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A STUDY ON NEW MATHEMATICAL MODEL FOR OPTIMAL CONTROL STRATEGIES OF INTEGRATED PEST MANAGEMENT

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A state- dependent impulsive SI epidemic model for integrated pest management (IPM) is proposed and investigated. We shall examine impulsive control problem in the management of an epidemic to control a pest population. We introduce a small amount of pathogen into a pest population with the expectation that it will generate epidemic and that it will subsequently be endemic such that the number of pests is no larger than the given economic threshold (ET), so that the pests cannot cause economic damage. This is the biological control strategy given in the present paper. The combination strategy of pulse capturing and pulse releasing is implemented in the model. **Firstly**, the impulsive control problem is to drive the pest population below a given pest level and to do so in a manner which minimizes a weighted sum of the cost of using the control. **Secondly**, we show the existence of periodic solution with the number of pests no larger than ET, and by using the Analogue of the Poincaré Criterion we prove that it is asymptotically stable under a planned impulsive control strategy. The main feature of the present paper is to apply an SI infectious disease model to IPM, and some pests control strategies given.

Keywords: *Biological and Mechanical Control, Economic Threshold, Impulsive Effect, SI Epidemic Model.*

1. Introduction

Pest outbreaks often cause serious ecological and economic problems, therefore effectively controlling insects and other arthropods has become an increasingly complex issue over the past three decades. Minimizing losses due to insect pests and insect vectors of important plant, animal, and human diseases remains an essential component of the programs. A wide range of pest control strategies is available to farmers. Integrated pest management (IPM) is a suitable approach to managing pests by combining biological, cultural, physical, mechanical and chemical tools in a way that minimize economic, health and environmental risks. IPM which was introduced in the late 1950s but more widely practiced during the 1970s and 1980s[12,13] promotes the use of several measures such as insecticide applications, crop rotation, biological control, harvest management, and the use of pest-resistant varieties of crops to reduce pest populations below economic levels.

The use of pathogens such as bacteria, fungi, and viruses is potentially one of the most important methods in pest control. For example, *Bacillus thuringiensis*,which is available in commercial preparations, is used in the control of a large number of pests. [3,4] An advantage of using insect pathogens is that they are safe to man and are usually safe to beneficial insects. In the present mathematical model, biological control is the purposeful introduction and establishment of the infective individuals with pathogen which are bred in laboratories or which have been captured from the field, and are infected by a pathogen, specifically for the purpose of suppressing pests such that they no longer cause economic damage. It is important that we can manipulate the number of the released infectives which is needed to generate an epidemic so that the pest population at the end of an epidemic will be below a desirable level. Another important method for pest control is mechanical or physical, which means that the pests can be selectively harvested, or trapped, or killed by man. This method is typically used in the control of pests. Wherever possible, different pest control techniques should work together rather than against each other. In some cases, this can lead to synergy-where the combined effect of different techniques is greater than would be expected from simply adding the individual effects together.

Impulsive systems are found in almost every domain of applied sciences and have been studiedin many investigations.[6] However, more work has to be done on models with fixed-moment pulse action; that is, assuming pulse effect occurs every T time period regardless of the number of pests present. The main purpose of this paper is to construct a simple SI mathematical model with state-dependent impulsive effect according to the fact of biological and mechanical control, find the optimal IPMstrategies and investigate the dynamics of the impulsive system.

2. Model Formulation and Preliminary Information

The basic model we considered is based on the following SI epidemic model

$$\begin{cases} \frac{ds}{dt} = \text{bs - βSI - dS}, & (1) \\ \frac{dI}{dt} = \text{βSI - dI-rl}, & \end{cases}$$

where S(t) and I (t) denote the members of the population susceptible to the dis-ease, and of infective members, respectively. Positive constants b, d, β and r denote the birth rate, physical death rate, contact rate and death rate for diseases, respectively. A similarly detailed description of the model and its dynamics may be found in Refs. 22 and 23. Since infective pests (1) cause less damage, we control the susceptible pests. [1,4] We rewrite system (1) as

$$\begin{cases} \frac{ds}{dt} = \alpha s - \beta SI, & (2) \\ \frac{dI}{dt} = \beta SI - \theta I, \end{cases}$$

where $\alpha = b - d$ and $\theta = d + r$ are positive constants.

we can see that the system (2) has :

