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ON s(gg)*- CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce a new class of closed sets called semi generalization of generalized star closed (briefly s(gg)* - closed) sets in Topological Spaces. We study the relation of this set with some other closed sets and some of the properties have been investigated.

Key words: s(gg)* - closed set, (gg)* - open
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I. Introduction

The concept of generalized closed sets [1] in Topological spaces was introduced by N. Levine in 1970. D. E. Cameron and M. Stone introduced regular semi open sets [2] and regular open sets [7] respectively. In 2018, T. Shyla Isac Mary and I.Christal Bai introduced (gg)* - closed sets[7] in Topological spaces. In this paper we introduce a new class of closed set called s(gg)* - closed sets in Topological spaces. In section 2, we recall some of the existing closed and open sets. In section 3, the concept of s(gg)* - closed set is introduced. In section 4, some of the properties of s(gg)* - closed sets are studied and the references are given at the end of the paper.

II. PRELIMINARIES

Throughout this paper (X, τ) represent nonempty topological spaces on which no separation axioms are assumed unless

otherwise mentioned. For a subset A of X, the closure of A and interior of A are denoted by *cl* (A) and *int* (A) respectively.

Definition 2.1 A subset A of a topological space (X, τ) is called a

(1) semi - open set [2] if $A \subseteq cl(int(A))$ and a semi - closed set if $int(cl(A)) \subseteq A$.

(2) α- open set[10] if $A \subseteq int(cl(int(A)))$ and α - closed set if $cl(int(cl(A))) \subseteq A$.
(3) regular open set[9] if $A = int(cl(A))$ and a regular closed set if $cl(int(A)) = A$.

Definition 2.2 A subset A of a topological space X is called a

(1) generalized - closed set (briefly g - closed) [1] if *cl* (A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X .

(2) generalized pre - regular closed set (briefly gpr - closed)[8] if *pcl (A)* $\subseteq$U whenever A $\subseteq$ U and U is regular open in X.

(3) generalized semi - pre regular - closed set (briefly gspr - closed) [9] if *spcl*(A) $\subseteq$U whenever A $\subseteq$ U and U is regular open in X.

(4) regular semi open[5] if there is a regular open set U such that U $\subseteq$ A $\subseteq$ *cl* (U)

(5) (gg)*- closed[7] if *rcl* (A) $\subseteq$ U whenever A $\subseteq$ U and U is gg - open.

The complements of the above closed sets are their open sets and vice versa.

3. s(gg)*- CLOSED SETS

Definition 3.1 A subset A of a topological space (X, τ) is called s(gg)*- closed if *scl* (A) $\subseteq$ U whenever A $\subseteq$ U and U is (gg)* - open.

Example 3.2 Let X= {a, b, c, d}, and τ = { φ, { c}, {d}, {c, d}, {a, c, d}, X}

(gg)* - closed = { φ, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X}

s(gg)*-closed={ φ, {a},{b},{c},{d},{a, b}, {b, c},{a,c},{a,d} {b, d}, {a, b, c}, {a,c,d},{a, b, d}, {b, c, d}, X}

Theorem 3.3

Every semi closed set is $s(gg)^*$ - closed.

Proof:

Let A be a semi closed set in X such that $A \subseteq U$ and U is $(gg)^*$ -open.

Then $scl(A) = A \subseteq U$ and U is $(gg)^*$ -open.

Hence $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gg)^*$ -open.

Hence A is $s(gg)^*$ - closed.

Remark 3.5

The Converse of the above theorem need not be true as shown in the following example.

Example 3.6

Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$

$s(gg)^*$ - closed= $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, X\}$

Semi-closed = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$

The set $\{a, c, d\}$ is $s(gg)^*$ - closed but not semi-closed.

Theorem 3.7 Every closed set is $s(gg)^*$ - closed.

Proof:

Let A be a closed set in X such that $A \subseteq U$ and U is $(gg)^*$ -open.

Since A is closed, $cl(A) = A \in U$ and we have $scl(A) \subseteq cl(A) scl(A) \subseteq cl(A) \subseteq U$

i.e, $scl(A) \subseteq U$.Therefore, $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gg)^*$ -open.

Hence A is a $s(gg)*$ - closed .

Remark 3.8 The Converse of the above theorem need not be true as given in the following example.

Example 3.9 $X = \{a, b, c, d\}, \tau = \{\emptyset, \{c\}, \{d\}, \{c, d, \}, X\}. \tau_c = \{\emptyset, \{a, b, d\}, \{a, b, c\}, \{a, b\}, X\}$,

$s(gg)^*$ - closed = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}\{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\},$

$\{a, c, d\}, \{a, b, d\}, \{b, c, d\}, X\}$

The set $\{a\}$ is $s(gg)^*$ - closed but not closed.

