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Mathematical Modelling And Stability Analysis Of Two Prey And One Predator Population Model In Fisheries Management System With Harvesting

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Abstract

This study develops and analyses a nonlinear mathematical model describing the interaction between two prey species and a common predator within a fisheries management framework under the influence of harvesting. The model incorporates logistic growth for prey populations, predator–prey interaction terms, and constant harvesting rates for each species. Local stability analysis is conducted using the Jacobian matrix method at various equilibrium points, while global stability is established via appropriate Routh-Hurwitz stability. The biological feasibility and boundedness of solutions are discussed, ensuring ecological realism. Numerical simulations are presented to illustrate the influence of harvesting rates on population dynamics, demonstrating scenarios of coexistence, predator extinction, and prey dominance. The results provide theoretical insight into sustainable harvesting strategies in multi-species fisheries systems.

Keyword

Two-prey–one-predator model; Fisheries management; Harvesting; Stability analysis; Equilibrium points; Routh Hurwitz criteria; Numerical simulation

Mathematics Subject Classification (MSC 2020)

92D25 (Population dynamics); 92D40 (Ecology); 34D20 (Stability); 37N25 (Dynamical systems in biology); 65P99 (Numerical methods for dynamical systems); 62P10**:** Mathematics of biological, medical, and related sciences.

1. Introduction

Fisheries play a crucial role in global food security, livelihoods, and economic development, yet overexploitation and poor management have led to the depletion of many marine and freshwater species. Effective fisheries management requires a comprehensive understanding of species interactions, environmental constraints, and human exploitation. Mathematical modelling provides a valuable framework for exploring these complex interactions and predicting the outcomes of various management strategies.

Predator–prey models have long been used to study ecological systems, beginning with the classical Lotka–Volterra framework. Over the years, these models have been extended to incorporate more realistic ecological features such as multiple prey species, competition, disease, harvesting, and environmental fluctuations. In particular, multi-species fisheries systems often involve the interaction of multiple prey species and a shared predator, making the two-prey–one-predator configuration an ecologically relevant and mathematically rich case for study.

An example of such a fishing population interaction model can be found in Vembanad Lake, the largest lake in the state of Kerala, India, known for its diverse fish populations and vital role in supporting the livelihoods of thousands of fisherfolk. Among its commercially significant species is the Pearl Spot (*Etroplus suratensis*) and Mullets, a native cichlid highly valued for its taste and nutritional quality. This species coexists with other prey fish and is preyed upon by larger predatory fish such as Barramundi (*Lates calcarifer*). Due to high market demand, the Pearl Spot has been subjected to intensive harvesting, raising concerns about population decline, predator-prey imbalances, and the sustainability of the lake's fisheries.

The interaction between predator prey populations in fish populations where pearl spot and Mullet fish as prey and Barramundi fish as predator formed in mathematical modeling understanding the dynamics from harvesting fish and obtain income of fisherman and taking into ecological aspects in maintaining the ecosystem, ecological aspects refer to the various ways in which living organisms interact with their environment and with each other. Mathematical Model is suitable for this paper predator prey model with pearl spot fish as prey and Barramundi fish as predator that follows logistic growth rate model. V. Volterra in 1926 and also known as the Lotka-Volterra model which describes interactions between predator populations or competitive interactions between populations. The Leslie–Gower predator–prey model is a modification of the classical Lotka–Volterra system where predator growth depends on the ratio of prey to predators, representing the idea that predator growth is limited not only by prey availability but also by intraspecific competition among predators.

Harvesting is an important component in fisheries models, as it directly impacts population sizes and alters the dynamics of species interactions. The inclusion of harvesting in mathematical models enables the assessment of sustainable yield, extinction risk, and long-term stability of fish populations. Understanding how different harvesting strategies affect the stability of ecological equilibria can guide policymakers and fisheries managers in designing effective conservation and exploitation policies.

The predator–prey model considered in this study is based on the logistic growth assumption and incorporates harvesting in a two prey one predator population system. The modelling process begins with determining the equilibrium points, followed by a stability analysis of these points to examine the system's behaviour over time. Numerical simulations are performed using Matlab software to illustrate the theoretical results. The simulations are conducted under the assumption that all three populations two prey and one predator possess economic value and contribute to the income of local fishing communities. The goal is to ensure that these populations can coexist, maintain ecological sustainability, and remain stable despite continuous harvesting pressure. Based on the description, the purpose of this study is to analyze the stability of the predator prey models in fishery populations and harvesting in all the multi species predator prey populations.

