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Multipliers In Td-Algebras

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Abstract: In this paper, we present the concept of a Multiplier on a Td-algebra and get a few properties of Multipliers of Td-algebra.

Index Terms - Td-algebras, multiplier, commutative, positive implicative.

I. Introduction

Y.Imai and adequate.Iseki[1, 2, 4,and 5] introduced instructions of summary algebras: BCK-algebras and BCI-algebras.it is recognized that the magnificence of BCK-algebras is a proper subclass of the class of BCI-algebras.Hu. Q. P and Lix [4]are introducedon right BCH-algebras. due to the fact then, several authors have studied BH-algebras. mainly, Q. Zhang, E. H. Roh and H.S. Kim [8] brought on d-algebras. The belief of d-algebra, every other generalization of the belief of BCK-algebra, was introduced byusing Negger[8].

They have got studied the houses of multipliers on them in addition to properties of those algebraic systems using the belief of a multiplier on them. but the houses of multipliers on d-algebra, an crucial magnificence of algebras containing the class of BCK-algebras, have not been investigated to this point. So with this motivation, on this paper we recommended the idea about a multiplier on an Td-algebra and acquire a few houses of multipliers about Td-algebra.

II. Preliminaries

Definition 2.1[1, 2, 5] An algebra (X, *, 0) of type (2,0) is known as a BCK-algebra if it satisfies the subsequent conditions:

1. $((\alpha * \beta) * (\alpha * z)) * (z * \beta) = 0.((\alpha * \beta) * (\alpha * z)) * (z * \beta) = 0$
2. $(\alpha * (\alpha * \beta)) * \beta = 0(\alpha * (\alpha * \beta)) * \beta = 0$
3. $\alpha * \alpha = 0\alpha * \alpha = 0$
4. $\alpha * \beta = 0, \beta * \alpha = 0 \Rightarrow \alpha = \beta\ \alpha * \beta = 0, \beta * \alpha = 0 \Rightarrow \alpha = \beta$
5. $0 * \alpha = 0\ for\ \ all\ \alpha, \beta, z \in X0 * \alpha = 0\ for\ \ all\ \alpha, \beta, z \in X$

Definition 2.2. [8] A nonempty set X with a consistent and a two fold operation * is called a d - polynomial math in case it fulfills the taking after sayings.

1 $\alpha * \alpha = 0\alpha * \alpha = 0$

2. $0 * \alpha = 0\ 0 * \alpha = 0$

3. $\alpha * \beta = 0, \beta * \alpha = 0 \Rightarrow \alpha = y\ \alpha * \beta = 0, \beta * \alpha = 0 \Rightarrow \alpha = y$ for all $\alpha, \beta \in X\alpha, \beta \in X$

Remark: 2.3. It's far obvious from above definitions that every BCK-algebra is a d-algebra. the following suggests that converse isn't always authentic, in preferred.

Example 2.4. . let X= {0, l, m, n, p, q} be a hard and fast with the binary operation * described by using

*	0	l	m	n	p	q
0	0	0	0	0	0	0
l	l	0	0	0	0	0
m	m	m	0	0	0	0
n	n	n	l	0	0	0
p	p	m	l	l	0	0
q	q	q	n	n	l	0

Then (X, $*,0)$ $*,0)$ is BH-algebra.
A BH – algebra X is stated to be medial if $l*(l*m)=ml*(l*m)=m$ for all $l,m\in X.l,m\in X.$

Definition 2.5 Permit S be a nonempty subset of a d-algebra X, then S is referred to as sub algebra of X if $\alpha*\beta\in S\ \alpha*\beta\in S$ for all $\alpha,\beta\in S\ \alpha,\beta\in S$.

Definition 2.6 allow X is a d-algebra and U,V are any non empty subsets of X. We outline a subset U*V of X as follows. U*V={α*β| α∈U,β∈V}

Definition: 2.7 let (X,*) be a d-algebra. Let τ be the collection of subsets of X. τ is said to be a topology on X. If

1. $X,\emptyset\in\tau X,\emptyset\in\tau$.
2. Arbitrary union of members of τ is in τ
3. Finite intersection of members of τ is in τ.

