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Semiopen Sets And Semilocally Closed Sets In Generalised Topology And Minimal Structure Spaces

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Abstract: The aim of this paper is to introduce semiopen sets and semilocally closed sets in generalized topology and minimal structure spaces. Further we investigate some properties of these sets on these spaces.
Index Terms - **$\tau_g m_X$-semiopen, $\tau_g m_X$-semilocally closed set**

I. INTRODUCTION

The concept of minimal structure space was introduced by V. Popa and T. Nori in 2000. A. Csarszar introduced the concept of generalised topological spaces. He also introduced the concepts of generalised continuous functions on generalised topological spaces. In 2011, S. Buadong and et al introduced the notion of the generalised topology and minimal structure space (briefly GTMS space). In 2022, A. A. basumatary and et al studied on semiopen sets and semilocallly closed sets in generalised topology and minimal structure spaces. In this paper, we introduce semiopen sets, semiclosed sets and semilocally closed sets in GTMS spaces.

I. PRELIMINARIES

Definition 1 Let (X,τ) be a topological spaces and A a subset of X. Then A is said to be semi-open set if A $\subset$ Cl(Int(A)).

Definition 2 Let X be a non empty set and τ_g a collection of subsets of X. Then τ_g is called a generalized topology (briefly GT) on X if and only if $\varphi \in \tau_g$ and $G_i \in \tau_g$ for i ∈ I ≠ φ implies $\cup_{i\in I} G_i \in \tau_g$. We call the pair (X, τ_g) a generalized topological space (briefly GTS) on X.

The elements of τ_g are called τ_g-open sets and the complements are called τ_g-closed sets.

The closure of a subset A in a generalized topological space (X, τ_g), denoted by τ_g-Cl(A), is the intersection of generalized closed sets containing A and the interior of A, denoted by τ_g-Int(A), is the union of generalized open sets contained in A.

Theorem 1 Let (X, τ_g) be a generalized topological space. Then

(1) τ_g-Cl(A) = X \ τ_g-Int(X \ A);

(2) τ_g-Int(A) = X \ τ_g-Cl(X \ A).

Proposition 1 Let (X, τ_g) be a generalized topological space. For subsets A and B of X, the following properties hold:

(1) τ_g-Cl(X \ A) = X \ τ_g -Int(A) and τ_g -Int(X \ A) = X \ τ_g -Cl(A);

(2) if X \ A ∈ τ_g, then τ_g-Cl(A) = A and if A ∈ τ_g, then τ_g-Int(A) = A;

(3) if A ⊆ B, then τ_g-Cl(A) ⊆ τ_g-Cl(B) and τ_g-Int(A) ⊆ τ_g-Int(B);

(4) A ⊆ τ_g-Cl(A) and τ_g-Int(A) ⊆ A;
(5) τ_g-Cl(τ_g-Cl(A)) =τ_g-Cl(A) and τ_g-Int(τ_g-Int(A)) =τ_g-Int(A)

Definition 3 Let X be a nonempty set and P(X) the power set of X. A subfamily m_X of P(X) is called a minimal structure (briefly m-structure) on X if $\emptyset \in m_X$ and X ∈ m_X.
By (X, m_X), we denote a nonempty set X with an m-structure m_X on X and it is called an m-space. Each member of $m_X$is said to be m_X -open and the complement of an m_X -open set is said to be m_X-closed.

Definition 4 Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the m_X-closure of A denoted by m_X-Cl(A) and the m_X-interior of A denoted by m_X -Int(A), are defined as follows:
(1) m_X -Cl(A) = ∩{F : A ⊆ F, X \ F ∈ m_X},
(2) m_X -Int(A) = ∩{U : U ⊆ A, U ∈ m_X}.

Lemma 1 Let X ≠ φ and m_X a m-structure on X and A subset of X. Then m_X -Cl(X\A) = X\ m_X -Int(A) and m_X -Int(X\A) = X\ m_X -Cl(A).

Definition 5 Let X be a nonempty set and let τ_g be a generalized topology and m_X a minimal structure on X. A triple (X,τ_g, m_X) is called a generalized topology and minimal structure space (briefly GTMS space).

