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On Fuzzy Semi-Hyper Connected Spaces

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Abstract: In this paper the concept of fuzzy semi- hyper connected spaces in fuzzy topological Spaces are introduced. Several characterizations of fuzzy semi- hyper connected space and the relations of fuzzy semi-hyper connected space and other fuzzy topological spaces are studied.

Keywords: Fuzzy semi-open set, Fuzzy semi-nowhere dense set, fuzzy semi-first category, fuzzy semi-dense, fuzzy semi-Baire space and fuzzy semi hyper-connected space.

I. Introduction

Among the various fields of Mathematics, the first to be considered in the context of fuzzy sets was general topology. In the year 1968, C.L.Chang [3] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces. Since C.L.Chang applied fuzzy set theory to topology, many topological notions were introduced in fuzzy setting. Fuzzy topology is a generalization of topology in classical Mathematics, but it also has its own marked characteristics. Also, it deepens the understanding pf basic structure of classical Mathematics, offering new methods and results and obtains significant results of classical Mathematics. Moreover, fuzzy set theory finds its applications in various fields of Science and Engineering such as Nonlinear Dynamical Systems, Control of Chaos, Quantum Physics, Meteorology, Medicine and Computer Science.

Indeed, possibilities for application include any field that examines how one processes an information or makes decisions, recognizes patterns or diagnoses problems of any field in which the complexity of the necessary knowledge requires some form of simplification. Successful applications have, in fact, been made in the varied fields engineering, psychology, artificial intelligence, medicine, ecology, decision theory, pattern recognition, information retrieval, sociology and meteorology. While the diversity of successful applications has thus been expanding rapidly, the theory of fuzzy sets in particular and the Mathematics of uncertainty and information in general, have been achieving a secure identity as valid and useful extensions of classical Mathematics.

The concepts of hyper connected spaces have been studied in classical topology [10]. In this paper we introduced fuzzy semi-hyper connected spaces and Investigation on several characterizations of fuzzy semi - hyper connected space and other fuzzy topological spaces.

2. Preliminaries

Definition 2.1[1]: Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define $int(\lambda) = \vee \{\mu/\mu \le \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge \{\mu/\lambda \le \mu, 1 - \mu \in T\}$. For any fuzzy set in a fuzzy topological space (X,T), it is easy to see that $1 - cl(\lambda) = int(1-\lambda)$ and $1 - int(\lambda) = cl(1-\lambda)$.

Definition 2.2[2]: A fuzzy set λ in a fuzzy topological space X is called fuzzy semi-open if $\lambda \le cl\ int(\lambda)$ and fuzzy semi-closed if $int\ cl(\lambda) \le \lambda$.

Definition 2.3[1]: Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T). We define the fuzzy semi-closure and the fuzzy semi-interior of λ as follows:

(1) $scl(\lambda) = \wedge\{\mu / \lambda \le \mu, 1 - \mu\ \varepsilon\ T\}$ is fuzzy semi-closed set of X.

(2) $sint(\lambda) = \vee\{\mu / \mu \le \lambda, \mu\ \varepsilon\ T\}$ is fuzzy semi-open set of X.

Definition 2.4[1]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy semi-closed set if λ = scl(λ) and fuzzy semi-open set if λ = sint(λ).

Lemma 2.1[1]: Let λ be a fuzzy set of a fuzzy topological space (X, T). Then

(1) 1-scl(λ) = sint(1-λ)

(2) 1-sint(λ) = scl(1-λ)

Definition 2.5 [11]: Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called a fuzzy semi-nowhere dense set if there exists no non-zero fuzzy semi-open set μ in (X,T) such that μ < scl(λ). That is, sint scl (λ) = 0.

Definition 2.6 [11]: A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy semi-dense if there exists no fuzzy semi-closed set μ in (X, T) such that λ < μ < 1. That is, scl(λ) = 1.

Definition 2.7 [11]: Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called fuzzy semi-first category if $\lambda = \vee_{1=1}^{\infty}(\lambda_i)$, where λ_i 's are fuzzy semi-nowhere dense sets in (X,T). Any other fuzzy set in (X, T) is said to be of fuzzy semi-second category.

Definition 2.8[11]: Let λ be a fuzzy semi-first category set in a fuzzy topological space (X, T). Then 1 - λ is called a fuzzy semi-residual set in (X, T).