Definition 2.8 Allow (X,*) be a d-algebra and τ a topology on X. Then X=(X,*,0,τ) is known as a topological d-algebra, (it's miles denoted by way of Td-algebra) if the operation "*" is non-stop or equivalently for any α, β ∈ X and for any open set W of α*β there exist open units U and V respectively such that U*V is a subset of .

III. MULTIPLIERS ON TD- ALGEBRAS

Inside the sequel will denote Td-algebra with steady 0 and binary operation until otherwise specific . We now show our outcomes.

Definition 3.1. A character map $f:X\to X\ f:X\to X$ satisfying $f(x*y)=f(x)*yf(x*y)=f(x)*y$, for all $x,y\in Xx,y\in X$, is called a multiplier on XX.

Example 3.2. Let $X=\{0,\alpha,\beta\}X=\{0,\alpha,\beta\}$ with the binary operation $**$ defined by

*	0	α	β
0	0	0	0
α	β	0	β
β	α	α	0

Then XX is a Td-algebra. Let $f:X\to Xf:X\to X$ be defined by $(x)=\{0\ if\ x=0,1\ 1\ if\ x=2$ $(x)=\{0\ if\ x=0,1\ 1\ if\ x=2$. Then ff is a expand on XX.

Remark 3.3: If XX is a Td-algebra with binary operation $**$, then we outline a binary relation $\leq\leq$ on XX by $x \leq y x \leq y$ if and most effective if $x * y = 0 x * y = 0$, $x, y \in X x, y \in X$

Proposition 3.4. Allow XX is a Td-algebra and f a multiplier on XX, then

1. $\mathcal{F}(0) = 0 \mathcal{F}(0) = 0$
2. $\mathcal{F}(\alpha) \leq \alpha \;\; \forall \alpha \in X \; and \; \mathcal{F}(\alpha) \leq \alpha \;\; \forall \alpha \in X \; and$
3. If $X \leq Y X \leq Y$ then $\mathcal{F}(\alpha) \leq \beta \;\; \forall\, \alpha, y \in X \mathcal{F}(\alpha) \leq \beta \;\; \forall\, \alpha, y \in X$

Proof. 1. $\mathcal{F}(0) = \mathcal{F}(0 * \mathcal{F}(0)) = \mathcal{F}(0) * \mathcal{F}(0) = 0 * 0 = 0$

$\mathcal{F}(0) = \mathcal{F}(0 * \mathcal{F}(0)) = \mathcal{F}(0) * \mathcal{F}(0) = 0 * 0 = 0$

Let $\alpha \in X, \alpha \in X$, Then $0 = \mathcal{F}(0) = \mathcal{F}(\alpha * \alpha) 0 = \mathcal{F}(0) = \mathcal{F}(\alpha * \alpha)$

$$= \mathcal{F}(\alpha) * \alpha \;\; so \;\; \mathcal{F}(\alpha) \leq \alpha$$

2. Let $\alpha, \beta \in X \alpha, \beta \in X$ and $x \leq \beta x \leq \beta$. Then $\alpha * y = 0 \;\; so \;\; 0 = \mathcal{F}(0) = \mathcal{F}(\alpha * \beta) = \mathcal{F}(\alpha) * \beta$

$\alpha * y = 0 \;\; so \;\; 0 = \mathcal{F}(0) = \mathcal{F}(\alpha * \beta) = \mathcal{F}(\alpha) * \beta$ Thus $\mathcal{F}(\alpha) \leq \beta \mathcal{F}(\alpha) \leq \beta$.

Proposition 3.5. Permit $\mathcal{F}\mathcal{F}$ and $\mathcal{G}\mathcal{G}$ is expand on XX, formerly their composition $\mathcal{F}\mathcal{G}\mathcal{F}\mathcal{G}$ is a expand on XX.

Proof: Let $\alpha, \beta \in X \alpha, \beta \in X$ Next .

$$(\mathcal{F}\mathcal{G})(\alpha * \alpha) = \mathcal{F}(\mathcal{G}(\alpha * \beta)) = \mathcal{F}(\mathcal{G}(\alpha) * \beta)(\mathcal{F}\mathcal{G})(\alpha * \alpha) = \mathcal{F}(\mathcal{G}(\alpha * \beta)) = \mathcal{F}(\mathcal{G}(\alpha) * \beta)$$

$$\mathcal{F}(\mathcal{G}(\alpha)) * \beta = (\mathcal{F}\mathcal{G})(\alpha) * \beta \mathcal{F}(\mathcal{G}(\alpha)) * \beta = (\mathcal{F}\mathcal{G})(\alpha) * \beta$$

So, $\mathcal{F}\mathcal{G}\mathcal{F}\mathcal{G}$ be a expand on XX.

