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# Generating Relations: Taylor's And Maclaurine's Series Expansions Of Hypergeometric Functions Of Four Variables

**S.S. Chauhan**

Dept. of Mathematics, Dayanand Vedic College, Orai (Jalaun) U.P.

&

**Shailendra Singh Chauhan**

Dept. of Physics, Nehru Mahavidhalay, Lalitpur, U.P.

**Abstract**

Chandel and Sharma [13] have established generating relations, integral representations and recurrence relations of hypergeometric functions of four variables, Also Majid and Younish [16] have established certain generating functions of some quadruple hypergeometric series. In the present paper, we shall derive self generating relations cum Taylor's series and Maclaurine's series expansions involving hyergeometric functions of four variables.

**2020 Mathematics Subject Classification :** 33C50

**Keywords :** Hypergeometric functions of four variables, Generatig Functions, Taylor's and Maclaurine's series expansions.

## 1 Introduction.

Exton ([4],[6],[7]) introduced twenty one quadruple hypergeometric functions. Sharma and Parihar ([2],[11]) introduced eighty three hypergeometric functions fo four variables but out of which nineteen had already been included in the conjecture of Exton ([4],[6],[7]) in some different notations (see Remark of Chandel and Kumar [9]). Also Chandel and Kumar [10] introduced seven more hypergeometric functions of four variables suggested by hypergeometric

functions of three variables, $H_A$, $H_B$, $H_C$ of Srivastava ([2],[3]).

Further, Chandel and Sharma ([12],[13]) introduced and studied following ten hypergeometric functions of four variables suggested by hypergeometric

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(1.1) $H_A^{(4\,1)}(a, b, c, d; e, e', e''; x, y, z, u)$

$$= \sum_{m,n,p,q=0}^{\infty} \frac{(a)_{m+n+p}(b)_{m+q}(c)_{p+q}(d)_n}{(e)_{m+p}(e')_n(e'')_q} \frac{x^m y^n z^p u^q}{m!\, n!\, p!\, q!}$$

(1.2) $H_B^{(4\,1)}(a, b, c, d; e_1, e_2, e_3, e_4; x, y, z, u)$

$$= \sum_{m,n,p,q=0}^{\infty} \frac{(a)_{m+n+q}(b)_{m+p}(c)_{p+q}(d)_n}{(e_1)_m(e_2)_n(e_3)_p(e_4)_q} \frac{x^m y^n z^p u^q}{m!\, n!\, p!\, q!}$$

(1.3) $G_A^{(4\,1)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{m,n,p,q=0} \frac{(a)_{n+p+q-m}(b)_{m+p}(c)_n(d)_q}{(e)_{n+p-m}(e')_q} \frac{x^m y^n z^p u^q}{m!\, n!\, p!\, q!}$$

(1.4) $G_A^{(4\,2)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{m,n,p,q=0} \frac{(a)_{n+p-m}(b)_{m+p+q}(c)_{n+q}(d)_q}{(e)_{n+p-m}(e')_q} \frac{x^m y^n z^p u^q}{m!\, n!\, p!\, q!}$$

(1.5) $G_A^{(4\,3)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{m,n,p,q=0} \frac{(a)_{n+p-m}(b)_{m+p}(c)_{n+q}(d)_q}{(e)_{n+p-m}(e')_q} \frac{x^m y^n z^p u^q}{m!\, n!\, p!\, q!}$$

(1.6) $G_B^{(4\,1)}(a, b_1, b_2, b_3, b_4, ; e, e'; x, y, z, u)$

$\infty$

$$= \sum_{m,n,p,q,=0} \frac{(a)_{n+p-m+q}(b_1)_m(b_2)_n(b_3)_p(b_4)_q}{(e)_{n+p-m}} \frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}$$

(1.7) $G_B^{(42)}(a, b_1, b_2, b_3, b_4, ; e, e'; x, y, z, u)$

$$= \sum_{m,n,p,q,=0}^{\infty} \frac{(a)_{n+p-m}(b_1)_{m+q}(b_2)_n(b_3)_p(b_4)_q}{(e)_{n+p-m}(e')_q} \frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}$$