2. Predator prey Mathematical Model

The results indicate that the fishery predator–prey model for the populations A, C, G uses a logistic equation for the prey and predator. Harvesting occurs in both predator and prey populations, with parameters h_i (i=1,2 ,3) representing the harvesting effort on each population. Following we assume additional predator–prey interactions with Holling Type II functional response as equation

$$\frac{dA}{dt} = rA\left(1 - \frac{A}{k}\right) - \frac{a_1 AG}{1+h_1 A} - V_1 A - qA$$

$$\frac{dC}{dt} = rC\left(1 - \frac{C}{k}\right) - \frac{a_2 CG}{1+h_2 C} - V_2 C - qC$$

$$\frac{dG}{dt} = -JG + \delta AG + \delta CG + ds - V_3 G$$

Where,

A→ pearl spot fish as prey
C→ Mullet fish as prey
G→ Barramundi fish as predator
$a_{1,} a_2$ → Attack rate for prey and predator
V_1, V_2, V_3 → Conversion coefficient of harvesting effort
J→ natural death rate in predator
q→ natural death rate in prey
δ → Interaction between prey(pearl spot and Mullets fish) and Predator(Barramundi fish)
ds→ Conversion rate s with positive parameter
k→ carrying capacity
r → Intrinsic growth rate

3. Positiveness and Boundedness of the system

In this phase, we seek to establish the requirements for obtaining both a positive and bounded solution of the system.

$$\frac{dA}{dt} = rA\left(1 - \frac{A}{k}\right) - \frac{a_1 AG}{1+h_1 A} - V_1 A - qA$$

Where

$$\varphi(A, G) = rA\left(1 - \frac{A}{k}\right) - \frac{a_1 AG}{1+h_1 A} - V_1 A - qA$$

By integrating the area [0, t], we obtain

A(t)= A(0) exp$\int \varphi(A, G)dt > 0$ ∀ as A(0)≥ 0

Second set of equation becomes'

$$\frac{dC}{dt} = rC\left(1 - \frac{C}{k}\right) - \frac{a_1 CG}{1+h_2 C} - V_2 C - qC$$

Where

$$\phi(C, G) = rC\left(1 - \frac{C}{k}\right) - \frac{a_1 CG}{1+h_2 C} - V_2 C - qC$$

By integrating the area [0, t], we obtain

C(t)= C(0) exp$\int \phi(C, G)dt > 0$ ∀ as C(0)≥ 0

Third set of equation becomes'

$$\frac{dG}{dt} = -JG + \delta AG + \delta CG + ds - V_3 G$$

Where

$$\psi(A, C, G) = -JG + \delta AG + \delta CG + ds - V_3 G$$

By integrating the area [0, t], we obtain

G(t)= G(0) exp$\int \psi(A, C, G)dt > 0$ ∀ as G(0)≥ 0

As a result, to conclude that system solutions are always positive.

Proposition 1

It follows that in the positive octant domain $R_+^3 = \{A(t), C(t), G(t) \epsilon R^3 : A \geq 0, C \geq 0, G \geq 0\}$for all solution of system are bounded.

Theorem 4.1:
Both preys are always bounded above when $A > 0; and\ C > 0$.
Proof:
If A (0) =0, the outcomes is straightforward. If w (0)>0, then w (t)>0 for all t by adding equation (2.1).
We get

$$\frac{dw}{dt} \leq r\,A\left(1 - \frac{A}{k}\right) \quad \frac{dx}{dt} \leq rC\left(1 - \frac{C}{k}\right)$$

$$\lim_{t \to \infty} \sup A(t) \leq k ,$$

$$\lim_{t \to \infty} \sup A(t) \leq k ,$$

Theorem 4.2:
predator are bounded above.

Proof:

If G(0) =0 The outcomes are evident.