Definition 3.6. A Td-algebra XX is said to be positive implicative if $(x * y) * z = (x * z) * (y * z) \;\; forall\, x, y, z \in X (x * y) * z = (x * z) * (y * z) \;\; forall\, x, y, z \in X$.

Let $M(X)M(X)$ denotes the collection of all multipliers on X. Obviously. $0: X \to X 0: X \to X$ defined by $0(X) = 0 0(X) = 0$ for all $x \in X x \in X$ and $I: X \to X I: X \to X$ defined by $I(x) = x I(x) = x$ for all $x \in X x \in X$ are in $M(X)M(X)$. So $M(X)M(X)$ is non-contempt.

Definition 3.7. Cause XX be a effective implicative Td-algebra and $M(X)M(X)$ be the collection of all multipliers on XX. We characterize a binary operation $**$ on $M(X)M(X)$ by. $(\mathcal{F} * \mathcal{G})(x) = \mathcal{F}(x) * \mathcal{G}(x) (\mathcal{F} * \mathcal{G})(x) = \mathcal{F}(x) * \mathcal{G}(x)$ for $x \in X x \in X$ and $\mathcal{F}, \mathcal{G} \in M(X) \mathcal{F}, \mathcal{G} \in M(X)$.

Theorem 3.8. Let XX be a effective implicative Td-algebra. Then $(M(\alpha), *, 0)(M(\alpha), *, 0)$ is a effective implicative Td-algebra.

Proof: Let X be implicative Td-algebra . Let $\mathcal{G}, \mathcal{F} \in M(X). \mathcal{G}, \mathcal{F} \in M(X)$.
Then

$$(\mathcal{G} * \mathcal{F})(\alpha * \beta) = (\mathcal{G}(\alpha * \beta)) * (\mathcal{F}(\alpha * \beta))$$

$$= (\mathcal{G}(\alpha) * \beta) * (\mathcal{F}(\alpha) * \beta)$$

$$= (\mathcal{G}(\alpha) * \beta) * (\mathcal{F}(\alpha) * \beta)$$

$$= (\mathcal{G}(\alpha) * \mathcal{F}(\alpha)) * ((\mathcal{G} * \mathcal{F})(\alpha)) * \beta$$

So $\mathcal{G} * \mathcal{F} \in M(X) \mathcal{G} * \mathcal{F} \in M(X)$. Let $\mathcal{F} \in M(X) \mathcal{F} \in M(X)$. Then

$$(O * \mathcal{F})(\alpha) = O(\alpha) * \mathcal{F}(\alpha) = 0 * \mathcal{F}(\alpha) = 0 = O(\alpha)$$

for all $\alpha \in X. \alpha \in X.$ So $O * \mathcal{F} = 0 O * \mathcal{F} = 0$ for all $\mathcal{F} \in M(X) \mathcal{F} \in M(X)$.

Now for $\mathcal{F} \in M(X) \mathcal{F} \in M(X)$, we have $(\mathcal{F} * \mathcal{F})(\alpha) = \mathcal{F}(\alpha) * \mathcal{F}(\alpha) = 0 = O(\alpha)$

$(\mathcal{F} * \mathcal{F})(\alpha) = \mathcal{F}(\alpha) * \mathcal{F}(\alpha) = 0 = O(\alpha)$ for all $\alpha \in X. \alpha \in X.$ So $\mathcal{F} * \mathcal{F} = 0 \mathcal{F} * \mathcal{F} = 0$

Let $\mathcal{G}, \mathcal{F} \in M(X) \mathcal{G}, \mathcal{F} \in M(X)$ be such that $\mathcal{F} * \mathcal{G} = O \text{ and } \mathcal{G} * \mathcal{F} = 0 \mathcal{F} * \mathcal{G} = O \text{ and } \mathcal{G} * \mathcal{F} = 0$