(1.8) $G_B^{(43)}(a, b_1, b_2, b_3, b_4, ; e, e'; x, y, z, u)$

$$= \sum_{m,n,p,q,=0}^{\infty} \frac{(a)_{n+p-m}(b_1)_{n+q}(b_2)_m(b_3)_p(b_4)_q}{(e)_{n+p-m}(e')_q} \frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}$$

(1.9) $G_C^{(14)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{m,n,p,q,=0}^{\infty} \frac{(a)_{m+p+q}(b)_{m+n}(c)_{n-p}(d)_q}{(e)_{n+p-m}(e')_q} \frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}$$

(1.10) $G_C^{(24)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{m,n,p,q,=0}^{\infty} \frac{(a)_{m+p+q}(b)_{m+p}(c)_{n-p}(d)_q}{(e)_{n+p-m}(e')_q} \frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}$$

Recently Chandel and Sharma [13] have established generating relations, integral representations and recurrence relations of above hypergeometric functions of four variables. Also Magid and Younish [16] have established certain generating functions of some quadruple hypergeometric series.

In the present paper, we shall derive self generating relations cum Taylor's series and Maclaurine's series expansions involving above hypergeometric functions of four variables.

## 2 Generating Relations.

Consider

$$\frac{\partial}{\partial x} H_{A1}^{(4)}(a, b, c, d; e, e', e''; x, y, z, u) = \sum_{m=1}^{\infty} \sum_{n,p,q=0}^{\infty} \frac{(a)_{m+n+p}(b)_{m+q}(c)_{p+q}(d)_n}{(e)_{m+p}(e')_n(e'')_q} \frac{x^{m-1}\, y^n\, z^p\, u^q}{(m-1)!\, n!\, p!\, q!}$$

$$= \sum_{m,n,p,q=0}^{\infty} \frac{(a)_{m+n+p+1}(b)_{m+q+1}(c)_{p+q}(d)_n}{(e)_{m+p+1}(e')_n(e'')_q} \frac{x^m\, y^n\, z^p\, u^q}{m!\, n!\, p!\, q!}$$

$$= \frac{ab}{e} \sum_{m,n,p,q=0}^{\infty} \frac{(a+1)_{m+n+p}(b+1)_{m+1}(c)_{p+q}(d)_n}{(e+1)_{m+p}(e')_n(e'')_q} \frac{x^m\, y^n\, z^p\, u^q}{m!\, n!\, p!\, q!}$$

Similarly, by induction, we can derive

(2.1) $$\frac{\partial^r}{\partial x^r} H_{A1}^{(4)}(a, b, c, d; e, e', e''; x, y, z, u) = \sum_{m,n,p,q=0}^{\infty} \frac{(a)_r(b)_r\,(a+r)_{m+n+p}(b+r)_{m+q}(c)_{p+q}(d)_n}{(e)_r\,(e+r)_{m+p}(e')_n(e'')_q} \frac{x^m\, y^n\, z^p\, u^q}{m!\, n!\, p!\, q!}$$

$$= \frac{(a)_r(b)_r}{(e)_r} H_{A1}^{(4)}(a+r, b+r, c, d; e+r, e', e''; x, y, z, u)$$

Therefore further, by making an appeal to Taylor's series expansion, we establish the following self generating relations for $H_{A1}^{(4)}$

(2.2) $$H_{A1}^{(4)}(a, b, c, d; e, e', e''; x+t, y, z, u) = \sum_{r=0}^{\infty} \frac{(a)_r(b)_r\, t^r}{(e)_r\, r!} H_{A1}^{(4)}(a+r, b+r, c, d; e+r, e', e''; x, y, z, u)$$

Applying the same techniques, we derive the following results:

