IJCRT.ORG ISSN : 2320-2882
INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

# ON I-CAUCHY SEQUENCES OF FUNCTIONS IN P-ADIC LINEAR 2-NORMED SPACES

[1] Dontula Shankaraiah, [2] Nagula Manjula

[1] Department of Mathematics,
[1] CVR College of Engineering, Ibrahimpatnam, Ranga Reddy District–501510, TS, INDIA
[2] Department of Mathematics,
[2]Sreenidhi Institute of Science and Technology, Hyderabad, TS, India.

***Abstract***
In this paper, we introduce the concepts of $I$ and $I^*-$convergence of sequences of functions and the concepts of $I$ and $I^*-$Cauchy sequences of functions in $p$-adic linear 2-normed spaces. Also we investigate the relation between these concepts in $p$-adic linear 2-normed spaces.
**Keywords:** 2-normed spaces, *p*-adic 2-norm, *p*-adic linear 2-normed space, $I-$Cauchy sequence, $I^*-$Cauchy sequence.

**2000 Mathematics Subject Classification:** 11B68, 11F80, 46A15, 11S80, 46A65.

## 1. Introduction

Gähler [1] introduced the concept of linear 2-normed spaces in 1965 and it has since been expanded in different fields. Lewandowska published a lot of papers on 2-normed sets and convergent sequences...etc. (see ([4], [7], [8], [9], [12]). P.Kostyrko et al [6] were very first to introduce ideal convergence of sequences, which is a generalization of statistical convergence. The concepts of $I-$cluster point and $I-$limit point of a sequence in metric space were first discussed in [5] and established some results between these concepts. The relation between $I-$cluster points and ordinary limit points of sequences in 2-normed spaces was studied by A.Sahiner et al [13] and Gurdal [2].Gurdal and Isil Acik [3, 11] proposed the concept of *I*-Cauchy sequences in 2-normed spaces. $I-$Convergence and $I-$Cauchy Sequences of Functions in 2-normed spaces were introduced by Mukaddes Arslan and Erdinc Dundar [16].

The connection between the concepts in *p*-adic numbers, p-adic analysis and linear 2-normed spaces was first introduced by Mehmet Acikgoz ([10]). Certain *p*-adic linear 2-normed space properties were introduced by B.Surender Reddy [14]. The concept of ideal convergent sequences in *p*-adic linear 2-normed spaces introduced by B.Surender Reddy and D.Shankaraiah [15].

This paper's major goal is to introduce the notion of $I$ and $I^*-$convergence of sequences of functions, $I$ and $I^*-$Cauchy sequences of functions in $p$-adic linear 2-normed spaces. Furthermore, we establish how these ideas relate to one another in $p$-adic linear 2-normed spaces.

## 2. Preliminaries

**Definition 2.1:** Let $K$ be field of real or complex numbers and $X$ be a linear space of dimension higher than 1 over $K$. Suppose $N(\bullet,\bullet)$ be a real valued function on $X \times X$ which is non-negative and fulfills the following criteria:

1). $N(x,y) > 0$ and $N(x,y) = 0$ if and only if $x$ and $y$ are linearly dependent vectors,

2). $N(x,y) = N(y,x)$ for all $x, y \in X$,

3). $N(\lambda x, y) = |\lambda| N(x,y)$ for all $\lambda \in K$ and $x, y \in X$,

4). $N(x+y,z) \leq N(x,z) + N(y,z)$ for all $x,y,z \in X$.

Then $N(\bullet,\bullet)$ is called a 2-norm on $X$ and the pair $(X, N(\bullet,\bullet))$ is called a linear 2-normed space.

**Definition 2.2:** Let $K$ be field of real or complex numbers and $X$ be a linear space of dimension higher than 1 over $K$. Suppose $N(\bullet,\bullet)_p$ be a real valued function on $X \times X$ which is non-negative and fulfills the following criteria:

1). $N(x,z)_p = 0$ if and only if $x$ and $z$ are linearly dependent vectors.

2). $N(xy,z)_p = N(x,z)_p . N(y,z)_p$ for all $x,y,z \in X$,

3). $N(x+y,z)_p \leq N(x,z)_p + N(y,z)_p$ for all $x,y,z \in X$,

4). $N(\lambda x, z)_p = |\lambda|\ N(x,z)_p$ for all $\lambda \in K$ and $x, z \in X$.

Then $N(\bullet,\bullet)_p$ is called a $p$-adic 2-norm on $X$ and the pair $(X, N(\bullet,\bullet)_p)$ is called $p$-adic linear 2-normed space.

