items in the inventory. If we can reduce the service time by increasing service persons or facilities and the total cost goes near to cost of classical EOQ model.

Key words Inventory system, Demand rate, Service rate, Replenishment rate, Lead time, Queuing system Service time, re-order point.

1. Introduction

Inventory control is the process of managing and regulating the Storage, demand, supply, and distribution of stock. To maintain appropriate quantities of stock to meet customer demand is important for the Inventory Management. Therefore, the objective of Inventory optimization is to have the right item with optimal quantity and with cost effectiveness. Key elements of Inventory optimization are demand, Inventory policies and Replenishment.

Demand is the numbers of units which are going to be withdraw from the inventory. Because of shortages or delays it may happen that demand may not be fulfilled and it is known as lost sale. There are two different approaches to check stock of inventory items. The first one is fixed order quantity system or continuous review systems and second is periodic review system. In the continuous review system, the inventory level (item on hand plus on order) drops to a given level, s or below, an order is placed for a fixed quantity Q, known as (s, Q) policy and the inventory level (item on hand plus on order) drops to a given level, s or below, and order is placed for a sufficient quantity to bring the inventory level up to a predetermined maximum level, known as (s, S) policy. In the periodic review system, the inventory level (item on hand plus on order) is reviewed regularly time intervals of length T and sufficient quantity to be bring the inventory level up to a predetermined maximum level S, is known as (T, S) policy and the inventory level (item on hand plus on order) is reviewed regularly at time interval of length T with inventory level is at level, s or below, an order is placed for a sufficient quantity to bring inventory level up to a pre-determined level, S , but if the inventory level is above s, no order is placed, is known as (T, s, S) policy. Stock replenishment is the rate at which inventory travels along the supply chain from the manufacturer to the supplier to warehousing, picking, and shipment locations. Stock replenishment principles are safety stock (buffer stock) and reorder point. The replenishment of stock may occur instantaneous or gradually.

Inventory model is a mathematical model which helps the business to optimize the inventory to be held in the warehouse to maintain production process or fulfil the demand for the raw materials, intermediate goods, finished products, and spare parts in different constraints. The components of inventory models are demand, purchase (or production) costs, carrying (or holding) costs, ordering (or set-up) costs, shortage (or stock out) costs, Lead time, order cycle, stock replenishment, re-order level, buffer stock, revenue, salvage cost, discount rate, and so on. The order cycle is the time period between two successive replenishments. The time delay between placing an order and receipt of delivery is called delivery lag or lead time. The reorder level (ROL) is the inventory level at which one should place the new order or start a production.

In everyday life, we can see a number of people arrive at Mall, Shops, Banks, Cinema and many other places for getting service for getting different items, for deposit or withdraw money, for getting ticket for movie and for many other things. If people arrive too frequently and because of finite capacity of server customer have to wait until they are served. Which makes a queue. Including all such terms we have a new term which is known as "Queuing system".

A queuing system can be completely described by following terms.

a) Arrival pattern
b) Service pattern
c) Queue discipline
d) Customer's behavior

Generally, the customers arrive in a random fashion which is not worth making the predication. Thus, the arrival pattern can be described in terms of probabilities and probability distribution for inter-arrival times or the distribution of number of customers arriving in unit time.

If it is known, how many customers can be served at a time, we can specify the distribution of service time. In the most situation the service time is random variable and having the distribution for all arrivals. For the inventory point of view if the items are not deteriorated the Queue discipline will be in SIRO (Service in Random order). Let denote λ as the average rate of customers entering the queuing system and μ as the average rate of serving customers. If average number of arrivals exceeds the maximum average service rate of the system, as time goes customers will not be served any how at some time point and hence to get steady state condition we must have,

$$\lambda < \mu \quad \text{for one server in the system}$$
$$\lambda < c\mu \quad \text{for c servers in the system}$$

Thus, the traffic intensity or utilization factor for server facility (expected fraction of time the servers are busy) becomes less than 1.

That is,

$$\rho = \frac{\lambda}{\mu} < 1 \quad or \quad \rho = \frac{\lambda}{c\mu} < 1$$

Most often we are interested in queuing models is to find the probability distribution for the total number of customers in the system at time t.