Let (X,τ_g, m_X) be a generalized topology and minimal structure space and A a subset of X. The closure and interior of A in τ_g are denote by τ_g-Cl(A) and τ_g-Int(A), respectively And the closure and interior of A in m_X are denote by m_X-Cl(A) and m_X-Int(A), respectively.

Definition 6 Let (X,τ_g, m_X) be a generalized topology and minimal structure space. A subset A of X is said to be a $\tau_g m_X$-closed set if τ_g-Cl(m_X-Cl(A)) = A. And a subset A of X is said to be a $m_X\tau_g$-closed set if m_X-Cl(τ_g-Cl(A)) = A.
The complement of a $\tau_g m_X$-closed set is said to be $\tau_g m_X$-open. And the complement of $m_X\tau_g$-closed set is said to be $m_X\tau_g$-open

Lemma 2 Let (X,τ_g, m_X) be a generalized topology and minimal structure space and A ⊆ X. Then A is $\tau_g m_X$-closed if and only if m_X-Cl(A) = A and τ_g-Cl(A) = A.

Lemma 3 Let (X,τ_g, m_X) be a generalized topology and minimal structure space and A ⊆ X. Then A is $m_X\tau_g$-closed if and only if m_X-Cl(A) = A and τ_g-Cl(A) = A.

Proposition 2 Let (X, τ_g, m_X) be a generalized topology and minimal structure space and A ⊆ X. Then A is $\tau_g m_X$-closed if and only if A is $m_X\tau_g$-closed.

Definition 7 Let (X, τ_g,m_X) be a generalized topology and minimal structure space and A a subset of X. Then A is said to be closed if A is $\tau_g m_X$-closed.
The complement of a closed set is said to be an open set.

Proposition 3 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. Then A is open if and only if A = τ_g-Int(m_X-Int(A)).

Proposition 4 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. If A and B are open, then A ∪ B is open.

Definition 8 A subset of A of a GTMS-space (X, τ_g, m_X) is said to be $\tau_g m_X$-locally closed if A = B ∩ C where B is τ_g-open and C is m_X-closed.

II. SEMIOPEN SETS IN GTMS SPACE

Definition 9 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. A subset A of X is said to be $\tau_g m_X$-semiopen if A ⊂ τ_g-Cl(m_X-Int(A)) and $m_X\tau_g$-semiopen if A ⊂ m_X-Cl(τ_g-Int(A)). If A subset of X is said to be $\tau_g m_X$-semiclosed set ($m_X\tau_g$-semiclosed set) if the complement X \ A of A is a $\tau_g m_X$-semiopen set ($m_X\tau_g$-semiopen set). The set of all $\tau_g m_X$-semiopen sets of (X, τ_g, m_X) denoted by $\tau_g m_X$-SO(X) ($m_X\tau_g$-SO(X) resp.) and the set of all $\tau_g m_X$-semiclosed sets of (X, τ_g, m_X) is denoted by $\tau_g m_X$-SC(X) ($m_X\tau_g$-SC(X) resp.).

Remark 1 $\tau_g m_X$-preopen and $\tau_g m_X$-semiopen are independent to each other in general as can be seen from the following example.

Example 1 Let X = {a,b,c}.We define a generalised topology and minimal structure space: τ_g={φ,{b},{b,c}}; m_X={φ,{a},{a,c},X}. Here, {b} is $\tau_g m_X$-semiopen but not $\tau_g m_X$-semiopen.
m_X={φ,{a},{a,c},X}. Here, {b} is $\tau_g m_X$-semiopen but not $\tau_g m_X$-semiopen.

Remark 2 A set which is $\tau_g m_X$-semiopen need not be $m_X\tau_g$-semiopen in general as can be seen from the following example:

Example 2 Let X = {a,b,c}.We define a generalised topology and minimal structure space: τ_g={φ,{a},{a,c}}; m_X={φ,{b},{b,c},X. Here, {b} is $\tau_g m_X$-semiopen but not $m_X\tau_g$-semiopen.

Theorem 2 Every $\tau_g m_X$-open is $\tau_g m_X$-semiopen.

Proof The proof is obvious.

Theorem 3 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. Then the arbitrary union of $\tau_g m_X$-semiopen sets is a $\tau_g m_X$-semiopen set.