**Proposition 2.3:** If a sequence $\{x_n\}$ in a $p$-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ is convergent to $x \in X$, then $\lim_{n\to\infty} N(x_n, z)_p = N(x,z)_p$ for each $z \in X$.

**Proposition 2.4:** If $\lim_{n\to\infty} N(x_n,z)_p$ exists then $\{x_n\}_{n\geq 1}$ is a Cauchy sequence w.r.t. $N(\bullet,\bullet)_p$.

**Definition 2.5:** A sequence $\{x_n\}_{n\geq 1}$ is a null sequence in $p$-adic linear 2-normed space if $\lim_{n\to\infty} N(x_n,z)_p = 0$ for all $z \in X$.

A family of sets $I \subseteq 2^Y$ (power sets of $Y \subset N$) is said to be an ideal if

a). $\Phi \in I$,

b). $P, Q \in I \Rightarrow P \bigcup Q \in I$ and

c). $P \in I, Q \subseteq P \Rightarrow Q \in I$.

A family of non empty sets $F \subset 2^Y$ is a filter on $Y$ if and only if $\Phi \notin F$, $P \bigcap Q \in F$ for each $P, Q \in F$, and any subset of an element of $F$ is in $F$. An ideal $I$ is called non-trivial if $I \neq \Phi$ and $Y \notin I$. Clearly $I$ is a non-trivial ideal if and only if $F = F(I) = \{Y - P : P \in I\}$ is a filter in $Y$, called the filter associated with the ideal $I$. A non-trivial ideal $I$ is called admissible if and only if $\{\{x\} : x \in Y\} \subset I$.

An admissible ideal $I \subset 2^Y$ is said to have the property (AP) if for any sequence $\{P_1, P_2, P_3, ...\}$ of mutually disjoint sets of $I$ there is a sequence $\{Q_1, Q_2, Q_3, ...\}$ of sets such that each symmetric difference $P_i \Delta Q_i$, $i = 1,2,3....$, is finite and $Q = \bigcup_{i=1}^{\infty} Q_i \in I$.

**Lemma 2.6:** Let $\{P_i\}_{i=1}^{\infty}$ be a countable collection of subsets of $N$ such that $P_i \in F(I)$ for each $i$, where $F(I)$ is a filter associated by an admissible ideal $I$ with property (AP). Then there is a set $P \subset N$ such that $P \in F(I)$ and the set $P - P_i$ is finite for all $i$.

**Definition 2.7:** Let $I \subset 2^N$ be a non trivial ideal in $N$. A sequence $\{x_n\}$ in a $p$-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ is $X$ is said to be $I-$convergent to $X$ if for each $\varepsilon > 0$ and non- zero $z$ in $X$ the set $A(\varepsilon) = \{n \in N : N(x_n - x, z)_p \geq \varepsilon\} \in I$. If $\{x_n\}$ is $I-$convergent to $x \in X$, then we write $I - \lim_{n\to\infty} N(x_n - x, z)_p = 0$ or $I - \lim_{n\to\infty} N(x_n, z)_p = N(x,z)_p$ for each non-zero $z \in X$. The number $x$ is called $I-$limit of the sequence $\{x_n\}$.

## 3. $I$ and $I^*$ – Cauchy sequences of functions in p-adic linear 2-normed spaces

In this section, we present the concepts of $I$ and $I^*$ – convergence of sequences of functions, as well as the concepts of $I$ and $I^*$ – Cauchy sequences of functions in $p$-adic linear 2-normed spaces. Additionally, we establish how these ideas relate to one another in $p$-adic linear 2-normed spaces.

Throughout this paper, $\left(X_1, N_1(\bullet,\bullet)_p\right)$ and $\left(X_2, N_2(\bullet,\bullet)_p\right)$ are two $p$-adic linear 2-normed spaces, $\{f_n\}$ and $\{g_n\}$ are two sequences of functions and $f$ is a function from $X_1$ to $X_2$.

**Definition 3.1:** The sequence of functions $\{f_n\}$ is said to be convergent to $f$ if for each $\varepsilon > 0$ and for each $z \in X_2$ there exists $n_0 \in N$ such that $\lim_{n\to\infty} N_2(f_n(x) - f(x), z)_p < \varepsilon \ \ \forall n \ge n_0$ and $\forall x \in X_1$.
$(\forall z \in X_2)(\forall x \in X_1)(\forall \varepsilon > 0)(\forall n_0 \in N)(\forall n \ge n_0) N_2\left(f_n(x) - f(x), z\right)_p < \varepsilon$. This can be expressed as $f_n(x) \xrightarrow{N_2(\bullet,\bullet)_p} f(x)$ for each $x \in X_1$ or we write $f_n \xrightarrow{N_2(\bullet,\bullet)_p} f$.