This implies $(\mathcal{F} * \mathcal{G})(\alpha) = O \text{ and } (\mathcal{G} * \mathcal{F})(\alpha) = 0 (\mathcal{F} * \mathcal{G})(\alpha) = O \text{ and } (\mathcal{G} * \mathcal{F})(\alpha) = 0$ for all $x \in X$ $x \in X$.
That is

$$\mathcal{F}(\alpha) * \mathcal{G}(\alpha) = O \text{ and } \mathcal{G}(\alpha) * \mathcal{F}(\alpha) = 0 \mathcal{F}(\alpha) * \mathcal{G}(\alpha) = O \text{ and } \mathcal{G}(\alpha) * \mathcal{F}(\alpha) = 0$$

Which implies $\mathcal{F}(\alpha) = \mathcal{G}(\alpha) \mathcal{F}(\alpha) = \mathcal{G}(\alpha)$ for all $\alpha \in X \alpha \in X$. Thus $\mathcal{F} = \mathcal{G} \mathcal{F} = \mathcal{G}$. Hence $M(X) M(X)$ is a Td-algebra.

Now we show that it is a effective implicative. Let $\mathcal{F}, \mathcal{G} \text{ and } h \in M(X) \mathcal{F}, \mathcal{G} \text{ and } h \in M(X)$.

Then $((\mathcal{F} * \mathcal{G}) * h)(\alpha) = ((\mathcal{F} * \mathcal{G})(\alpha)) * h(\alpha) = (\mathcal{F}(\alpha) * \mathcal{G}(\alpha)) * h(\alpha)$

$((\mathcal{F} * \mathcal{G}) * h)(\alpha) = ((\mathcal{F} * \mathcal{G})(\alpha)) * h(\alpha) = (\mathcal{F}(\alpha) * \mathcal{G}(\alpha)) * h(\alpha)$

$$= (\mathcal{F}(\alpha) * h(\alpha)) * (\mathcal{G}(\alpha) * h(\alpha)) = ((\mathcal{F} * h)(\alpha)) * ((\mathcal{G} * h)(\alpha))$$

$= (\mathcal{F}(\alpha) * h(\alpha)) * (\mathcal{G}(\alpha) * h(\alpha)) = ((\mathcal{F} * h)(\alpha)) * ((\mathcal{G} * h)(\alpha))$

$$= ((\mathcal{F} * h) * (\mathcal{G} * h))(\alpha) = ((\mathcal{F} * h) * (\mathcal{G} * h))(\alpha)$$

For all $x \in X x \in X$

Hence $(\mathcal{F} * \mathcal{G}) * h = (\mathcal{F} * \mathcal{G}) * (\mathcal{F} * h) (\mathcal{F} * \mathcal{G}) * h = (\mathcal{F} * \mathcal{G}) * (\mathcal{F} * h)$

Thus $M(X) M(X)$ is an implicative Td-algebra.

Theorem 3.9 . Let XX be a Td-algebra and $\mathcal{F}\mathcal{F}$ a multiplier on XX. If f is 1-1 then ff is the identity map on XX..

Proof : Let $\mathcal{F}\mathcal{F}$ be 1-1 . Let $\alpha \in X \alpha \in X$. Then $\mathcal{F}(\alpha * \mathcal{F}(\alpha)) = \mathcal{F}(\alpha) * \mathcal{F}(\alpha) = 0 = \mathcal{F}(0)$

$\mathcal{F}(\alpha * \mathcal{F}(\alpha)) = \mathcal{F}(\alpha) * \mathcal{F}(\alpha) = 0 = \mathcal{F}(0)$

Suddenly $\alpha * f(\alpha) = 0, \alpha * f(\alpha) = 0,$ which $\alpha \le \mathcal{F}(\alpha). \alpha \le \mathcal{F}(\alpha).$ Since $\mathcal{F}(\alpha) \le \alpha. \mathcal{F}(\alpha) \le \alpha.$ by pro 3.4(2)

for all $\alpha\alpha$, there fore $\mathcal{F}(\alpha) = \alpha \mathcal{F}(\alpha) = \alpha$. Hence $\mathcal{F}\ \mathcal{F}$ is the identity map.