(2.3) $H_{A1}^{(4)}(a, b, c, d; e, e', e''; x, y+t, z, u)$

$\_\_\_\_\_\_(er()b_r)rH_A(41)(a \quad b, c, d + r; e, e' + r, e"; x, y, z, u)$

(2.4) $H_A{}^{(4}1^{)}(a, b, c, d; \qquad e', e"; x, y, z + t, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)_r (b)_r}{(e)} {}_{r!} H_{A_1}^{(4)}(a + \quad r \quad r, b, c + r, d; e + r, e', e"; x, y, z, u)$$

(2.5) $H_A{}^{(4}1^{)}(a, b, c, d; e, e', e"; x, y, z, u + t)$

$$= \sum_{r=0}^{\infty} (\_\_\_\_\_\_ b()er"()c_r)r\ trr!\ H_A(41)(a, b + r, c + r, d; e, e', e" + r; x, y, z, u),$$

(2.6) $H_B{}^{(4}1^{)}(a, b, c, d; e_1, e_2, e_3, e_4; x + t, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)r(b)r\ tr}{\_\_\_\_\_\_ (e)_r r!} H_B{}^{(4)}_1 (a + r, b + r, c, d; e_1 + r, e_2, e_3, e_4; x, y, z, u),$$

(2.7) $H_B{}^{(4}1^{)}(a, b, c, d; e_1, e_2, e_3, e_4; x, y + t, z, u)$

$$= \sum_{r=0}^{\infty} (\_\_\_\_\_\_ a()er2()dr)r\ trr!\ H_B(41)(a + r, b, c, d + r; e_1, e_2 + r, e_3, e_4; x, y, z, u),$$

(2.8) $H_B{}^{(4}1^{)}(a, b, c, d; e_1, e_2, e_3, e_4; x, y, z + t, u)$

$$= \sum_{r=0}^{\infty} (\_\_\_\_\_\_ a()er3()cr)r\ trr!\ H_B(41)(a, b + r, c + r, d; e_1, e_2, e_3 + r, e_4; x, y, z, u),$$

(2.9) $H^{(4)}_{B1}(a, b, c, d; e_1, e_2, e_3, e_4; x, y, z, u + t)$

$$= \sum^{\infty} (\_\_\_\_\_\_ a()er4()cr)r\ trr!\ H_B(41)(a + r, b, c + r, d; e_1, e_2, e_3, e_4 + r; x, y, z, u),$$

$r=0$

(2.10) $G_A{}^{(4}1^{)}(a, b, c, d; e, e'; x, y+t, z, u)$

$\underline{\hspace{3em}} r(c)rGA(41)(a \quad b, c+r, d; e-r, e'; x, y, z, u),$

$(e)r$

(2.11) $G^{(4)}_{A_1}(a, b, c, d; \quad e'; x, y, z+t, u)$

$$= \sum^{\infty} \frac{(a)_r (b)_r}{(e)} \, r! \, G^{(4)}_{A_1}(a+r, b+ \; r, c, d; e+r, e'; x, y, z, u),$$

$r$

$r=0$

(2.12) $G^{(4)}_{A_1}(a, b, c, d; e, e'; x, y, z, u+t)$

$\infty$

$= \sum (\underline{\hspace{5em}} a()er'()d_r)r\ trr!\ G_A(41)(a+r, b, c, d+r; e, e'+r; x, y, z, u),$

$r=0$

(2.13) $G^{(4)}_{A_2}(a, b, c, d; e, e'; x, y+t, z, u)$

$\infty$

$= \sum (\underline{\hspace{5em}} a()er()c_r)r\ trr!\ G_A(42)(a_1+r, b, c+r, d; e+r, e'; x, y, z, u),$

$r=0$

(2.14) $G^{(4)}_{A_2}(a, b, c, d; e, e'; x, y, z+t, u)$

$\infty$

$= \sum (\underline{\hspace{5em}} a()er()b_r)r\ trr!\ G_A(42)(a+r, b+r, c, d; e+r, e'; x, y, z, u),$

$r=0$

(2.15) $G^{(4)}_{A_2}(a, b, c, d; e, e'; x, y, z, u+t)$

$\infty$

$r \quad r$ $= \sum (\underline{\hspace{8em}} b)_r((ec'))_r(d)\ tr^r!\ G_{A(4}2_{)}(a, b+r, c+r, d+r; e,$ $e'+r; x, y, z, u),$