**Definition 3.2:** The sequence of functions $\{f_n\}$ is said to be $I$ – pointwise convergent to $f$ if for each $\varepsilon > 0$ and each non zero $z \in X_2$ the set $A(\varepsilon, z) = \left\{n \in N : N_2(f_n(x) - f(x), z)_p \ge \varepsilon\right\} \in I$
Or $I - \lim_{n\to\infty} N_2(f_n(x) - f(x), z)_p = 0$, for each $x \in X_1$ or $I - \lim_{n\to\infty} N_2(f_n(x), z)_p = N_2(f(x), z)_p$, for each non-zero z in $X_2$. The function $f$ is called $I$ – limit of the sequence of functions $\{f_n\}$.

**Lemma 3.3:** Let $I \subset 2^N$ be an admissible ideal with property (AP). If a sequence of functions $\{f_n\}$ is $I$ – pointwise convergent to $f$ then there exists a set $P \in F(I)$, $P = \{m_1 < m_2 < ..... < m_k < .......\}$ such that $\lim_{k\to\infty} N_2(f_{m_k}(x) - f(x), z)_p = 0$, for each non-zero $z \in X_2$.

**Proof:** Let a sequence of functions $\{f_n\}$ be $I$ – pointwise convergent to $f$. Then for each $\varepsilon > 0$ and non-zero $z \in X_2$, the set $A(\varepsilon, z) = \{n \in N : N_2(f_n(x) - f(x), z)_p \ge \varepsilon\} \in I$. For each $i \in N$ and non-zero $z \in X_2$, define a sets $P_i = \{n \in N : N_2(f_n(x) - f(x), z)_p < \frac{1}{i}\}$ and $H_i = N - P_i =$
$\{n \in N : N_2(f_n(x) - f(x), z)_p \ge \frac{1}{i}\}$. Then for each $i \in N$, $H_i \in I$ and $P_i \in F(I)$. By Lemma (2.6), we have $P \in F(I)$ such that $P = \{m_1 < m_2 < .... < m_k < ........\}$ and $P - P_i$ is finite set for all $i$. Now define the sequence $\{g_n\}$ such that $g_n = f_n$ for each $n \in P$ and $g_n = f$ for $n \notin P$. Then $\lim_{n\to\infty} N_2(g_n(x) - f(x), z)_p = 0$ which implies that $\lim_{k\to\infty} N_2(f_{m_k}(x) - f(x), z)_p = 0$, for each non-zero $z \in X_2$.

Now we introduce the concept of $I^*$ – convergence of sequence of functions in $p$-adic linear 2-normed spaces.

**Definition 3.4:** A sequence of functions $\{f_n\}$ is said to be $I^*$ – pointwise convergent to $f$ if and only if there exists a set $M \in F(I)$, $M = \{m_1 < m_2 < ..... < m_k < ......\}$ such that $\lim_{k\to\infty} N_2(f_{m_k}(x) - f(x), z)_p = 0$, for each non-zero $z \in X_2$.

If $\{f_n\}$ is $I^*$ – pointwise convergent to $f$, then we write $I^* - \lim_{n\to\infty} N_2(f_n(x) - f(x), z)_p = 0$ or
$I^* - \lim_{n\to\infty} N_2(f_n(x), z)_p = N_2(f(x), z)_p$, for each non-zero z in $X_2$. The function $f$ is called $I^*$ – limit of the sequence of functions $\{f_n\}$.

From Lemma (3.3), it follows that if $I$ is an admissible ideal with property (AP) and a sequence of functions $\{f_n\}$ is $I$ – pointwise convergent to $f$ then $\{f_n\}$ is $I^*$ – pointwise convergent to $f$.

**Lemma 3.5:** Let $I \subset 2^N$ be an admissible ideal. If $I^* - \lim_{n\to\infty} N_2(f_n(x) - f(x), z)_p = 0$ then

$I-\lim_{n\to\infty} N_2(f_n(x)-f(x),z)_p = 0$ for each non-zero $z \in X_2$ .