Let $f f$ be a multiplier on XX. We define $ker\ (f) ker\ (f)$ by

$ker\ ker\ (\mathcal{F}) = \{\alpha : \alpha \in X\ \ and\ \mathcal{F}(\alpha) = 0\}. ker\ ker\ (\mathcal{F}) = \{\alpha : \alpha \in X\ \ and\ \mathcal{F}(\alpha) = 0\}.$

Proposition 3.10. Commission XX be a Td-algebra and $\mathcal{F}\ \mathcal{F}$ a multiplier on XX. Then

1) $ker\ (\mathcal{F}) ker\ (\mathcal{F})$ is a sub algebra of XX and
2) If $\mathcal{F}\mathcal{F}$ is 1-1, then $ker\ ker\ (\mathcal{F}) = \{0\} ker\ ker\ (\mathcal{F}) = \{0\}$

Proof: 1) Let $\alpha, \beta \in ker\ ker\ (\mathcal{F}) . \alpha, \beta \in ker\ ker\ (\mathcal{F})$. Then $\mathcal{F}(\alpha) = 0 \mathcal{F}(\alpha) = 0$ and $\mathcal{F}\ (\beta) = 0\ \mathcal{F}\ (\beta) = 0$. So

$\mathcal{F}(\alpha * \beta) = \mathcal{F}(\alpha) * \beta = 0 * \beta = 0 \mathcal{F}(\alpha * \beta) = \mathcal{F}(\alpha) * \beta = 0 * \beta = 0$. Thus $\alpha * \beta \in ker\ ker\ (\mathcal{F})$, $\alpha * \beta \in ker\ ker\ (\mathcal{F})$, which implies $ker(\mathcal{F}) ker(\mathcal{F})$ is a sub algebra of XX.

2) Let f be one to one. Let $\alpha \in ker\ ker\ (\mathcal{F}) . \alpha \in ker\ ker\ (\mathcal{F})$. So $\mathcal{F}(\alpha) = 0 = \mathcal{F}(0)$ $\mathcal{F}(\alpha) = 0 = \mathcal{F}(0)$. Thus $\alpha = 0 \alpha = 0$. So $ker\ ker\ (\mathcal{F}) = \{0\}. ker\ ker\ (\mathcal{F}) = \{0\}$.

Definition 3.11. Let Td-algebra XX is said commutative if $\alpha * (\alpha * \beta) = \beta * (\beta * \alpha) \forall\ \ \alpha, \beta \in X.$ $\alpha * (\alpha * \beta) = \beta * (\beta * \alpha) \forall\ \ \alpha, \beta \in X.$

Proposition 3.12. Sanction XX be a commutative Td- algebra pleasurable $\alpha = 0 = \alpha \alpha = 0 = \alpha$, $\alpha \in X\ \alpha \in X$. Let $\mathcal{F}\mathcal{F}$ be as multiplier on XX. If $\alpha \in Ker(\mathcal{F}) \alpha \in Ker(\mathcal{F})$ and $\beta \le \alpha, \beta \le \alpha$, then $\beta \in ker\ (\mathcal{F}) \beta \in ker\ (\mathcal{F})$.

Proof: Let $\alpha \in ker\ ker\ (\mathcal{F}) \alpha \in ker\ ker\ (\mathcal{F})$ and $\beta \le \alpha \beta \le \alpha$. Then $\mathcal{F}(\alpha) = 0 \mathcal{F}(\alpha) = 0$ and $(\beta * \alpha) = 0$

$(\beta * \alpha) = 0$
Now $\mathcal{F}(\beta) = \mathcal{F}(\beta * 0) = \mathcal{F}(\beta * (\beta * \alpha)) = \mathcal{F}(\alpha * (\alpha * \beta))$

$\mathcal{F}(\beta) = \mathcal{F}(\beta * 0) = \mathcal{F}(\beta * (\beta * \alpha)) = \mathcal{F}(\alpha * (\alpha * \beta))$

$= \mathcal{F}(\alpha) * (\alpha * \beta) = 0 * (\alpha * \beta) = 0 \qquad = \mathcal{F}(\alpha) * (\alpha * \beta) = 0 * (\alpha * \beta) = 0$

So $\alpha \in ker\ (\mathcal{F}) \alpha \in ker\ (\mathcal{F})$

Theorem 3.13. Allow XX be a Td-algebra pleasant $\alpha = 0 = \alpha \alpha = 0 = \alpha$ for every $\alpha \in X \alpha \in X$. Let $\mathcal{F}\mathcal{F}$ be a multiplier on XX, which is likewise an endomorphism on XX. Then $ker\ (\mathcal{F}) ker\ (\mathcal{F})$ is a Td-perfect of XX.