We obtained the equation. (2.1)

$$\frac{dG}{dt} < 0 \text{ if } \frac{ds}{J + v_3} < 0$$

Theorem 4.3:

The system of the equation (2.1) has bounded trajectories

Proof:

We differentiate with respect to t, we obtain

$$\frac{dH(t)}{dt} = \frac{dA(t)}{dt} + \frac{dC(t)}{dt} + \frac{dG(t)}{dt}$$

$$\frac{dH(t)}{dt} = r\,A\left(1 - \frac{A}{k}\right) + rC\left(1 - \frac{C}{K}\right) - JG - V_3 Gw + \rho A + \rho C + \rho G + (\rho + r)A + (\rho + r)C ++(\rho - J - V_3)G - \frac{rA^2}{k} - \frac{rC^2}{k}$$

Where $\rho\ is\ a\ positive\ constant\ for$

$$\rho > J or\ \rho > V_3\ or\ \rho(J + V_3) given\ \epsilon > 0\ there\ exists\ to\ such\ that\ t\ on\ t \geq t_0$$

$$m = \min\{((\rho + r) + (\rho + r) + (\rho - J) + (\rho - V_3)\}$$

$$hence \ \frac{d}{dt}(le^{\rho t}) \leq (m+\epsilon)e^{\rho t} \ \ L(t) \leq L(t_0)e^{-\rho(t-t_0)} + \left(\frac{m+l}{\rho}\right)\left(1-e^{-\rho(t-t_0)}\right)$$

Let $t \rightarrow 0, then\ letting\ \epsilon \rightarrow 0$

$$\lim_{t\to\infty} \sup L(t) \leq \frac{m}{\rho}$$

4. Equilibrium points

The steady-state equations define the parametric model's a state of equilibrium point (2.1).

$$\frac{dH}{dt} = \frac{dA}{dt} = \frac{dC}{dt} = \frac{dG}{dt} = 0$$

1) Trivial Equilibrium point
   (A=0,C=0,G=0)
2) Mullet fish prey free and Barramundi fish predator free equilibrium points
   (A=k, C=0, G=0)

3) Pearl spot fish and Mullet fish prey free equilibrium point
   (A=0, C=0, G= $\frac{ds}{J+V_3}$)
4) Mullet fish prey free and Barramundi fish predator free equilibrium point
   (A= k(1$-\frac{V_1+qA}{rA}$) $, C=0, G=0$)
5) Pearl spot fish prey free and Barramundi fish predator free equilibrium point
   (A=0,C= k(1$-\frac{V_2+qC}{rC}$) $, G=0$)
6) Non trivial equilibrium point
   (A= $\frac{J-ds+V_3ds}{\delta(J+V_3)}, \ C = \frac{J+V_3ds}{\delta(J+V_3)} - ds, G = \frac{ds}{J+V_3}$)

The resultant system of equation (2.1) is the Jacobian matrix provided by V

$$J=\begin{bmatrix} T_{11} & 0 & -\frac{a_1A}{1+h_1A} \\ 0 & T_{22} & -\frac{a_2C}{1+h_2C} \\ \delta G & \delta G & T_{33} \end{bmatrix}$$

Where,

$$T_{11} = r\left(1-\frac{2A}{K}\right) - \frac{(1+h_1A)a_1G - a_1AG(1+h_1)}{(1+h_1A)^2} - V_1 - q$$
$$T_{22} = r\left(1-\frac{2C}{K}\right) - \frac{(1+h_2C)a_2G - a_2CG(1+h_2)}{(1+h_2C)^2} - V_2 - q$$
$$T_{33} = -J + \delta A + \delta C + ds - V_3$$

5. Stability analysis

Theorem 5.1

Given the nonlinear differential equation(2.1) of the system, therefore the equilibrium point is (0,0,0) is locally asymptotically stable.

Proof:

The Jacobian matrix is

$$J_1 = \begin{bmatrix} r-V_1-q & 0 & 0 \\ 0 & r-V_2-q & 0 \\ 0 & 0 & -J+ds-V_3 \end{bmatrix}$$

Where,

A= $V_1 + q$

B= $V_2 + q$

The eigenvalues are $\lambda_1 =$ r−A >0, if A> r, $\lambda_2 =$ r−B >0, if B> r , $\lambda_3 = -J + ds - V_3 > 0$ are locally asymptotically stable.