**Proof:** Let $I \subset 2^N$ be an admissible ideal and $I^* - \lim_{n\to\infty} N_2(f_n(x)-f(x),z)_p = 0$. Then there exists a set $H \in I$ such that for $M = N - H = \{m_1 < m_2 < ..... < m_k < ......\}$ we have

$\lim_{k\to\infty} N_2(f_{m_k}(x)-f(x),z)_p = 0$, for each non-zero $z \in X_2$. (3.6)

Let $\varepsilon > 0$. By virtue of (3.6) there exists $k_0 \in N$ such that $\lim_{k\to\infty} N(f_{m_k}(x)-f(x),z)_p < \varepsilon$ for each $k > k_0$.

Then obviously

$A(\varepsilon,z) = \{n \in N : N_2(f_n(x)-f(x),z)_p \geq \varepsilon\} \subset H \bigcup \{m_1 < m_2 < ....... < m_{k_0}\}$ (3.7)

The set on the right hand side of (3.7) belongs to I (since I is an admissible). So $A(\varepsilon) \in I$. Therefore $I-\lim_{n\to\infty} N_2(f_n(x)-f(x),z)_p = 0$, for each non-zero $z \in X_2$.

From Lemma (3.3) and Lemma (3.5) we obtain the following Lemma which gives the equivalence between $I-$convergence and $I^*-$convergence in $p$-adic linear 2-normed spaces.

**Lemma 3.8:** Let $I \subset 2^N$ be an admissible ideal with property (AP), $\{f_n\}$ be a sequence of functions and $f$ be a function from $X_1$ to $X_2$. Then $\{f_n\}$ is $I-$pointwise convergent to $f$ if and only if $\{f_n\}$ is $I^*-$pointwise convergent to $f$.

Now we introduce the concepts $I$ and $I^*-$ Cauchy sequences in $p$-adic linear 2-normed spaces. Also, we establish the relation between these concepts in $p$-adic linear 2-normed spaces.

**Definition 3.9:** Let $I \subset 2^N$ be an admissible ideal. The sequence of functions $\{f_n\}$ is said to be $I-$Cauchy sequence, if for each $\varepsilon > 0$ and non-zero $z \in X_2$ there exists a number $l(\varepsilon,z)$ such that $\{k \in N : N_2(f_k(x)-f_l(x),z)_p \geq \varepsilon\} \in I$.

**Definition 3.10:** Let $I \subset 2^N$ be an admissible ideal. Then the sequence of functions $\{f_n\}$ is said to be $I^*-$Cauchy sequence if there exists a set $M = \{m_1 < m_2 < ..... < m_k < ........\} \subset N, M \in F(I)$ such that the sequence $\{f_{m_k}\}$ is a Cauchy sequence. i.e., $\lim_{k,l\to\infty} N_2(f_{m_k} - f_{m_l}, z)_p = 0$ for each non-zero $z \in X_2$.

**Theorem 3.11:** Let $I$ be an admissible ideal. If the sequence of functions $\{f_n\}$ is $I^*-$Cauchy sequence then $\{f_n\}$ is $I-$Cauchy sequence.

**Proof:** Let $I$ be an admissible ideal and $\{f_n\}$ be $I^*-$Cauchy sequence. Then by definition there exists the set $M = \{m_1 < m_2 < ..... < m_k < .....\} \subset N$, $M \in F(I)$ such that $N_2(f_{m_k}(x)-f_{m_l}(x),z)_p < \varepsilon$ for every non-zero $z \in X_2$ and $k,l > k_0(\varepsilon)$. Let $l(\varepsilon) = m_{k_0+1}$. Then for every $\varepsilon > 0$, we have $N_2(f_{m_k}(x)-f_l(x),z)_p < \varepsilon$, for every non-zero $z \in X_2$ and $k > k_0$. Now put $H = N - M$. It is clear that $H \in I$ and

$$A(\varepsilon,z) = \{n \in N : N_2(f_n(x)-f_l(x),z)_p \geq \varepsilon\} \subset H \bigcup \{m_1 < m_2 < ..... < m_k < ......\} . \quad (3.12)$$

Then the set on the R.H.S. of (3.12) belongs to $I$ because $I$ is an admissible ideal. Therefore for every $\varepsilon > 0$ we find $l(\varepsilon)$ such that $A(\varepsilon,z) \in I$ i.e., $\{f_n\}$ is $I-$Cauchy sequence.

Now we will prove that $I^*-$convergence implies $I-$Cauchy condition in a $p$-adic linear 2-normed spaces.