Definition 2.9[11]: A fuzzy topological space (X, T) is called a fuzzy semi-first category space if the fuzzy set 1_x is a fuzzy semi-first category set in (X, T). That is, $1_x = \vee_{1=1}^{\infty}(\lambda_i)$, where λ_i's are fuzzy semi-nowhere dense sets in (X,T). Otherwise (X, T) will be called a fuzzy semi-second category space.

Definition 2.10 [11]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy semi-Baire space if $\text{sint}[\vee_{1=1}^{\infty}(\lambda_i)] = 0$, where λ_i's are fuzzy semi-nowhere dense sets in (X, T).

Definition 2.11 [12]: A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy σ −nowhere dense set if λ is a fuzzy F_σ-set in (X,T) such that $\text{int}(\lambda) = 0$.

3. Fuzzy semi-hyper connected spaces

Motivated by the classical concept introduces by [7]. Now we are define:

Definition 3.1: A fuzzy topological space (X, T) is said to be a fuzzy semi-hyper connected space in which every non-empty fuzzy semi-open set is semi-dense set in (X, T).

Example 3.2:

Let X = {a, b, c}. The fuzzy sets α, β and γ are defined on X as follows:

α : X →[0,1] defined as α(a) = 0.8; α(b) = 0.9; α(c) = 0.6;

β : X →[0,1] defined as β(a) = 0.5; β(b) = 0.8; β(c) = 0.7;

γ : X→[0,1] defined as γ(a) = 0.5; γ(b) = 0.7; γ(c) = 0.7,

Then T = {0, α, β, γ, (α ∨ β), (α∨γ), (α∧β), (α∧γ), 1} is a fuzzy topology on X. Now $\text{scl}(\alpha) = 1, \text{scl}(\beta) = 1,\ \text{scl}(\gamma) = 1,\ \text{scl}(\alpha\vee\beta) = 1, \text{scl}(\alpha\vee\gamma) = 1, \text{scl}\ (\alpha\wedge\beta) = 1$ and $\text{scl}(\alpha\wedge\gamma) = 1$. Hence, every fuzzy semi-open set in (X, T) is a fuzzy semi-dense set in (X, T).

Example 3.3:

Let X = {a, b, c}. The fuzzy sets λ, μ and γ be the fuzzy sets defined on X as follows:

λ : X →[0,1] defined as λ(a) = 0.1; λ(b) = 0.3; λ(c) = 0.8,

μ : X →[0,1] defined as μ(a) = 0.7; μ(b) = 0.6; μ(c) = 0.4,

γ : X→[0,1] defined as γ(a) = 0.6; γ(b) = 0.5; γ(c) = 0.3.

Then T = {0, λ, μ, γ, λ∨μ, λ∨γ, λ∧μ, λ∧γ, 1} is a fuzzy topology on X. Now $\text{scl}(\lambda) = 1,\ \text{scl}(\mu) = 1 - \lambda\wedge\mu,\ \text{scl}(\gamma) = 1,\ \text{scl}(\lambda\vee\mu) = 1,\ \text{scl}(\lambda\vee\gamma) = 1,\ \text{scl}\ (\lambda\wedge\mu) = 1 - \mu,\ \text{scl}(\lambda\wedge\gamma) = 1 - \mu$. This implies that λ, γ, λ∨μ, λ∨γ are fuzzy semi-dense set but μ, λ∧μ, λ∧γ are not fuzzy semi-dense set in (X, T).

Therefore, (X,T) is not a fuzzy semi-hyper connected space.

Theorem 3.4[11]: If λ be a fuzzy semi-dense and fuzzy semi-open in a fuzzy topological space (X,T) and if μ ≤ 1-λ, then μ is a fuzzy semi-nowhere dense set in (X,T).

Proposition 3.5: If λ is a fuzzy semi-open set in a fuzzy semi-hyper connected space in a fuzzy topological space (X, T) and if μ ≤ 1-λ, then μ is a fuzzy semi-nowhere dense set in (X, T).

Proof: Let λ be a fuzzy semi-open in a fuzzy semi-hyper connected space (X, T). Therefore λ is a fuzzy semi-dense in a fuzzy semi-hyper connected space in (X, T). Then by theorem 3.4., μ is a fuzzy semi-nowhere dense set in (X, T).