Proof Let $\{A_\alpha\}_{\alpha\in J}$ be a family of $\tau_g m_X$-semiopen sets in X. Since A_α is a $\tau_g m_X$-semiopen set, we have $A_\alpha \subset \tau_g$-Cl(m_X-Int(A_α)) ∀ α ∈ J.Therefore, $\cup_{\alpha\in J} A_\alpha \subset \cup_{\alpha\in J}\, \tau_g$-Cl($m_X$-Int($A_\alpha$)) ⊂ τ_g-Cl($\cup_{\alpha\in J}$ (m_X-Int(A_α)) ⊂ τ_g-Cl(m_X-Int($\cup_{\alpha\in J} A_\alpha$)). This implies that $\cup_{\alpha\in J} A_\alpha$ is a $\tau_g m_X$-semiopen set.

Theorem 4 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. Then the arbitrary union of $m_X\tau_g$-semiopen sets is a $m_X\tau_g$-semiopen set.

Proof Refer the semivious theorem.

Theorem 5 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. Then the arbitrary intersection of $\tau_g m_X$-semiclosed ($m_X\tau_g$-semiclosed) sets is a $\tau_g m_X$-semiclosed set ($m_X\tau_g$-semiclosed).

Proof Let $\{F_\alpha\}_{\alpha\in J}$ be a family of $\tau_g m_X$-semiclosed sets in X. Since F_α is a $\tau_g m_X$-semiclosed set, we have X \ F_α is a $\tau_g m_X$-semiopen set. By Thm $\cup_{\alpha\in J}$ (X \ F_α) is a $\tau_g m_X$-semiopen set.Therefore, $\cup_{\alpha\in J}$ (X \ F_α) = X \ $\cap_{\alpha\in J} F_\alpha$.This implies that $\cap_{\alpha\in J} F_\alpha$ is a $\tau_g m_X$-semiclosed set.

Theorem 6 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. Let A be subset of X. Then the following properties hold:

(1) $\tau_g m_X$-Int_s(A) = ∪{U : U ⊂ A and A ∈ $\tau_g m_X$-SO(X)}.
(2) $\tau_g m_X$-Int_s(A) is the largest $\tau_g m_X$-semiopen subset of X contained in A.
(3) A is $\tau_g m_X$-semiopen if and only if A = $\tau_g m_X$-Int_s(A).
(4) $\tau_g m_X$-Int_s($\tau_g m_X$-Int_s(A)) = $\tau_g m_X$-Int_s(A).

Proof (1) Let x ∈ $\tau_g m_X$-Int_s(A). Then there exists U ∈ $\tau_g m_X$-SO(X) containing x such that x ∈ U ⊂ A. Therefore, $\tau_g m_X$-Int_s(A) ⊂ ∪{U : U ⊂ A and A ∈ $\tau_g m_X$-SO(X)}. Let x ∈ ∪{U : U ⊂ A and A ∈ $\tau_g m_X$-SO(X)}. Then there exists a U in ∪{U : U ⊂ A and A ∈ $\tau_g m_X$-SO(X)} containing x such that x ∈ $\tau_g m_X$-Int_s(A). Therefore, ∪{U : U ⊂ A and A ∈ $\tau_g m_X$-SO(X)} ⊂ $\tau_g m_X$-Int_s(A).Hence, $\tau_g m_X$-Int_s(A) = ∪{U : U ⊂ A and A ∈ $\tau_g m_X$-SO(X)}.

(2) By defn, $\tau_g m_X$-Int_s(A) is denoted as the union of all $\tau_g m_X$-semiopen sets contained in A. That is, $\tau_g m_X$-Int_s(A) = ∪{U : U ⊂ A and A ∈ $\tau_g m_X$-SO(X)}. Since the arbitrary union of $\tau_g m_X$-semiopen sets is a $\tau_g m_X$-semiopen set, we have $\tau_g m_X$-Int_s(A) is a $\tau_g m_X$-semiopen set. Therefore, it is clear that for all U such that U is $\tau_g m_X$-semiopen and U ⊂ A implies U ⊂ $\tau_g m_X$-Int_s(A). Hence, $\tau_g m_X$-Int_s(A) is the largest $\tau_g m_X$-semiopen subset of X contained in A.