Theorem 5.2

Given the nonlinear differential equation(2.1) of the system, therefore the equilibrium point is (K,0,0) is locally asymptotically stable.

Proof:

The Jacobian matrix is

$$J_2 = \begin{bmatrix} -r - V_1 - q & 0 & -\frac{a_1 k}{1+h_1 k} \\ 0 & r - V_2 - q & 0 \\ 0 & 0 & -J + ds - V_3 \end{bmatrix}$$

Where,

A= $V_2 + q$

The eigenvalues are $\lambda_1 = -r - V_1 - q > 0$, $\lambda_2 =$ r−B >0, if B> r , $\lambda_3 = -J + ds - V_3 > 0$ are locally asymptotically stable.

Theorem 5.3

Given the nonlinear differential equation(2.1) of the system, therefore the equilibrium point is (0,0,$\frac{ds}{J+V_3}$) is locally asymptotically stable.

Proof:

The Jacobian matrix is

$$J_3 = \begin{bmatrix} r - V_1 - q & 0 & 0 \\ 0 & r - V_2 - q & 0 \\ \frac{\delta ds}{J+V_3} & \frac{\delta ds}{J+V_3} & -J + ds - V_3 \end{bmatrix}$$

Where,

A= $V_1 + q$
B= $V_2 + q$

The eigenvalues are $\lambda_1 = r - A > 0$, if A>r, $\lambda_2 =$ r−B >0, if B> r , $\lambda_3 = -J + ds - V_3 > 0$ are locally asymptotically stable.

Theorem 5.4

Given the nonlinear differential equation(2.1) of the system, therefore the equilibrium point is (k(1−$\frac{V_1+qA}{rA}$) ,0,0) is locally asymptotically stable.

Proof:

The Jacobian matrix is

$$J_4 = \begin{bmatrix} T_{11} & 0 & \frac{-a_1 k\left(1-\frac{V_1+qA}{rA}\right)}{\left(1+h_1 k\left(1-\frac{V_1+qA}{rA}\right)\right)} \\ 0 & r - V_2 - q & 0 \\ 0 & 0 & -J + \delta k\left(1 - \frac{V_1+qA}{rA}\right) + ds - V_3 \end{bmatrix}$$

Where,

$$T_{11} = r\left(1 - 2\left(1 - \frac{V_1+qA}{rA}\right) - \frac{\left(1+h_1 \mathrm{k}\left(1-\frac{V_1+qA}{rA}\right)\right)a_1 G - a_1 \mathrm{k}\left(1-\frac{V_1+qA}{rA}\right)G(1+h_1)}{(1+h_1\left(1-\frac{V_1+qA}{rA}\right))^2}\right) - V_1 - q$$

$$\mathrm{A}= \left(1 - 2\left(1 - \frac{V_1+qA}{rA}\right) - \frac{\left(1+h_1 \mathrm{k}\left(1-\frac{V_1+qA}{rA}\right)\right)a_1 G - a_1 \mathrm{k}\left(1-\frac{V_1+qA}{rA}\right)G(1+h_1)}{(1+h_1\left(1-\frac{V_1+qA}{rA}\right))^2}\right) + V_1 + q$$

B= $V_2 + q$

The eigenvalues are $\lambda_1 = r - A > 0$,if A>r, $\lambda_2 =$ r−B >0, if B> r , $\lambda_3 = -J + \delta k\left(1 - \frac{V_1+qA}{rA}\right) + ds - V_3 > 0$ are locally asymptotically stable.

Theorem 5.5

Given the nonlinear differential equation(2.1) of the system, therefore the equilibrium point is (0,k(1−$\frac{V_2+qC}{rC}$) ,0) is locally asymptotically stable.