2. Literature review

Karush (1957) showed that inventory with random demand and lead times can be modelled as a queuing system. Berman O., Kaplan E.H., and Shimshak D.G. (1993) considered the inventory during the provision of service and inventory depleted according to the demand rate when there are no customers waiting in the queue and when customers are waiting in a queue it is depleted according to service rate. Edward A. Silver (1976) worked on the quantity received from a supplier may not match the quantity ordered—due to defects, shortages, shipping issues, etc. Silver's extended Economic Order Quantity (EOQ) model tackles this by considering that only the mean and standard deviation of the received amount matter in determining the optimal order quantity. Ha (1997) worked on single-item make-to-stock production system and considered poison demand and exponential production times and used an M/M/1/S queuing system for modeling the system. M. Schwarz, C. Sauer, H. Daduna, R. Kulik, R. Szekli (2006), analysed the queuing with inventory. They considered M/M/1 systems with inventory under continuous-review and different inventory management policies with lost sales. Arda and Hennet (2006) analyzed inventory control of a multi-supplier strategy in a two-level supply chain with random arrival for customers and random delivery time for suppliers, the system was represented as a queuing network. Isotupa (2006), considering a lost sales (s, Q) inventory system with two customer groups, they illustrated the model by Markov processes. Jung Woo Baek and Seung Ki Moon (2014), provides a queueing-theoretic model for production-inventory systems with lost sales, develops exact analytical results for performance evaluation, and offers managerial insights into balancing production, inventory levels, and customer service. Seyedhoseini et al. (2015) applied queuing theory to propose a mathematical model for inventory systems with substitute flexibility.

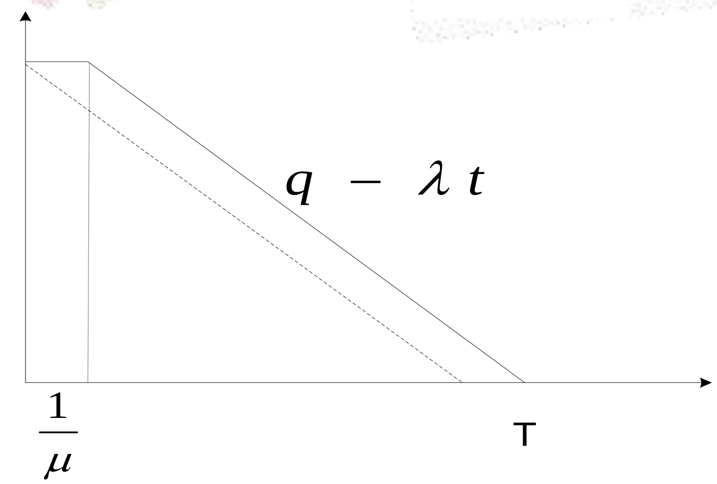
In classical inventory models, the customer is served immediately by providing the item on his arrival and he leaves the place. That item is depleted and can be considered as the arrival of the customer. Also, depletion starts immediately when replenishment is done. But in some cases, like a car, depletion does not start immediately when it reaches the warehouse but some services need to be done on it. This is known as service time. If there is q item inventory for which the average service time is required before starting the sale, then it also includes service time. Therefore, we have assumed that in which the item is served before the customer arrives and the customer has to wait at the place, the queue gradually builds up. Thus, the model has been designed to study what could be the optimal quantity of the item when the queue is formed during uniform quantity. And a hypothetical situation has also been considered to check its effectiveness and sensitivity of the model.

3. Notations and Assumptions for the model:

The model is developed under the very stringent assumption and the notations used for the derivation are as under:

- q = the order quantity
- q* = Economic order quantity to be determined.
- C_3 = Replenishment cost per order which is known and constant
- C_1 = Holding cost per unit per unit time which is known and constant
- L = Lead time which is zero.
- The stockout cost is zero.
- $\lambda =$ Demand rate is constant and known
- $\frac{1}{\mu}$ = Total Service time to serve q items.
- T = Total Cycle time.
- The time horizon if infinite.
- $TC(q)$ = Average total cost per cycle
- The replenishment rate is infinite

Consider the following figure. Where inventory has taken time $\frac{1}{\mu}$ for the process to be done on all items before it starts for depletion. After the time $\frac{1}{\mu}$ items are depleted at the rate of λ and the inventory becomes zero at time T.