Theorem 3.10

Every regular closed set is $s(gg)^*$- closed.

Proof:

Let A be a regular closed set in X such that $A \subseteq U$ and U is $(gg)^*$ -open.

Since A is regular closed $rcl(A) = A \subseteq U$

But $scl(A) \subseteq rcl(A)$

Therefore, $scl(A) \subseteq U$ and U is $(gg)^*$ -open .

Therefore, A is $s(gg)^*$- closed.

Remark 3.11 The Converse of the above theorem need not be true as shown in the following example.

Example 3.12

Let$X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, X\}$

$s(gg)^*$-closed =$\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

regular closed = $\{\emptyset, X, \{a, b, c\}, \{a, b, d\}\}$

The set $\{a, c, d\}$ is $s(gg)^*$- closed but not closed.

Theorem 3.13 Every α-closed set is $s(gg)^*$- closed.

Proof:

Let A be a α-closed set in X such that $A \subseteq U$ and U is $(gg)^*$ -open.

Since A is α- closed $\alpha cl(A) = A \subseteq U$

But $scl(A) \subseteq \alpha cl(A)$

Therefore, $scl(A) \subseteq U$,whenever $A \subseteq U$ and U is $(gg)^*$ -open.

Therefore, A is $s(gg)^*$- closed.

Remark 3.14

The Converse of the above theorem need not be true.

Example 3.15 Let $X = \{a, b, c, d\}, \tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, X\}$

$s(gg)^*$-closed =$\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

α- closed = $\{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$

Then $\{b, c, d\}$ is s $s(gg)^*$-closed but not α- closed.

Theorem 3.16

Every $s(gg)^*$- closed set is $gspr$-closed.

Proof:

Let A be a $s(gg)^*$- closed set in X.

Let U be a regular open set in X such that $A \subseteq U$

Since every regular open set is $(gg)^*$ -open and A is $s(gg)^*$- closed, $scl(A) \subseteq U$

But $spcl(A) \subseteq scl(A) \subseteq U$.

This implies $spcl(A) \subseteq U$

Therefore, A is $gspr$-closed .

Remark 3.17

The Converse of the above theorem need not be true as shown in the following example.

Example 5.17

Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{c, d\}, \{a, c, d\}, X\}$

$(gg)^*$- closed = $\{\emptyset, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

$(gg)^*$- open = $\{\emptyset, \{a, c, d\}\{c, d\}, \{d\}, \{c\}, \{a\}, X\}$

$s(gg)^*$- closed = $\{\emptyset, \{a\}, \{b\}, \{a, b\}\{a, c\}\{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

$gspr$-closed-

$\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, X\}$

The set $\{c\}$ is $gspr$-closed but not $s(gg)^*$- closed other.

4. Characteristics of $s(gg)^*$- closed sets

Theorem 4.1

Union of any two $s(gg)^*$- closed sets of X is $s(gg)^*$- closed.

Proof:

Let A and B be $s(gg)^*$- closed sets in X.

Let U be a $(gg)^*$- open set in X such that $A \cup B \subseteq U$.

Then $A \subseteq U$ and $B \subseteq U$.

Since A and B are $s(gg)^*$- closed sets in X, $scl(A) \subseteq U$ and $scl(B) \subseteq U$

We have $scl(A \cup B) = scl(A) \cup scl(B) \subseteq U$.

This implies $scl(A \cup B) \subseteq U$

Hence $A \cup B$ is $s(gg)^*$- closed .

Theorem 4.2 A subset A of X is $s(gg)^*$- closed set in X if and only if $scl(A) - A$ contains no non-empty $(gg)^*$- closed set.

Proof :

Let F be a non-empty $(gg)^*$-closed set in X such that $F \subseteq scl(A) - A$.

That is $F \subseteq scl(A) \cap A^C$

Therefore $F \subseteq scl(A)$ and $F \subseteq A^C$ and so $A \subseteq F^C$.

Now since A is s$(gg)^*$-closed, and F^C is (gg)*-open, scl$(A) \subseteq F^C$.

This implies $F \subseteq [scl(A)]^C$. Also we have $F \subseteq scl(A)$.

Therefore $F \subseteq scl(A) \cap [scl(A)]^C = \emptyset$.

This is a contradiction.

Therefore $scl(A) - A$ contains no non-empty $(gg)^*$-closed set.

Conversely, suppose that $scl(A) - A$ contains no non-empty $(gg)^*$-- closed set.