(3) Let A = $\tau_g m_X$-Int_s(A). By (1), $\tau_g m_X$-Int_s(A) = ∪{U : U ⊂ A and A ∈ $\tau_g m_X$-SO(X)}. Since the arbitrary union of $\tau_g m_X$-semiopen sets is a $\tau_g m_X$-semiopen set, we have $\tau_g m_X$-Int_s(A) is a $\tau_g m_X$-semiopen set. Therefore, A is $\tau_g m_X$-semiopen. Conversely, suppose A is $\tau_g m_X$-semiopen. By(1), it is clear that for any $\tau_g m_X$-semiopen set U ⊂ A implies U ⊂ $\tau_g m_X$-Int_s(A). For A ⊂ A implies A ⊂ $\tau_g m_X$-Int_s(A). But $\tau_g m_X$-Int_s(A) ⊂ A. Therefore, A = $\tau_g m_X$-Int_s(A). Hence A is $\tau_g m_X$-semiopen if and only if A = $\tau_g m_X$-Int_s(A).

(4) By defn, $\tau_g m_X$-Int_s(A) is denoted as the union of all $\tau_g m_X$-semiopen sets contained in A. Therefore, $\tau_g m_X$-Int_s(A) ⊂ A. Since $\tau_g m_X$-Int_s(A) is a $\tau_g m_X$-semiopen set, we have $\tau_g m_X$-Int_s($\tau_g m_X$-Int_s(A)) = $\tau_g m_X$-Int_s(A).

Theorem 7 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. Let A and B be the subsets of X. Then the following properties hold:

(1) If A ⊂ B, then $\tau_g m_X$-Int_s(A) ⊂ $\tau_g m_X$-Int_s(B).
(2) $\tau_g m_X$-Int_s(A) ∪ $\tau_g m_X$-Int_s(B) ⊂ $\tau_g m_X$-Int_s(A ∪ B).
(3) $\tau_g m_X$-Int_s(A ∩ B) ⊂ $\tau_g m_X$-Int_s(A) ∩ $\tau_g m_X$-Int_s(B).

Proof (1) Let x ∈ $\tau_g m_X$-Int_s(A). Then there exists a $\tau_g m_X$-semiopen set U such that x ∈ U ⊂ A. Since A ⊂ B, we have x ∈ U ⊂ A ⊂ B. Therefore, x ∈ U ⊂ B. This implies x ∈ $\tau_g m_X$-Int_s(B). Hence, $\tau_g m_X$-Int_s(A) ⊂ $\tau_g m_X$-Int_s(B).

(2) We know that, A ⊂ A ∪ B. By (1), $\tau_g m_X$-Int$_s$(A) ⊂ $\tau_g m_X$-Int$_s$(A ∪ B). Also, we know that, B ⊂ A ∪ B. By (1), $\tau_g m_X$-Int$_s$(B) ⊂ $\tau_g m_X$-Int$_s$(A ∪ B). Hence, $\tau_g m_X$-Int$_s$(A) ∪ $\tau_g m_X$-Int$_s$(B) ⊂ $\tau_g m_X$-Int$_s$(A ∪ B).

(3)We know that, A ∩ B ⊂ A. By (1), $\tau_g m_X$-Int$_s$(A ∩ B) ⊂ $\tau_g m_X$-Int$_s$(A). Also, we know that, A ∩ B ⊂ B. By (1), $\tau_g m_X$-Int$_s$(A ∩ B) ⊂ $\tau_g m_X$-Int$_s$(B). Hence, $\tau_g m_X$-Int$_s$(A ∩ B) ⊂ $\tau_g m_X$-Int$_s$(A) ∩ $\tau_g m_X$-Int$_s$(B).

Theorem 8 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. Let A be subset of X. Then the following properties hold:

(1) $\tau_g m_X$-Cl$_s$(A) = ∩ {F : A ⊂ F and F ∈ $\tau_g m_X$-SC(X)}.
(2) $\tau_g m_X$-Cl$_s$(A) is the smallest $\tau_g m_X$-semiclosed subset of X containing A.
(3) A is $\tau_g m_X$-semiclosed if and only if A = $\tau_g m_X$-Cl$_s$(A).
(4) $\tau_g m_X$-Cl$_s$($\tau_g m_X$-Cl$_s$(A)) = $\tau_g m_X$-Cl$_s$(A).