**Theorem 3.13:** Let $I$ be an admissible ideal and $I^* - \lim_{n\to\infty} N_2(f_n(x)-f,z)_p = 0$ where $\{f_n\}$ is a sequence of functions. Then $\{f_n\}$ is $I-$ Cauchy sequence in $X$. In other words, let $I$ be an admissible ideal and if a sequence of functions $\{f_n\}$ is $I^*-$convergent to $f$, then $\{f_n\}$ is $I-$Cauchy sequence.

**Proof:** Let $I$ be an admissible ideal. Suppose $\{f_n\}$ is a sequence of functions is $I^*-$convergent to $f$ .Then there exists the set $M \in F(I)$, $M = \{m_1 < m_2 < ..... < m_k < ......\} \subset N$, such that $\lim_{k \to \infty} N_2(f_{m_k}(x) - f, z)_p = 0$ for each non-zero $z \in X_2$. It shows that there exists $k_0 = k_0(\varepsilon)$ such that $N_2(f_{m_k}(x) - f, z)_p < \frac{\varepsilon}{2}$ for every $\varepsilon > 0$, non-zero $z \in X_2$ and $k > k_0$.

$$N_2(f_{m_k}(x) - f_{m_l}(x), z)_p = N_2(f_{m_k}(x) - f + f - f_{m_l}(x), z)_p$$
$$= N_2((f_{m_k}(x) - f) - (f_{m_l}(x) - f), z)_p$$
$$\leq N_2(f_{m_k}(x) - f, z)_p + N_2(f_{m_l}(x) - f, z)_p$$
$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$, for every $\varepsilon > 0$, non-zero z in $X_2$ and $k > k_0$, $l > k_0$.

$\Rightarrow \lim_{k,l \to \infty} N_2(f_{m_k} - f_{m_l}, z)_p = 0$

$\Rightarrow \{f_{m_k}\}$ is Cauchy sequence.

$\Rightarrow \{f_n\}$ is a $I^* -$Cauchy sequence.

According to Theorem (3.11), $\{f_n\}$ is $I -$Cauchy sequence.

The following corollary, which gives the relation between $I -$convergence and $I -$Cauchy sequence in a $p$-adic linear 2-normed spaces, is derived from Theorem (3.13) and Lemma (3.8).

**Corollary 3.14:** Let $I \subset 2^N$ be an admissible ideal with property (AP). If $I - \lim_{n \to \infty} N_2(f_n - f, z)_p = 0$, for each non- zero $z \in X_2$ then a sequence of functions $\{f_n\}$ is $I -$Cauchy sequence i.e., if $\{f_n\}$ is $I -$convergence to f then $\{f_n\}$ is $I -$Cauchy sequence.

The following theorem, which states that the equivalence of $I -$Cauchy sequence and $I^* -$Cauchy sequence in the case $I$ has the property (AP).

**Theorem 3.15:** Let $I$ be an admissible ideal with property (AP). Then $\{f_n\}$ is $I -$Cauchy sequence if and only if $\{f_n\}$ is $I^* -$Cauchy sequence.

**Proof:** Let $\{f_n\}$ be $I^* -$Cauchy sequence. Then by Theorem (3.11), $\{f_n\}$ is also $I -$Cauchy sequence. Let $\{f_n\}$ be $I -$Cauchy sequence. Now we have to prove that $\{f_n\}$ is a $I^* -$Cauchy sequence. There exists $l(\varepsilon)$ such that $A(\varepsilon, z) = \{n \in N : N_2(f_n - f_l, z)_p \geq \varepsilon\} \in I$, for each $\varepsilon > 0$ and non-zero $z \in X_2$. Let $P_i = \{n \in N : N_2(f_n - f_{m_i}, z)_p < \frac{1}{i}\}, i = 1,2,3,......,$ where $m_i = N(\frac{1}{i})$.

Since $H_i = N - P_i = \{n \in N : N_2(f_n - f_{m_i}, z)_p \geq \frac{1}{i}\} \in I$, for each $i \in N$ and non-zero $z \in X_2$, therefore $P_i \in F(I), i \in N$. Since $I$ has (AP) property then by Lemma (3.4) there exists a set $P \subset N$ such that $P \in F(I)$ and the set $P - P_i$ is finite for all $i$. Now we have to show that $\lim_{n,m \to \infty} N_2(f_n - f_m, z)_p = 0$; $m, n \in P$ and for each non-zero $z \in X_2$. For this, let $\varepsilon > 0$ and $j \in N$ be such that $j > \frac{2}{\varepsilon}$. If $m, n \in P$ then $P - P_j$ is a finite set, so there exists $k = k(j)$ such that $m \in P_j$ and $n \in P_j$ for all $m, n > k(j)$. Therefore $N_2(f_n - f_{m_j}, z)_p < \frac{1}{j}$ and $N_2(f_m - f_{m_j}, z)_p < \frac{1}{j}$, for all $m, n > k(j)$ and non-zero $z \in X_2$.