Suppose $scl(A)$ is not contained in U.

Let U be a (gg)*-open set in X such that $A \subseteq U$.

Then $scl(A) \cap U^C$ is a non - empty (gg)*-closed set contained in $scl(A) - A$.

Which is a contradiction.

Hence A is a $s(gg)^*$-closed set.

Theorem 4.3

Let $A \subseteq B \subseteq scl(A)$ and A is $s(gg)^*$-closed set in X, then B is also $s(gg)^*$-closed.

Proof:

Let U be a $(gg)^*$-open set in X such that $A \subseteq U$.

Now if $A \subseteq B \subseteq scl(A)$, then $scl(A) \subseteq scl(B) \subseteq scl(A)$.

Therefore $scl(B) = scl(A)$. Since A is $s(gg)^*$-closed, $scl(A) \subseteq U$.

Therefore $scl(B) = scl(A) \subseteq U$.

Hence B is $s(gg)^*$- closed.

Theorem 4.4

If A is $(gg)^*$-open subset of X and $s(gg)^*$-closed set in X . Then A is a semi closed set in X.

Proof:

Let A be a $(gg)^*$-open subset of X and a $s(gg)^*$-closed set in X.

Since A is $s(gg)^*$- closed , $scl(A) \subseteq A$.

But $A \subseteq scl(A)$.

Therefore $A = scl(A)$.

Hence A is semi closed.

Theorem 4.5

Let $A \subseteq B \subseteq X$, where B is $(gg)^*$-open and $s(gg)^*$-closed in X. If A is $s(gg)^*$-closed in B. Then A is $s(gg)^*$-closed in X.

Proof:

Let U be a $(gg)^*$-open set in X such that $A \subseteq U$.

Since $A \subseteq U \cap B, U \cap B$ is $(gg)^*$-open in B and A is $s(gg)^*$-closed in $B, scl(A) \subseteq U \cap B$.

Now $scl(A) \cap B \subseteq U \cap B$. Since $A \subseteq B, scl(A) \subseteq scl(B)$.

Since B is $(gg)^*$-open and $s(gg)^*$- closed in X, by theorem 4.4, B is semi closed. Therefore $scl(B) = B$.

This implies $scl(A) \subseteq B$.

Thus $scl(A) = scl(A) \cap B \subseteq U \cap B \subseteq U$.

Hence A is $s(gg)^*$-closed in X.

Theorem 4.6

For every point x of the space X the set $X - \{x\}$ is either $s(gg)^*$-closed (or) $(gg)^*$-open.

Proof:

Suppose that $X - \{x\}$ is not $(gg)^*$-open .Then X is the only $(gg)^*$-open set containing $X - \{x\}$.

That is $X - \{x\} \subseteq X$.

This implies $scl(X - \{x\}) \subseteq scl(X) \subseteq X$.

Therefore $X - \{x\}$ is a $s(gg)^*$- closed set in X.

Theorem 4.7

A subset A of a space X is $s(gg)^*$-closed if and only if for each $A \subseteq F$ and F is $(gg)^*$-open, there exists a $s$emi-closed set G such that $A \subseteq G \subseteq F$.

Proof:

Suppose A is a $s(gg)^*$-closed set and $A \subseteq F$ and F is $(gg)^*$-open.

Then $scl(A) \subseteq F$.

If G = $scl(A)$, then G is $s$emi-closed set and $A \subseteq G \subseteq F$.

Conversely, assume that F is a $(gg)^*$-open set containing A. Then there exists a $s$emi-closed set M such that $A \subseteq G \subseteq F$.

Since $scl(A)$ is the smallest semi-closed set containing A, we have $A \subseteq scl(A) \subseteq G$.

Also, since $G \subseteq F, scl(A) \subseteq F$.

Hence A is a $s(gg)^*$-closed set in X.

Theorem 4.8 If A is semi-closed and B is $s(gg)^*$-closed subset of a space X then $A \cup B$ is $s(gg)^*$-closed.

Proof:

Let F be a $(gg)^*$-open set containing $A \cup B$.

Then $A \subseteq F$ and $B \subseteq F$. Since B is $s(gg)^*$-closed and $B \subseteq F$,

we have $scl(B) \subseteq F$.

Then $A \cup B \subseteq A \cup (scl(B)) \subseteq F$.

Since A is s-closed, we have $A \cup (scl(B))$ is s-closed.

Hence there exist a s-closed set $A \cup (scl(B))$ such that $A \cup B \subseteq A \cup (scl(B)) \subseteq F$. Therefore $A \cup B$ is $s(gg)^*$-closed.

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