$r=0$

(2.16) $G^{(4)}_{A_3}(a, b, c, d; e, e'; x, y+t, z, u)$

$\infty$

$= \sum (\underline{\hspace{5em}} a()er()c_r)r\ trr!\ G_A(43)(a+r, b, c+r, d; e+r, e'; x, y, z, u),$

$r=0$

(2.17) $G_{A^{(4}3)}(a, b, c, d; e, e'; x, y, z+t, u)$

$\underline{\qquad\qquad} r(b)rGA(43)(a \quad b+r, c, d; e+r, e'; x, y, z, u),$

$(e)r$

(2.18) $G_{A_3}^{(4)}(a, b, c, d; \quad e'; x, y, z, u+t)$

$$= \sum^{\infty} \frac{(c)_r (d)_r}{(e')} r! \, G_{A_3}^{(4)} \quad (a, b, c+r, d+r; e, e'+r; x, y, z, u),$$

$r$

$r=0$

(2.19) $G_{B_1}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y+t, z, u)$

$\infty$

$= \sum (\underline{\qquad\qquad} a)(re()b_r 2)r \; trr! \; G_B(41)(a+r, b_1, b_2+r, b_3, b_4; e+r, e'; x, y, z, u),$

$r=0$

(2.20) $G_{B_1}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y, z+t, u)$

$\infty$

$= \sum (\underline{\qquad\qquad} a)(re()b_r 2)r \; trr! \; G_B(41)(a+r, b_1, b_2+r, b_3, b_4; e+r, e'; x, y, z, u),$

$r=0$

(2.21) $G_{B_1}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y, z, u+t)$

$\infty$

$= \sum (\underline{\qquad\qquad} a)(re(')b_r 4)r \; trr! \; G_B(41)(a+r, b_1, b_2, b_3, b_4+r; e, e'+r; x, y, z, u),$

$r=0$

(2.22) $G_{B_2}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y+t, z, u)$

$\infty$

$= \sum (\underline{\qquad\qquad} a)(re()b_r 2)r \; trr! \; G_B(41)(a+r, b_1, b_2+r, b_3, b_4; e+r, e'; x, y, z, u),$

$r=0$

(2.23) $G_{B_2}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y, z+t, u)$

$\infty$

$= \sum (\underline{\qquad\qquad} a)(re()b_r 3)r \; trr! \; G_B(42)(a+r, b_1, b_2, b_3+r, b_4; e+r, e'; x, y, z, u),$

$r=0$

(2.24) $G_B{}^{(4}2^{)}(a, b_1, b_2, b_3, b_4; e, e'; x, y, z, u+t)$

$(\_\_\_\_\_\_ b1)r(b4)r\ tr\ GB(42)(a, b1+r, b_2, b_3, b_4+r; e, e'+r; x, y, z, u), (e')_r$

$r!$

(2.25) $G_{C_1}^{(4)}(a, b, c, d; e, e'; x+t, y, z, u)$

$$\sum^{\infty} \frac{(a)_r(b\ {}_r t^r \quad (4)\ )}{r} = \frac{}{(e)_r} \frac{}{!} G_{C_1}\ (a+r, b+ r, c, d; e+r, e'; x, y, z, u),$$

$r=0$

(2.26) $G_{C_1}^{(4)}(a, b, c, d; e, e'; x, y+t, z, u)$

$\infty$

$$= \frac{(b)r(c)r\ tr}{(e)_r} \frac{(4)}{r!} G_{C_1}\ (a, b+r, c+ \sum r, d; e+r, e'; x, y, z, u),$$

$r=0$

(2.27) $G_{C_1}^{(4)}(a, b, c, d; e, e'; x, y, z, u+t)$

$$\sum^{\infty} \frac{(a)_r(d\ {}_r t^r \quad (4)\ )}{r} = \frac{}{(e')_r} \frac{}{!} G_{C_1}\ (a+ r, b, c, d+r; e, e'+r; x, y, z, u),$$