Proof (1) Let x ∉ $\tau_g m_X$-Cl$_s$(A). Then there exists U ∈ $\tau_g m_X$-SO(X) containing x such that U ∩ A = φ. This implies X – U is a $\tau_g m_X$-semiclosed set containing A and x ∉ X – U. Therefore, $\tau_g m_X$-Cl$_s$(A) ⊂ ∩ {F : A ⊂ F and F ∈ $\tau_g m_X$-SC(X)}. Conversely, suppose x ∉ ∩ {F : A ⊂ F and F ∈ $\tau_g m_X$-SC(X)}. Then there exists a F ∈ $\tau_g m_X$-SC(X) such that A ⊂ F and x ∉ F. This implies X – F is a $\tau_g m_X$-semiopen set containing x and (X – F) ∩ A = φ. Therefore, x ∉ $\tau_g m_X$-Cl$_s$(A). This implies ∩ {F : A ⊂ F and F ∈ $\tau_g m_X$-SC(X)} ⊂ $\tau_g m_X$-Cl$_s$(A). Hence, $\tau_g m_X$-Cl$_s$(A) = ∩ {F : A ⊂ F and F ∈ $\tau_g m_X$-SC(X)}.

The proof of (2) – (4) is obivious.

Theorem 9 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. Let A and B be the subsets of X. Then the following properties hold:

(1) If A ⊂ B, then $\tau_g m_X$-Cl$_s$(A) ⊂ $\tau_g m_X$-Cl$_s$(B).
(2) $\tau_g m_X$-Cl$_s$(A) ∪ $\tau_g m_X$-Cl$_s$(B) ⊂ $\tau_g m_X$-Cl$_s$(A ∪ B).
(3) $\tau_g m_X$-Cl$_s$(A ∩ B) ⊂ $\tau_g m_X$-Cl$_s$(A) ∩ $\tau_g m_X$-Cl$_s$(B).

Proof. The proof can be established similarly.

Theorem 10 Let (X, τ_g, m_X) be a generalized topology and minimal structure space. Let A be subset of X. Then the following properties hold:

(1) $\tau_g m_X$-Int$_s$(X \ A) = X \ $\tau_g m_X$-Cl$_s$(A).
(2) $\tau_g m_X$-Cl$_s$(X \ A) = X \ $\tau_g m_X$-Int$_s$(A).

Proof (1)Now X \ $\tau_g m_X$-Cl$_s$(A) = X \ ∩ {F : A ⊂ F and F ∈ $\tau_g m_X$-SC(X)}. Therefore, X \ $\tau_g m_X$-Cl$_s$(A) = ∪ (X \ F), where X – F ⊂ X – A and X – F is $\tau_g m_X$-semiopen of X. This implies X \ $\tau_g m_X$-Cl$_s$(A) = $\tau_g m_X$-Int$_s$(X \ A).

(2)Now, X \ $\tau_g m_X$-Int$_s$(A) = X \ ∪{U : U ⊂ A and A ∈ $\tau_g m_X$-SO(X)}. Therefore, X \ $\tau_g m_X$-Int$_s$(A) = ∩ (X – U), where X – A ⊂ X \ U is $\tau_g m_X$-semiclosed of X. This implies $\tau_g m_X$-Cl$_s$(X \ A) = X \ $\tau_g m_X$-Int$_s$(A).

III. SEMILOCALLY CLOSED SETS

Definition 10 A subset of A of a GTMS-space (X, τ_g, m_X) is said to be $\tau_g m_X$-locally closed if A = B ∩ C where B is τ_g-semiopen and C is m_X-semiclosed. It is denoted by $\tau_g m_X$-SL$_C$(X).

Theorem 11 If A, B ∈ $\tau_g m_X$-SL$_C$(X), then A ∩ B ∈ $\tau_g m_X$-SL$_C$(X).