4. The Mathematical Model:

In each order the quantity q is ordered in the cycle [0, T] time and hence the inventory level at any time t, $0 \leq t \leq T$ will be,

$$I(t) = q - \lambda t$$

Because of service time, the products stay in the system long before leaving and this stay is delayed by the service time $\frac{1}{\mu}$, which affects the inventory depletion rate. Therefore, the Inventory level at any time t, $0 \leq t \leq T$ will be,

$$I(t) = \begin{cases} q & if\ 0 \leq t \leq \dfrac{1}{\mu} \\ q - \lambda\left(t - \dfrac{1}{\mu}\right) & if\ \dfrac{1}{\mu} < t \leq T \end{cases}$$

Where the total cycle time is $\frac{1}{\mu} + \frac{q}{\lambda}$ and at time T, $I(T) = q - \lambda T = 0$

Thus, the average inventory during time interval [0, T] will be,

$$\frac{1}{T}\int_0^T I(t)dt$$

$$I_1 = \int_0^{1/\mu} q\, dt = \frac{q}{\mu}$$

$$I_2 = \int_{1/\mu}^{T} \left[q - \lambda\left(t - \frac{1}{\mu}\right)\right] dt$$

Let $\left(t - \frac{1}{\mu}\right) = u$, then

$$I_2 = \int_0^{q/\lambda} [q - \lambda u]\, du = \frac{q^2}{2\lambda}$$

Thus, we have, Average Inventory,

$$\frac{1}{T}\int_0^T I(t)dt = \frac{1}{T}\left[\frac{q}{\mu} + \frac{q^2}{2\lambda}\right]$$

Average total cost per cycle is,

$$TC(q) = \left(\frac{q}{T\mu} + \frac{q}{2}\right)C_1 + \frac{\lambda}{q}C_3$$

Now,

$TC(q)$ will be minimum if $\frac{d(TC(q))}{dt} = 0$ and its solution $\frac{d^2(C(t))}{dt^2} > 0$

Let $\frac{d(TC(q))}{dq} = 0$

$$\left(\frac{1}{T\mu} + \frac{1}{2}\right)C_1 - \frac{\lambda}{q^2}C_3 = 0$$

$$q^2 = \frac{\lambda C_3}{C_1}\left(\frac{2T\mu}{T\mu + 2}\right)$$

$$q = \sqrt{\frac{\lambda C_3}{C_1}\left(\frac{2T\mu}{T\mu + 2}\right)}$$

$$q = \sqrt{\frac{\lambda C_3}{C_1}\left(\frac{2T}{T+\frac{2}{\mu}}\right)}$$

And $\frac{d^2(TC(q))}{dt^2} > 0$

The average inventory cost function is minimum at

$$q^* = \sqrt{\frac{\lambda C_3}{C_1}\left(\frac{2T}{T+\frac{2}{\mu}}\right)}$$

As $\frac{1}{\mu} \longrightarrow 0$,

$$q^* = Q^* = \sqrt{\frac{2\lambda C_3}{C_1}}$$

Which is EOQ model for the inventory

5. Sensitive Analysis

To check the effectiveness and utility of the current model along with the theoretical theory when we take the ratio of the quantity obtained by the current model to the quantity obtained by the basic EOQ model it tends to 1 as the service time tends to zero. That is, if we ignore the service time, it becomes the EOQ mode.

$$\lim_{\frac{1}{\mu}\to 0}\frac{q^*}{Q^*} = \lim_{\frac{1}{\mu}\to 0}\sqrt{\frac{T}{T+\frac{2}{\mu}}} = 1$$

Inventory with service will have mor total variable cost than the total variable cost in classical inventory because of service time and hence holding cost. If we reduce the service time by increasing service persons or facilities, we will have no difference between our model and classical inventory model.