$r=0$

(2.28) $G_{C_2}^{(4)}(a, b, c, d; e, e'; x+t, y, z, u)$

$\infty$

$(a)r(b)r\ tr \quad (4)$

$= \sum\_\_\_\_\_\_(e)_r r!\ G_C 2\ (a+r, b+r, c, d; e+r, e'; x, y, z, u),$

$r=0$

(2.29) $G_{C_1}^{(4)}(a, b, c, d; e, e'; x, y+t, z, u)$

$\infty$

$(a)r(c)r\ tr \quad (4)$

$= \sum\_\_\_\_\_\_(e)_r r!\ G_C 1\ (a+r, b, c+r, d; e+r, e'; x, y, z, u),$

$r=0$

and

(2.30) $G_{C_1}^{(4)}(a, b, c, d; e, e'; x, y, z, u+t)$

$\infty$

$$= \sum \frac{}{(e')_r} \frac{}{r!} G_{C_1}\ (a+ \quad (a)r(d)r\ tr \quad (4) \quad r, b, c, d+r; e, e'+r; x, y, z, u),$$

$r=0$

Similarly applying the same techniques we can derive self generating relations-cum-Taylor's series expansions for other hypergeometric functions of four variables due to Exton ([4],[6],[7]), Sharma-Parihar ([2],[11]) and QureshiQureshi Kalon; Srivastava [14] and Qureshi-Majid [19] also.

## 3. Marclaurine's Series Expansions.

Replacing $t$ by $x$ and $x$ by 0, in (2.2), we derive the following Maclaurine's series expansion:

(3.1) $H_A^{(4}1^{)}(a, b, c, d; e, e', e"; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)_r (b)_r x^r}{} H(a + r, b + r, c, d; e + r, e', e"; 0, y, z, u)\ (e)_r\ r!$$

Replacing $t$ by $y$ and $y$ by 0 in (2.3), we obtain

(3.2) $H_A^{(4}1^{)}(a, b, c, d; e, e', e"; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{{}_r\ b\ {}_r\, y}{(e')_r} \frac{}{r\ !} H_1^4(a + \quad (a)\ (\ )\ ^r \quad r, b, c, d + r; e, e' + r, e"; x, 0, z, u)$$

Replacing $z$ by zero and $t$ by $z$ in (2.4), we get

(3.3) $H_A^{(4}1^{)}(a, b, c, d; e, e', e"; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{{}_r\, z^r}{(e)_r} \frac{}{r\ !} H_{A_i}^{(r)}(a + \quad (a)_r (c) \quad r, b, c + r, d; e + r, e', e"; x, y, 0, u)$$

Replacing $u$ by *0* and $t$ by $u$ in (2.5), we establish

(3.4) $H_A^{(4}1^{)}(a, b, c, d; e, e', e"; x, y, z, u)$

$$= \sum_{r=0}^{\infty} (\_\_\_ b()er"()c_r)r\ urr!\ H_A(r1)(a, b + r, c + r, d; e, e', e" + r; x, y, z, 0).$$

Replacing $x$ by *0* and $t$ by $x$ in (2.6), we arrive at

(3.5) $H_{B1}^{(4)}(a, b, c, d; e_1, e_2, e_3, e_4; x, y, z, u)$

$(\_\_\_\_\_\_ a()er1()br)r\ xrr!\ H_B(41)(a+r, b+r, c, d; e_1+r, e_2, e_3, e_4; 0, y, z, u).$

Replacing *y* by 0 and *t* by *y* in (2.7), we derive

(3.6) $H_B^{(4}1^{)}(a, b, c, d; e_1, e_2, e_3, e_4; x, y, z, u)$

$\infty$

$= \sum (\_\_\_\_\_\_ a()er2()dr)r\ yr!r\ H_B(41)(a+r, b, c, d+r; e_1, e_2+r, e_3, e_4; x, 0, z, u).$