Proof:

The Jacobian matrix is

$$J_5 = \begin{bmatrix} r - V_2 - q & 0 & 0 \\ 0 & T_{22} & \frac{-a_1 k\left(1-\frac{V_2+qC}{rC}\right)}{\left(1+h_1 k\left(1-\frac{V_2+qC}{rC}\right)\right)} \\ 0 & 0 & -J + \delta k\left(1 - \frac{V_2+qC}{rC}\right) + ds - V_3 \end{bmatrix}$$

Where,

$$T_{22} = r\left(1 - 2\left(1 - \frac{V_2+qC}{rC}\right) - \frac{\left(1+h_2 \mathrm{k}\left(1-\frac{V_2+qC}{rC}\right)\right)a_2 G - a_2 \mathrm{k}\left(1-\frac{V_2+qC}{rC}\right)G(1+h_2)}{(1+h_2\left(1-\frac{V_2+qC}{rC}\right))^2}\right) - V_2 - q$$

$$\mathrm{B}= r\left(1 - 2\left(1 - \frac{V_2+qC}{rC}\right) - \frac{\left(1+h_2 \mathrm{k}\left(1-\frac{V_2+qC}{rC}\right)\right)a_2 G - a_2 \mathrm{k}\left(1-\frac{V_2+qC}{rC}\right)G(1+h_2)}{(1+h_2\left(1-\frac{V_2+qC}{rC}\right))^2}\right) + V_2 + q$$

A= $V_1 + q$

The eigenvalues are $\lambda_1 =$ r−$A > 0$,if A>r , $\lambda_2 = r - B > 0$, if b>r, $\lambda_3 = -J + \delta k\left(1 - \frac{V_2+qC}{rC}\right) + ds - V_3 > 0$ are locally asymptotically stable

Theorem 5.6

Given the nonlinear differential equation(2.1) of the system, therefore the equilibrium point is $\left(\frac{J-ds+V_3ds}{\delta(J+V_3)}, \frac{J+V_3ds}{\delta(J+V_3)} - ds, \frac{ds}{J+V_3}\right)$ is locally asymptotically stable.

Proof:

The Jacobian matrix is

$$J_6 = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$$

Where,

$$T_{11} = r\left(1 - \frac{2\frac{J-ds+V_3ds}{\delta(J+V_3)}}{K}\right) - \frac{\left(1+h_1\frac{J-ds+V_3ds}{\delta(J+V_3)}\right)a_1\left(\frac{ds}{J+V_3}\right) - a_1\left(\frac{J-ds+V_3ds}{\delta(J+V_3)}\right)\left(\frac{ds}{J+V_3}\right)(1+h_1)}{\left(1+h_1\frac{J-ds+V_3ds}{\delta(J+V_3)}\right)^2} - V_1 - q$$

$$T_{12} = 0$$

$$T_{13} = \frac{-a_1\left(\frac{J-ds+V_3ds}{\delta(J+V_3)}\right)}{\left(1+h_1\frac{J-ds+V_3ds}{\delta(J+V_3)}\right)}$$

$$T_{21} = 0$$

$$T_{22} = r\left(1 - \frac{2\frac{J+V_3ds}{\delta(J+V_3)} - ds}{K}\right) - \frac{\left(1+h_2\frac{J+V_3ds}{\delta(J+V_3)} - ds\right)a_2\left(\frac{ds}{J+V_3}\right) - a_2\left(\frac{J+V_3ds}{\delta(J+V_3)} - ds\right)\left(\frac{ds}{J+V_3}\right)(1+h_2)}{\left(1+h_2\frac{J+V_3ds}{\delta(J+V_3)} - ds\right)^2} - V_2 - q$$

$$T_{23} = \frac{-a_2\left(\frac{J+V_3ds}{\delta(J+V_3)} - ds\right)}{\left(1+h_2\frac{J+V_3ds}{\delta(J+V_3)} - ds\right)}$$

$$T_{31} = \delta\left(\frac{ds}{J+V_3}\right)$$

$$T_{32} = \delta\left(\frac{ds}{J+V_3}\right)$$

$$T_{33} = -J + \delta\left(\frac{J-ds+V_3ds}{\delta(J+V_3)}\right) + \delta\left(\frac{J+V_3ds}{\delta(J+V_3)}\right) + ds - V_3$$