Theorem 3.6 [11]: If λ is a fuzzy semi-closed set in a fuzzy topological space(X, T) and if $sint(\lambda) = 0$, then λ is a fuzzy semi-nowhere dense set in (X, T).

Proposition 3.7: If λ be a fuzzy semi-closed set in fuzzy semi-hyper connected space (X,T) and if $sint(\lambda) = 0$, then λ is a fuzzy semi-nowhere dense set in (X, T).

Proof: Let λ be a fuzzy semi-closed set in a fuzzy semi-hyper connected space. We know that every fuzzy semi-open set in a fuzzy semi-hyper connected space is fuzzy semi-dense. Then by theorem 3.6., λ is a fuzzy semi-nowhere dense set in (X, T).

Theorem 3.8[11]: If λ is a fuzzy semi-nowhere dense set in a fuzzy topological space(X, T), then 1-λ is a fuzzy semi-dense set (X, T).

Proposition 3.9: The complement of a fuzzy semi-closed set in a fuzzy semi-hyper connected space is fuzzy semi-dense set in (X, T).

Proof: By Proposition 3.7, a fuzzy semi-closed set in a fuzzy semi-hyper connected space is fuzzy semi-nowhere dense set in (X, T).

By theorem 3.8, 1-λ is a fuzzy semi-dense set in (X, T). Hence the complement of fuzzy semi-closed set in a fuzzy semi-hyper connected space is a fuzzy semi-dense set in (X, T).

Proposition 3.10: Every fuzzy semi-closed set in a fuzzy semi-hyper connected space is fuzzy semi-nowhere dense set in (X, T).

Proof: Suppose that if the fuzzy-closed set λ is not a fuzzy semi-closed nowhere dense set in a fuzzy semi-hyper connected space, therefore $sint\ scl(\lambda) \neq 0$, and the complement of λ is fuzzy semi-open but not a fuzzy semi-dense set in (X, T), this is a contradiction to the definition of fuzzy semi-hyper connected space. Hence every fuzzy semi-closed set in fuzzy semi-hyper connected space is a fuzzy semi-nowhere dense set in (X, T).

Proposition 3.11: In a fuzzy topological space(X, T), a fuzzy semi-hyper connected space is a fuzzy semi-second category space (X, T).

Proof: In example 3.2, (X, T) is a fuzzy semi-hyper connected space, then (1-α), (1-β), (1-γ), (1-α∨β), (1-α∨γ), (1-α∧β) and (1-α∧γ) are fuzzy semi-nowhere sets in (X, T). Now, $[(1-\alpha) \vee (1-\beta) \vee (1-\gamma) \vee (1-\alpha\vee\beta)\vee(1-\alpha\vee\gamma)\vee(1-\alpha\wedge\beta)\vee(1-\alpha\wedge\gamma)] \neq 1$.Therefore union of the fuzzy semi-nowhere dense sets is not equal to 1 this implies that (X, T) is a fuzzy semi-second category space.

Theorem 3.12[11]: In a fuzzy topological space(X, T) is a fuzzy semi-second category need not be a fuzzy semi-Baire space.

Proposition 3.13: In a fuzzy topological space (X, T) is a fuzzy semi-hyper connected space need not be a fuzzy semi-Baire space

Proof: Let (X, T) be a Space fuzzy semi-hyper connected space. By proposition 3.11, (X, T) is a fuzzy semi-second category space and by theorem. 3.12, (X, T) need not be a fuzzy semi-Baire space.

Definition 3.14 [9]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy semi-D Baire space if each fuzzy semi-first category set is fuzzy semi-nowhere dense sets in (X, T)

Example 3.15

Let X = {a, b, c}. The fuzzy sets λ, μ and γ are defined on X as follows:

λ : X →[0,1] defined as λ(a) = 0.7; λ(b) = 0.6; λ(c) = 0.6

μ : X →[0,1] defined as μ(a) = 0.8; μ(b) = 0.6; μ(c) = 0.7

γ : X→[0,1] defined as γ(a) = 0.6; γ(b) = 0.8; γ(c) = 0.6

Then T = {0, λ, μ, γ, μ∨γ, λ∨γ, μ∧γ, 1} is a fuzzy topology on X. Now $scl(\lambda) = 1,\ scl(\mu) = 1-\mu$, $scl(\gamma) = 1,\ scl(\mu\vee\gamma) = 1,\ scl(\lambda\vee\gamma) = 1,\ scl\ (\mu\wedge\gamma) = 1$. Hence, every fuzzy semi-open set in (X, T) is a fuzzy semi-dense in (X, T). Hence (X, T) is a fuzzy semi- hyperconnected space.