$r=0$

Replacing *z* by 0 and *t* by *z* in (2.8), we obtain

(3.7) $H_B^{(4}1^{)}(a, b, c, d; e_1, e_2, e_3, e_4; x, y, z, u)$

$$= \sum^{\infty} \frac{{}_r\ c\ {}_r z}{r} H^{(4)}(a, b+r, c+ \quad (b)\,()\,^r \quad (e_3)_r \quad ! \quad B1 \quad r, d; e_1, e_2, e_3 + r, e_4; x, y, 0, u).$$

$r=0$

Replacing *u* by 0 and *t* by *u* in (2.9), we have

(3.8) $H_B^{(4}1^{)}(a, b, c, d; e_1, e_2, e_3, e_4; x, y, z, u)$

$$= \sum^{\infty} \frac{{}_r\ c\ {}_r u^r}{r} H^{(4)}(a+ \quad (a)\,() \quad (e_4)_r \quad ! \quad B1 \quad r, b, c+r, d; e_1, e_2, e_3, e_4 + r; x, y, z, 0).$$

$r=0$

Replacing *y* by 0 and *t* by *y* in (2.10), we establish

(3.9) $G_{A_1}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$\infty$

$(a)r(c)r\ yr \quad (4)$

$= \sum \_\_\_\_\_\_ (e)_r r!\ G_A1\ (a+r, b, c+r, d; e+r, e'; x, 0, z, u).$

$r=0$

Replacing *z* by 0 and *t* by *z* in (2.10), we arrive at

(3.10) $G_{A_1}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$\infty$

$(a)r(b)r\ zr \quad (4)$

$= \sum$ ——— $(e)_r r!\ G_A 1\ (a + r, b + r, c, d; e + r, e'; x, y, 0, u).$

$r=0$

Repalcing $u$ by 0 and $t$ by $u$ in (2.10), we obtain

(3.11) $G_{A_1}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$\infty$

$(a)r(d)r\ ur \quad (4)$

$= \sum$ ——— $(e')_r r!\ G_A 1\ (a + r, b, c, d + r; e, e' + r; x, y, z, 0).$

$r=0$

Replacing $y$ by 0 and $t$ by $y$ in (2.13), we derive

(3.12) $G_{A_2}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$\infty$

$= \sum$ ——— $_r \frac{y^r}{r} G^{(4)}(a +$ $(a)_r(c)$ $A2 \quad r, b, c + r, d; e + r, e'; x, 0, z, u).$

$(e)r \quad !$

$r=0$

Replacing $t$ by $z$ and $z$ by 0 in (2.14), we get

(3.13) $G_{A_2}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$\infty$

$= \sum$ ( ——— $a()er()b_r)r\ zrr!\ G_A(42)(a + r, b + r, c, d; e + r, e'; x, y, 0, u).$

$r=0$

Replacing $u$ by 0 and $t$ by $u$ in (2.15), we obtain

(3.14) $G_{A_2}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$= \sum^{\infty} \frac{(b)_r(c)_r}{\phantom{x}}$ $_r \frac{u^r}{r} G^{(4)}(a, b + r, c +$ $(d)$ $A2 \quad r, d + r; e, e' + r; x, y, z, 0).$

$(e')r \quad !$

$r=0$

Replacing $y$ by 0 and $t$ by $y$, in (2.16), we establish

(3.15) $G_{A_3}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$\infty$

$= \sum$ ( ——— $a()er()c_r)r\ yrr!\ G_A(43)(a + r, b, c + r, d; e + r, e'; x, 0, z, u).$

$r=0$

Replacing $z$ by 0 and $t$ by $z$, in (2.17), we arrive at

(3.16) $G_A{}^{(4}3^{)}(a, b, c, d; e, e'; x, y, z, u)$

$(\_\_\_\_\_\_a)r(b)r\ zr\ GA(43)(a + r, b + r, c, d; e + r, e'; x, y, 0, u).$

$(e)r \quad r!$

Replacing $u$ by 0 and $t$ by $u$ in (2.18), we have

(3.17) $G_{A_3}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$\infty$

$= \sum (\_\_\_\_\_\_c()er'()d_r)r\ urr!\ G_A(43)(a, b, c + r, d + r; e, e' + r; x, y, z, 0).$