The eigenvalues are $\lambda_1 = r\left(1 - \frac{2\frac{J-ds+V_3ds}{\delta(J+V_3)}}{K}\right) - \frac{\left(1+h_1\frac{J-ds+V_3ds}{\delta(J+V_3)}\right)a_1\left(\frac{ds}{J+V_3}\right) - a_1\left(\frac{J-ds+V_3ds}{\delta(J+V_3)}\right)\left(\frac{ds}{J+V_3}\right)(1+h_1)}{\left(1+h_1\frac{J-ds+V_3ds}{\delta(J+V_3)}\right)^2} - V_1 - q > 0$, $\lambda_2 = r\left(1 - \frac{2\frac{J+V_3ds}{\delta(J+V_3)} - ds}{K}\right) - \frac{\left(1+h_2\frac{J+V_3ds}{\delta(J+V_3)} - ds\right)a_2\left(\frac{ds}{J+V_3}\right) - a_2\left(\frac{J+V_3ds}{\delta(J+V_3)} - ds\right)\left(\frac{ds}{J+V_3}\right)(1+h_2)}{\left(1+h_2\frac{J+V_3ds}{\delta(J+V_3)} - ds\right)^2} - V_2 - q > 0$, $\lambda_3 = -J + \delta\left(\frac{J-ds+V_3ds}{\delta(J+V_3)}\right) + \delta\left(\frac{J+V_3ds}{\delta(J+V_3)}\right) + ds - V_3 > 0$

The characteristic equation is $\Lambda_1(\lambda) = B_1\lambda^3 + B_2\lambda^2 + B_3\lambda + B_4$

$B_1 = 1 > 0$

$B_2 = -(T_{11} + T_{22} + T_{33}) > 0$

$B_3 = -(-T_{11}T_{22} - T_{11}T_{33} + T_{12}T_{21} + T_{13}T_{31} - T_{23}T_{32} + T_{22}T_{33}) > 0$

$B_4 = T_{11}T_{22}T_{33} + T_{11}T_{23}T_{32} + T_{12}T_{21}T_{33} - T_{12}T_{23}T_{31} - T_{13}T_{21}T_{32} + T_{13}T_{22}T_{31}) > 0$

By Routh Hurwitz's criterion, all the eigenvalues of J_6 have negative real parts if

i) $B_1 > 0$
ii) $B_3 > 0$
iii) $B_1B_2B_3 > B_3^2 + B_1^2B_4$

the system (2.1) is locally asymptotically stable.

6. Numerical solution

The value of parameter K=100,r= 2.5, r=2.6, $\delta = 0.8, d = 0.7, S = 0,6$ we assume from the fisheries management in harvesting effort.

First simulation as h= 0.2. With this set of parameter values the equilibria are (3.7568,0.7654,3,9863) from the pearl spot fish prey population (Figure 1).

Second simulation as h= 0.2. With this set of parameter values the equilibria are (2.7532,0.08654,0.2863) from the Mullet fish prey population (Figure 2).

Third simulation as h= 0.2. With this set of parameter values the equilibria are (1.2569,0.0432,0.3663) from the Barramundi fish predator population (Figure 3).

Fourth stimulation as h=0.2 with this set of parameter values the equilibria are (0.8976,0.2365.0.5612) from two prey and one predator population(Figure 4).

The following figure shows the solution curves for the prey predator population in fisheries management.

7. Conclusion

This study demonstrates the value of mathematical modelling in managing multi-species fisheries. Using a two-prey–one-predator model with logistic growth and harvesting, we showed how fishing pressure affects both ecological stability and economic outcomes. The Vembanad Lake case highlights the practical importance of balancing harvesting to ensure species coexistence, biodiversity conservation, and fisherfolk livelihoods. The case of Vembanad Lake, with Pearl Spot and Mullets as prey species and Barramundi as a predator, illustrates the real-world relevance of such models. Incorporating harvesting into the mathematical framework allows us to assess the impact of fishing intensity on long-term population persistence and income generation. Numerical simulations confirm that balanced harvesting strategies are critical to maintaining biodiversity, preventing extinction, and ensuring stable population dynamics.Overall, the model provides useful insights for sustainable fisheries management, with potential extensions to include environmental variability and market influences.