Proof: Manifestly, $0 \in ker\ ker\ (\mathcal{F})\ 0 \in ker\ ker\ (\mathcal{F})$. Therefore $ker\ (\mathcal{F}) ker\ (\mathcal{F})$ is nonempty. let

$\alpha * \beta \in ker\ ker\ (\mathcal{F})\ \alpha * \beta \in ker\ ker\ (\mathcal{F})$ and

$\beta \in ker\ (\mathcal{F}) \beta \in ker\ (\mathcal{F})$. Then $\mathcal{F}(\beta) = 0 \mathcal{F}(\beta) = 0$. Also $\mathcal{F}(\alpha * \beta) = 0\ \mathcal{F}(\alpha * \beta) = 0$, which implies

$$0 = \mathcal{F}(\alpha) * \mathcal{F}(\beta) = \mathcal{F}(\alpha) * 0 = \mathcal{F}(\alpha)\ 0 = \mathcal{F}(\alpha) * \mathcal{F}(\beta) = \mathcal{F}(\alpha) * 0 = \mathcal{F}(\alpha)$$

Thus $\alpha \in ker\ ker\ (\mathcal{F})\ .\alpha \in ker\ ker\ (\mathcal{F})$.

Let $\alpha \in ker\ ker\ (\mathcal{F})\ \alpha \in ker\ ker\ (\mathcal{F})$ and $\beta \in X \beta \in X$. Then $\mathcal{F}(\alpha * \beta) = \mathcal{F}(\alpha) * \beta = 0 * \beta = 0$

$\mathcal{F}(\alpha * \beta) = \mathcal{F}(\alpha) * \beta = 0 * \beta = 0$

So $\alpha * \beta \in ker\ (\mathcal{F}) \alpha * \beta \in ker\ (\mathcal{F})$. Hence $ker\ (\mathcal{F}) ker\ (\mathcal{F})$ is a Td-ideal of $X\ X$.

Definition 3.14. Cause XX be a Td-algebra and $\mathcal{F}\ \mathcal{F}$ a multiplier on XX. Then the set $Fix(\mathcal{F}) = \{\alpha : \alpha \in X\ \ and\ \ \mathcal{F}(\alpha) = \alpha\} Fix(\mathcal{F}) = \{\alpha : \alpha \in X\ \ and\ \ \mathcal{F}(\alpha) = \alpha\}$ is called the set of fixed factors of $\mathcal{F}\mathcal{F}$.

Proposition 3.15. If XX is a Td- algebra and $\mathcal{F}\ \mathcal{F}$ a multiplier on X. Then fix $\mathcal{F}\ \mathcal{F}$ is a sub algebra of X.

Proof: Because $\mathcal{F}(0) = 0 \mathcal{F}(0) = 0$, so $Fix(\mathcal{F}) Fix(\mathcal{F})$ is non-empty. Allow $\alpha, \beta \in Fix(\mathcal{F}) .\alpha, \beta \in Fix(\mathcal{F})$. Then

$\mathcal{F}(\alpha) = \alpha, \mathcal{F}(\beta) = \beta \mathcal{F}(\alpha) = \alpha, \mathcal{F}(\beta) = \beta$ Thus $\mathcal{F}(\alpha * \beta) = \mathcal{F}(\alpha) * \beta = \alpha * \beta$

$\mathcal{F}(\alpha * \beta) = \mathcal{F}(\alpha) * \beta = \alpha * \beta$. So $\alpha * \beta \in Fix(\mathcal{F})\ \alpha * \beta \in Fix(\mathcal{F})$. Hence $Fix(\mathcal{F}) Fix(\mathcal{F})$ is a sub algebra of XX.

Definition 3.16.If XX be Td- algebra and f a multiplier on X. f is referred to as idempotent if $f^{\circ}f = f.f^{\circ}f$ $f^{\circ}f = f.f^{\circ}f$ can be denoted by means of $f^2.f^2$.