Now $N_2(f_n - f_m, z)_p = N_2(f_n - f_{m_j} + f_{m_j} - f_m, z)_p$
$$= N_2((f_n - f_{m_j}) - (f_m - f_{m_j}), z)_p$$
$$\leq N_2(f_n - f_{m_j}, z)_p + N_2(f_m - f_{m_j}, z)_p$$

$$< \frac{1}{j} + \frac{1}{j} = \frac{2}{j} < 2(\frac{\varepsilon}{2}) = \varepsilon \text{, for } m,n > k(j) \text{ and each non-zero } z \in X_2 .$$

Hence, for any $\varepsilon > 0$ there exists $k = k(\varepsilon)$ such that for $m,n > k(\varepsilon)$ and $m,n \in P \in F(I)$, $N_2(f_n - f_m, z)_p < \varepsilon$, for each non-zero $z \in X_2$. This shows that the sequence $\{f_n\}$ is $I^* -$Cauchy sequence. Thus $\{f_n\}$ is $I -$Cauchy sequence if and only if $\{f_n\}$ is $I^* -$Cauchy sequence.

## REFERENCES


[1] S. Gähler, Linear 2-normerte raume, *Math. Nachr*, 28(1965), 1-45.

[2] M. Gurdal, On ideal convergent sequences in 2-normed spaces, *Thai Journal of Mathematics*, 4(1) (2006), 85-91.

[3] M. Gurdal and Isil Acik, On *I*-Cauchy sequences in 2-normed spaces, Mathematical Inequalities & Applications, Vol 11, Number 2(2008), 349-354

[4] H. Gunawan and Mashadi, On finite dimensional 2-normed spaces, *Soochow J. Math.*, 27(3) (2001), 321-329.

[5] P. Kostyrko, M. Macaj and T. Salat, *I*-convergence, *Real Anal. Exchange*, 26(2) (2000), 669-686.

[6] P. Kostyrko, M. Macaj, T. Salat and M. Sleziak, *I*-convergence and extremal *I*-limit points, *Math. Slovaca*, 55(2005), 443-464.

[7] Z. Lewandowska, Generalized 2-normed spaces, *Stuspskie Prace Matematyczno-Fizyczne*, 1(2001), 33-40.

[8] Z. Lewandowska, Linear operators on generalized 2-normed spaces, *Bull. Math. Soc. Sci. Math. Roumanie (N.S.)*, 42(90) (4) (1999), 353-368.

[9] Z. Lewandowska, On 2-normed sets, *Glasnik Mat. Ser. III*, 38(58) (1) (2003), 99-110.

[10] Mehmet Acikgoz, N. Aslan, N. Koskeroglu and S. Araci, p-adic approach to linear 2-normed spaces, *Mathematica Moravica*, 13(2) (2009), 7-22.

[11] Anar Nabiev, Serpil Pehlivan and M. Gurdal, On *I*-Cauchy sequences, Taiwanese Journal of Mathematics, Vol.11, No.2, pp.569-576, June 2007

[12] W. Raymond, R. Freese and J. Cho, *Geometry of Linear 2-Normed Spaces*, Nova Science Publishers, (2001).

[13] H. Sahiner, M. Gurdal, S. Saltan and H. Gunawan, Ideal convergence in 2-normed spaces, *Taiwanese Journal of Mathematics*, 11(5) (2007), 1477-1484.

[14] B.Surender Reddy, Equivalence of p-adic 2-norms in p-adic linear 2-normed spaces, International Journal of Open Problems in Computer Science and Mathematics, 3(5) (2010), 25-38.

[15] B.Surender Reddy and D.Shankaraiah, On Ideal convergent sequences in p-adic linear 2-normed spaces, General Mathematics Notes, Vol. 17, No. 1, 2013, (Press)

[16] Mukaddes Arslan and Erdinc Dundar, $I -$Convergence and $I -$Cauchy Sequences of Functions in 2-normed spaces, Konuralp Journal of Mathematics, Vol. 6, No. 1, 2018, 57-62.