$r=0$

Replacing $y$ by 0 and $t$ by $y$ in (2.19), we derive

(3.18) $G_{B_1}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)_r(b_2)_r}{(e)_r} \frac{y^r}{r!} G_{B1}^{(4)}(a + r, b_1, b_2 + r, b_3, b_4; e + r, e'; x, 0, z, u).$$

Replacing z by 0 and $t$ by $z$ in (2.20), we derive

(3.19) $G_{B_1}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)_r(b_2)_r}{(e)_r} \frac{z^r}{r!} G_{B1}^{(4)}(a + r, b_1, b_2 + r, b_3, b_4; e + r, e'; x, y, 0, u).$$

Replacing $t$ by $u$ and $u$ by 0 in (2.21), we get

(3.20) $G_{B_1}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y, z, u)$

$\infty$

$= \sum (\_\_\_\_\_\_a)(re(')b_r4)r\ urr!\ G_B(41)(a + r, b_1, b_2, b_3, b_4 + r; e, e' + r; x, y, z, 0).$

$r=0$

Replacing $y$ by 0 and $t$ by $y$, in (2.22) we have

(3.21) $G_{B_2}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y, z, u)$

$\infty$

$$= \sum_{r=0} \frac{(a)_r (b_2)_r y^r}{(e)_r r!} G_{B_2}^{(4)}(a+r, b_1, b_2+r, b_3, b_4; e+r, e'; x, 0, z, u).$$

Replacing $z$ by 0 and $t$ by $z$, in (2.23), we obtain

(3.22) $G_{B_2}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)_r (b_3)_r z^r}{(e)_r r!} G_{B_2}^{(4)}(a+r, b_1, b_2, b_3+r, b_4; e+r, e'; x, y, 0, u).$$

Replacing $u$ by 0 and $t$ by $u$, in (2.24), we derive

(3.23) $G_{B_2}^{(4)}(a, b_1, b_2, b_3, b_4; e, e'; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(b_1)_r (b_4)_r u^r}{(e')_r r!} G_{B_2}^{(4)}(a, b_1+r, b_2, b_3, b_4+r; e, e'+r; x, y, z, 0).$$

Replacing $x$ by 0 and $t$ by $x$, in (2.25), we establish

(3.24) $G_{C_1}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)_r (b)_r x^r}{(e)_r r!} G_{C_1}^{(4)}(a+r, b+r, c, d; e+r, e'; 0, y, z, u).$$

Replacing $y$ by 0 and $t$ by $y$, in (2.26), we get

(3.25) $G_{C_1}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(b)_r (c)_r y^r}{(e)_r r!} G_{C_1}^{(4)}(a, b+r, c+r, d; e+r, e'; x, 0, z, u).$$

Replacing $u$ by 0 and $t$ by $u$, in (2.27), we arrive at

(3.26) $G_{C_1}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)_r (d)_r u^r}{(e')_r r!} G_{C_1}^{(4)}(a+r, b, c, d+r; e, e'+r; x, y, z, 0).$$

Replacing $x$ by 0 and $t$ by $x$, in (2.28), we obtain

(3.27) $G_{C_2}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)_r (b)_r x^r}{(e)_r \, r!} G_{C_2}^{(4)}(a + r, b + r, c, d; e + r, e'; 0, y, z, u).$$

Replacing $y$ by 0 and $t$ by $y$, in (2.29), we have

(3.28) $G_{C_1}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)_r (c)_r x^r}{(e)_r \, r!} G_{C_1}^{(4)}(a + r, b, c + r, d; e + r, e'; x, 0, z, u).$$

Replacing $u$ by 0 and $t$ by $u$ in (2.30), we derrive

(3.29) $G_{C_1}^{(4)}(a, b, c, d; e, e'; x, y, z, u)$

$$= \sum_{r=0}^{\infty} \frac{(a)_r (d)_r u^r}{(e')_r \, r!} G_{C_1}^{(4)}(a + r, b, c, d + r; e, e' + r; x, y, z, 0).$$

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