Proof Let A, B ∈ $\tau_g m_X$-SL$_C$(X). Then A = U ∩ V and B = W ∩ X where U, W ∈ τ_g-semiopen and V, X ∈ m_X-semiclosed. Now, A ∩ B = (U ∩ V) ∩ (W ∩ X) = (U ∩ W) ∩ (V ∩ X). Therefore, A ∩ B ∈ $\tau_g m_X$-SL$_C$(X), Since U ∩ V ∈ τ_g-semiopen and V ∩ X ∈ m_X-semiclosed.

Remark 3 The converse of the above theorem is need not be true as shown in the following example.

Example 3 Let X = {a, b, c}. We define generalised topology and minimal structure space as follows: τ_g = { φ, {a}, {a, c}}; m_X = { φ, {b}, {b, c}, X}. Then $\tau_g m_X$-SL$_C$ = { φ, {a}, {c}, {a, c}, X}. Here, {a, b} ∩ {a, c} = {a} ∈ $\tau_g m_X$-SL$_C$, but {a, b} ∉ $\tau_g m_X$-SL$_C$.

Theorem 12 Let A be a subset of a GTMS space (X, τ_g, m_X). If A ∈ $\tau_g m_X$-L$_C$(X), then A ∈ $\tau_g m_X$-SL$_C$(X).

Proof Let A ∈ $\tau_g m_X$-L$_C$(X). Then A = B ∩ C, where B ∈ τ_g-open and C ∈ m_X-closed. Since every τ_g-open is τ_g-semiopen and m_X-closed is m_X-semiclosed, we have B is τ_g-semiopen and C is m_X-semiclosed. Hence A ∈ $\tau_g m_X$-SL$_C$(X).

Theorem 13 Let A be a subset of a GTMS space (X, τ_g, m_X). Then the following statements are equivalent:

(1) A ∈ $\tau_g m_X$-SL$_C$(X).
(2) A = B ∩ m_X-Cl$_s$(A).
(3) m_X-Cl$_s$(A) \ A is τ_g-semiclosed.
(4) A ∪ (X \ m_X-Cl$_s$(A)) is τ_g-semiopen.
(5) A ⊂ τ_g-Int$_s$(A ∪ (X \ m_X-Cl$_s$(A))

Proof (1) ⇒ (2) Let A ∈ $\tau_g m_X$-SL$_C$. Then A = B ∩ C, where B is τ_g- semiopen and C is m_X-semiclosed. Since A ⊂ C and m_X-Cl$_s$(A) is the smallest m_X-semiclosed set containing A, so m_X-Cl$_s$(A) ⊂ C. Now,A = B ∩ C ⊃ B ∩ m_X-Cl$_s$(A). Since A ⊂ B and A ⊂ m_X-Cl$_s$(A), therefore, A ⊂ B ∩ m_X-Cl$_s$(A). Hence, A = B ∩ m_X-Cl$_s$(A).

(2) ⇒ (3) Let A = B ∩ m_X-Cl$_s$(A). Now, m_X-Cl$_s$(A)\ A = m_X-Cl$_s$(A) \ (B ∩ m_X-Cl$_s$(A)) = (m_X-Cl$_s$(A) \ B) ∪ (m_X-Cl$_s$(A) \ m_X-Cl$_s$(A)) = m_X-Cl$_s$(A) \ B = m_X-Cl$_s$(A) ∩ (X \ B), which is τ_g-semiclosed, since X \ B is τ_g-semiclosed.

(3) ⇒ (4) A ∪ (X \ m_X-Cl$_s$(A)) = X \ (m_X-Cl$_s$(A) \A). By(3), m_X-Cl$_s$(A) \ A is τ_g-semiclosed, therefore X \ m_X-Cl$_s$(A) \ A) is τ_g-semiopen. Hence, A ∪ (X \ m_X-Cl$_s$(A) is τ_g-semiopen.