6. Hypothetical Numerical Example:

If the supply per year of the product is 12,000 units and the demand is fixed and known then the demand rate will be 1000 units per month. If the inventory holding cost is Rs. 0.20 per unit per month and the ordering cost per order is Rs. 350, the optimal quantity, that is

$$Q^* = \sqrt{\frac{2RC_3}{C_1}} = 1870.829\ units\ per\ order$$

And the total minimum cost per month, that is total variable cost will be,

$$TVC = \sqrt{2C_1C_3R} = Rs.4490$$

In the above example if we consider the arrival rate is equals to demand rate then for the different service rate sensitivity analysis can be obtained as under:

C_1	C_3	λ	μ	λ/μ	q^*	q^*/Q^*
0.20	350	1000	1000	1	1870.766	1.000033333
0.20	350	1000	1500	0.666667	1870.787	1.000022222
0.20	350	1000	2000	0.5	1870.798	1.000016667
0.20	350	1000	2500	0.4	1870.804	1.000013333
0.20	350	1000	3000	0.333333	1870.808	1.000011111
0.20	350	1000	3500	0.285714	1870.811	1.000009524
0.20	350	1000	4000	0.25	1870.813	1.000008333
0.20	350	1000	4500	0.222222	1870.815	1.000007407
0.20	350	1000	5000	0.2	1870.816	1.000006667
0.20	350	1000	5500	0.181818	1870.817	1.000006061
0.20	350	1000	6000	0.166667	1870.818	1.000005556
0.20	350	1000	6500	0.153846	1870.819	1.000005128
0.20	350	1000	7000	0.142857	1870.82	1.000004762
0.20	350	1000	7500	0.133333	1870.82	1.000004444
0.20	350	1000	8000	0.125	1870.821	1.000004167
0.20	350	1000	8500	0.117647	1870.821	1.000003922
0.20	350	1000	9000	0.111111	1870.822	1.000003704
0.20	350	1000	9500	0.105263	1870.822	1.000003509
0.20	350	1000	10000	0.1	1870.822	1.000003333

From the above table it can be seen that for the fixed value of T=30, the optimal quantity q* tends to the optimal quantity Q*= 1870.829 which was obtained by inventory method when the service time $\frac{1}{\mu}$ tends to zero.

7. Conclusion

In the situation of the inventory where after the replenishment of the products or items, the selling of it will not be started immediately but they all have to go through some process before selling, and it increase the time equals to average waiting time of customer and hence all items are in the inventory system increases the holding cost. If we can reduce the process time on the product, that is if we can decrease the waiting time as much as possible the new model becomes the first EOQ model of the inventory.

References

1. Arda Y, Hennet J (2006) ‘Inventory control in a multi-supplier system’, Int J Prod Econ 104:249–259
2. Berman O., Kaplan E.H., and Shimshak D.G. (1993) ‘Deterministic approximations for inventory management at service facilities’, IIE Transactions, 25:5, 98-104.
3. Edward A. Silver (1976) ‘Establishing the Order Quantity When the Amount Received Is Uncertain’, INFOR 14(1), 32-39
4. Ha AY (1997) ‘Stock rationing policy for a make-to-stock production system with two priority classes and backordering’ Nav Res Logist 457–472
5. Isotupa KPS (2006) An (s, Q) Markovian inventory system with lost sales and two demand classes. Math Compute Model 687–694
6. Jung Woo Baek and Seung Ki Moon (2014),’The M/M/1 queue with a production-inventory system and lost sales’, The Applied Mathematics and Computation, Volume 233, pages 534–544.
7. M. Schwarz, C. Sauer, H. Daduna, R. Kulik, R. Szekli (2006),’M/M/1 queuing system with inventory, Queuing System’ Queueing System, 54:55-78
8. S.M.Seyedhoseini, Reza Rashid, Iman Kamalpour, Erfan Zangeneh (2015), ‘Application of queuing theory in inventory systems with substitution flexibility’, The journal of Industrial Engineering International, 11:37-44.

9. William Karush (1957) ‘A queuing for an Inventory Problem’, The journal Operations Research, Volume 5, Issue 5, pages 693-703,