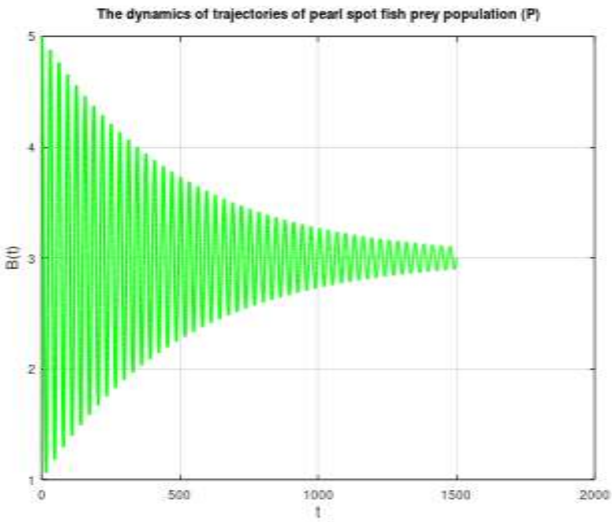

Figure 1 shows the dynamics of trajectories of solution curve in pearl spot fish prey population

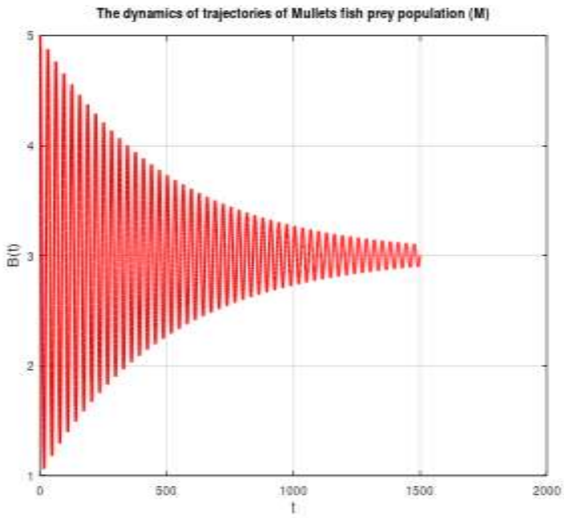

Figure 2 shows the dynamics of trajectories of solution curve in Mullet fish prey population

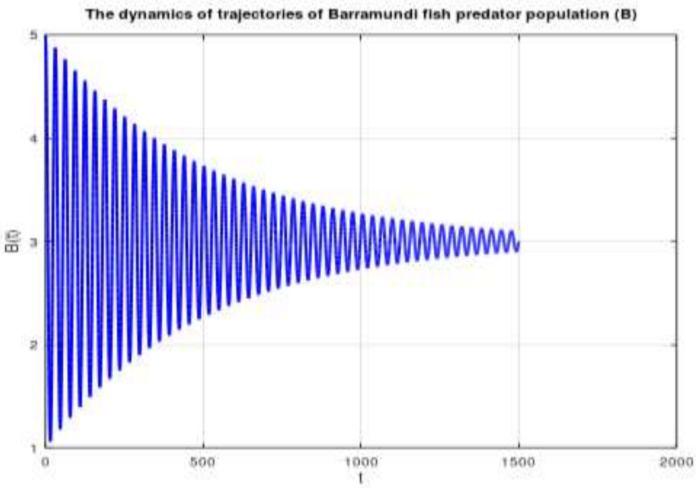

Figure 3 shows the dynamics of trajectories of solution curve in Barramundi fish predator population

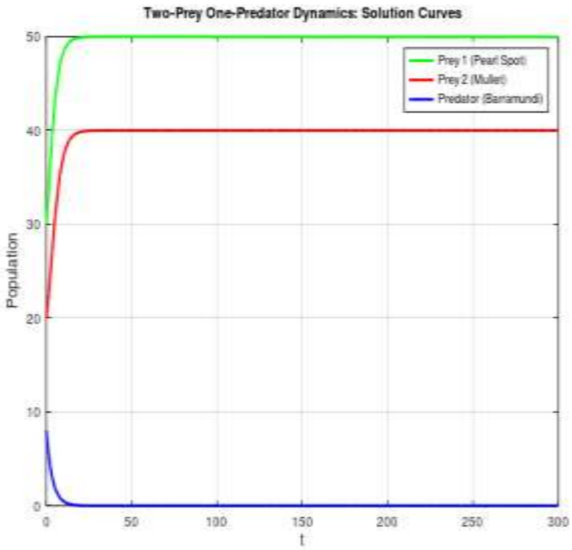

Figure 4 shows Two prey (pearl spot, Mullet fish) and one predator(Barramundi fish) dynamics of trajectories of solution curve

8. Acknowledgement

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9.Conflict of Interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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