(4) ⇒ (5) Let A ∪ (X \ m_X-Cl$_s$(A)) be τ_g-semiopen. Then A ∪ (X \ m_X-Cl$_s$(A)) = τ_g-Int$_s$(A ∪ (X \ m_X-Cl$_s$(A))). Hence A ⊂ τ_g-Int$_s$(A ∪ (X \ m_X-Cl$_s$(A))).
(5) ⇒ (1) By (5), we have A ⊂ τ_g-Int$_s$(A ∪ (X \ m_X-Cl$_s$(A))). Since, A ⊂ m_X-Cl$_s$(A), so A ⊂ τ_g-Int$_s$(A ∪ (X \ m_X-Cl$_s$(A))) ∩ m_X-Cl$_s$(A). Now, { τ_g-Int$_s$(A ∪ (X \ m_X-Cl$_s$(A))) ∩ m_X-Cl$_s$(A)} ⊂ {A ∪ (X \ m_X-Cl$_s$(A))) ∩ m_X-Cl$_s$(A)} = {m_X-Cl$_s$(A) ∩ A} ∪ {m_X-Cl$_s$(A) ∩ (X \ m_X-Cl$_s$(A))} = A. Consequently, A = τ_g-Int$_s$(A ∪ (X \ τ_g-Cl$_s$(A))) ∩ m_X-Cl$_s$(A). Since, τ_g-Int$_s$(A ∪ (X \ m_X-Cl$_s$(A))) is τ_g-semiopen and m_X-Cl$_s$(A) is m_X-semiclosed. Hence A ∈ $\tau_g m_X$-SL$_C$(X).
Theorem 14 If A ⊂ B ⊂ (X, τ_g, m_X) and B ∈ $\tau_g m_X$-SL$_C$(X), then there exists C ∈ $\tau_g m_X$-SL$_C$(X) such that A ⊂ C ⊂ B.
Proof Since B ∈ $\tau_g m_X$-SL$_C$(X), we have B = U ∩ m_X-Cl$_s$ (T) where U is τ_g-semiopen. Since A ⊂ B and B ⊂ U, we have A ⊂ U. Also, A ⊂ m_X-Cl$_s$(A). Therefore, A ⊂ U ∩ m_X-Cl$_s$(A). Now, C = U ∩ m_X-Cl$_s$ (A) ⊂ U ∩ m_X-Cl$_s$(B) = B. Since, U is τ_g-preopen and m_X-Cl$_s$(A) is m_X-semiclosed, so C ∈ $\tau_g m_X$-SL$_C$(X) such that A ⊂ C ⊂ B.
Theorem 15 Let A be a subset of (X, τ_g, m_X). Then m_X-Cl$_s$(A) \ A is τ_g-semiclosed if and only if A ∪ (X \ m_X-Cl$_s$(A)) is τ_g^c- semiopen.
Proof Let m_X-Cl$_s$(A) \ A be τ_g-semiclosed. Then X \ m_X-Cl$_s$(A) \ A is τ_g-semiopen. Now, X \ m_X-Cl$_s$(A) \ A) = A ∪ (X \ m_X-Cl$_s$(A)). Hence, A ∪ (X \ m_X-Cl$_s$(A)) is τ_g-semiopen. Conversely, let A ∪ (X \ m_X-Cl$_s$(A)) be τ_g-semiopen. Then X \ (A ∪ (X \ m_X-Cl$_s$(A))) is τ_g-semiclosed. Now, X \ (A ∪ (X \ m_X-Cl$_s$(A))) = m_X-Cl$_s$(A) \ A. Hence m_X-Cl$_s$(A) \ A is τ_g-semiclosed.
Theorem 16 Let A, B be two subsets of (X, τ_g, m_X). If A ∈ $\tau_g m_X$-SL$_C$(X) and B is τ_g-semiopen or m_X-preclosed then A ∩ B ∈ $\tau_g m_X$-SL$_C$(X).
Proof Let A ∈ $\tau_g m_X$-SL$_C$(X). Then A = U ∩ V, where U is τ_g-semiopen and V is m_X-semiclosed. Suppose B is τ_g-semiopen. Then U ∩ B is also τ_g-semiopen. Now, A ∩ B = (U ∩ V) ∩ B = (U ∩ B) ∩ V, since V is m_X-semiclosed. Hence, A ∩ B ∈ $\tau_g m_X$-SL$_C$(X). Again if B is m_X-preclosed, then B ∩ V is m_X-preclosed. Now, A ∩ B = (U ∩ V) ∩ B = U ∩ (B ∩ V). Hence, A ∩ B ∈ $\tau_g m_X$-SL$_C$(X).

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