Proposition 3.16: In a fuzzy topological space (X, T) is a fuzzy semi-hyper connected space, then (X, T) is fuzzy semi-D Baire space.

Proof

In example 3.15, (X, T) is a fuzzy semi-hyper connected space, then (1-λ), (1-μ), (1-γ), (1-μ∨γ), (1-λ∨γ), (1-μ∨γ) are fuzzy semi-nowhere dense sets in (X, T). Now,$[(1-\lambda) \vee (1-\mu) \vee (1-\gamma) \vee (1-\mu\vee\gamma)\vee(1-\lambda\vee\gamma)\vee(1-\mu\vee\gamma)] = 1-\mu\wedge\gamma$, the union of the fuzzy semi-nowhere dense sets in (X, T) is a fuzzy semi-first category set. Therefore (X, T) is s fuzzy semi-D Baire space.

Proposition 3.17: In a fuzzy topological space (X, T) is a fuzzy semi-hyper connected space then (X, T) is a fuzzy semi-Baire Space.

Proof: Let (X, T) be a fuzzy semi-hyper connected space. In example 3.15., (1-λ), (1-μ), (1-γ), (1-μ∨γ), (1-λ∨γ), (1-μ∨γ) are fuzzy semi-nowhere dense sets in (X, T) this implies $sint[(1-\lambda) \vee (1-\mu) \vee (1-\gamma) \vee (1-\mu\vee\gamma)\vee(1-\lambda\vee\gamma)\vee(1-\mu\vee\gamma)] = 1-\mu\wedge\gamma) = 0$. Hence (X, T) is a fuzzy semi-Baire space.

Also, $sint\ scl(1-\lambda) = 0;\ sint\ scl(1-\mu) = 0; sint\ scl(1-\gamma) = 0; sint\ scl(1-\mu\vee\gamma) = 0; sint\ scl(1-\lambda\vee\gamma) = 0; sint\ scl(1-\mu\wedge\gamma) = 0$. This implies that (1-λ), (1-μ), (1-γ), (1-μ∨γ), (1-λ∨γ) and (1-μ∧γ) are fuzzy semi-nowhere dense sets in (X, T). Then the union of fuzzy semi-nowhere dense sets in a fuzzy semi- first category set in (X, T). Hence (X, T) is a fuzzy semi-D Baire space.

Remark 3.18: The following arrow diagram to relation between fuzzy semi-hyper connected space, fuzzy semi-category space, fuzzy semi-D Baire Space and fuzzy semi-Baire space.

Fuzzy semi-hyper connected space ⟶ Fuzzy semi-second category space

↓

Fuzzy Semi D-Baire space

↓

Fuzzy Semi-Baire space.

Proposition 3.19: In a fuzzy topological space (X, T) is a fuzzy semi-hyper connected space then every fuzzy set λ is a fuzzy semi-dense (or) fuzzy semi-nowhere dense set in (X, T).

Proof:

Let (X, T) be a fuzzy semi-hyper connected space and λ be a fuzzy set of (X, T). Suppose that λ is not a fuzzy semi-nowhere dense set in (X, T). Then $\text{sint}\ \text{scl}(\lambda) \neq 0$. Since $\text{sint}[\text{scl}(\lambda)]$ is non-zero fuzzy semi-open set in (X, T), then $\text{scl}(\text{sint}[\text{scl}(\lambda)]) = 1$. Since $1 = \text{scl}(\text{sint}[\text{scl}(\lambda)]) \leq \text{scl}[\text{scl}(\lambda)] = \text{scl}(\lambda) \Rightarrow \text{scl}(\lambda) = 1$. Thus λ is a fuzzy semi-dense set in (X, T).

Conclusion:

In this paper, we introduce fuzzy semi-hyper connected spacers using fuzzy semi-open sets and semi-dense sets. In these spaces to handle imprecise boundaries makes them a significant tool and practical modeling in fuzzy systems.

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