Theorem 3.17. let X be a wonderful implicative Td-algebra which satisfies $\alpha * 0 = \alpha \alpha * 0 = \alpha$ for every $\alpha \in X \alpha \in X$. If

$\mathcal{F}_1, \mathcal{F}_2 \mathcal{F}_1, \mathcal{F}_2$ is two idempotent multipliers on XX. If $\mathcal{F}_1{}^{\circ}\mathcal{F}_2 = \mathcal{F}_2{}^{\circ}\mathcal{F}_1,\ \ then\ \mathcal{F}_1 * \mathcal{F}_2$

$\mathcal{F}_1{}^{\circ}\mathcal{F}_2 = \mathcal{F}_2{}^{\circ}\mathcal{F}_1,\ \ then\ \mathcal{F}_1 * \mathcal{F}_2$ is an idempotent multiplier on XX.

Proof: We know that, we get that $f_1 f_2 f_1 f_2$ is a multiplier on X. Now

$$((\mathcal{F}_1 * \mathcal{F}_2)^{\circ}(\mathcal{F}_1 * \mathcal{F}_2))(\alpha) = (\mathcal{F}_1 * \mathcal{F}_2)((\mathcal{F}_1 * \mathcal{F}_2)(\alpha)) = (\mathcal{F}_1 * \mathcal{F}_2)\big(\mathcal{F}_1(\alpha) * \mathcal{F}_2(\alpha)\big)$$

$$= (\mathcal{F}_1(\mathcal{F}_1(\alpha) * \mathcal{F}_2(\alpha))) * (\mathcal{F}(\mathcal{F}_1(\alpha) * \mathcal{F}_2(\alpha)))$$

$= (\mathcal{F}_1(\mathcal{F}_1(\alpha) * \mathcal{F}_2(\alpha))) * (\mathcal{F}(\mathcal{F}_1(\alpha) * \mathcal{F}_2(\alpha)))$

$=((\mathcal{F}_1{}^\circ(\mathcal{F}_1)(\alpha) * \mathcal{F}_2(\alpha) * ((\mathcal{F}_2{}^\circ\mathcal{F}_1)(\alpha)\mathcal{F}_2(\alpha))$

$((\mathcal{F}_1{}^\circ(\mathcal{F}_1)(\alpha) * \mathcal{F}_2(\alpha) * ((\mathcal{F}_2{}^\circ\mathcal{F}_1)(\alpha)\mathcal{F}_2(\alpha))$

$= (\mathcal{F}_1(\alpha)\mathcal{F}_2(\alpha)) * (((\mathcal{F}_1{}^\circ\mathcal{F}_2)(\alpha) * \mathcal{F}_2(\alpha))$

$= (\mathcal{F}_1(\alpha)\mathcal{F}_2(\alpha)) * (((\mathcal{F}_1{}^\circ\mathcal{F}_2)(\alpha) * \mathcal{F}_2(\alpha))$

$= (\mathcal{F}_1(\alpha) * \mathcal{F}_2(\alpha)) * ((\mathcal{F}_1(\mathcal{F}_2(\alpha)\mathcal{F}_2(\alpha)))$

$= ((\mathcal{F}_1 * \mathcal{F}_2)(\alpha)) * ((\mathcal{F}_1(0)) = ((\mathcal{F}_1 * \mathcal{F}_2)(\alpha)) * 0$

$= ((\mathcal{F}_1 * \mathcal{F}_2)(\alpha)) * ((\mathcal{F}_1(0)) = ((\mathcal{F}_1 * \mathcal{F}_2)(\alpha)) * 0$

$= (\mathcal{F}_1 * \mathcal{F}_2)(\alpha). = (\mathcal{F}_1 * \mathcal{F}_2)(\alpha).$

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Thus $(\mathcal{F}_1 * \mathcal{F}_2)((\mathcal{F}_1 * \mathcal{F}_2) = f_1 * \mathcal{F}_2(\mathcal{F}_1 * \mathcal{F}_2)((\mathcal{F}_1 * \mathcal{F}_2) = f_1 * \mathcal{F}_2$. Hence $f_1 * \mathcal{F}_2 f_1 * \mathcal{F}_2$ is idempotent.

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