(i) two steady states: O(0,0) saddle point, and $(S^*,I^*) = (\theta/\beta, \alpha/\beta)$ stable center;

(ii) a unique closed trajectory through any point in the first quadrant contained inside the point (S^*, I^*);

(iii) a first integral

$$f(S,I) = \beta S + \beta I - \theta InS - \alpha InI + f_0$$

Where $f_0 = \alpha\ (\ln \frac{\alpha}{\beta} - 1) + \theta\ (\ln \frac{\alpha}{\beta} - 1)$

Further, we have

(iv) $f(S^*, I^*) = 0$;

(v) $f(S, I) > 0$ for any point $(S, I) \in R^2+$, $(S, I) \neq (S^*, I^*)$;

(vi) for every positive constant r, the set $D_r = (S, I) \in R_+^2 : f(S, I) < r$) is a simply connect domain with smooth boundary $\partial D_r = \{(S, I)\} \in R_2^+ : f(S, I) = r \}$;

(vii) If $0 < r_1 < r_2$, then $\tau r1 \subset D_{r2}$.

In the following, the first integral f (S, I) plays a key role in the analysis of the state-dependent impulsive system discussed. Based on this, we find the existence of a periodic solution for the following state-dependent impulsive system geometrically, and which can help us to investigate the optimal control strategy in terms of cost and to analyze the stability of the periodic solution and to calculate its period.

Definition 2.1 The Lambert W function is defined to be multivaluated inverse of the function x - xe^x Satisfying.

Lambert W (x) exp (Lambert w (x)) = x.

Hence if x > -1, then the function x exp (x) has the positive derivative (x+1) exp(x). Lambert W (0,x) is the inverse function of x exp(x) on the interval [-1, ∞), and Lambert W (-1, x) is the inverse function of xexp(x) on the intrerval (-∞, -1]. For the nature of this study. both W (0,x) and w (-1, x) will be employed only for $x \in [-\exp(-1), 0)$ because both functions are real values for x in this interval. For more details of the Lambert W function,

Lemma 2.1 (Analogue of Poincare Criterion).[2,7] We consider the state dependent impulsive equation

$$\begin{cases} \dot{x} = P(x,y), \\ \dot{y} = Q(x,y), \end{cases} \quad \emptyset(x,y) \neq 0,$$
$$\begin{cases} \Delta x = A(x,y), \\ \Delta y = A(x,y), \end{cases} \quad \emptyset(x,y) = 0, \qquad (3)$$

If the multiplier μ satisfies the condition $|\mu| < 1$, then the T-periodic solution (ε(t), η(t)) of system (3) is orbitally asymptotically stable and enjoys the property of asymptotic phase, where

$$\mu = \prod_{i=1}^{q} \Delta_i \exp\left[\int_0^T \left(\frac{\partial P}{\partial x}(\varepsilon(t), \eta(t) + \frac{\partial Q}{\partial y}(\varepsilon(t), \eta(t))\right) dt\right]$$

$$\Delta_i = \frac{P + \left(\frac{\partial B}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial B}{\partial x}\frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial x}\right) + Q\left(\frac{\partial A}{\partial x}\frac{\partial \phi}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y}\right)}{P(\varepsilon(T_i)\,\eta(T_i))\frac{\partial \phi}{\partial x} + Q(\varepsilon(T_i)\,\eta(T_i))\frac{\partial \phi}{\partial y}},$$

and P,Q $\frac{\partial A}{\partial y}, \frac{\partial A}{\partial x}, \frac{\partial B}{\partial x}, \frac{\partial B}{\partial y}, \frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ are calculates at the point $(\varepsilon(T_i)\,\eta(T_i))$ and

$P_+ = P\left(\varepsilon(T_i^+)\eta(T_i^+)\right), Q_+ = Q\left(\varepsilon(T_i^+)\eta(T_i^+)\right)$

Now we develop system (2) as a more extensive form by introducing a proportion impulsive catching for the susceptible pest population and a constant releasing for the infective populations with pathogen as the moment when the number of the susceptible pest population reaches the economic injury level. That is we consider the following state-dependent impulsive differential equation.

$$\begin{cases} \frac{dS}{dt} = \alpha S - \beta SI, \\ \frac{dI}{dt} = \beta SI - \theta I, \end{cases} \quad S < S_E = EL, \qquad (4)$$
$$\begin{cases} \Delta S = -\delta S(t), \\ \Delta I = m \\ S(0^+) = S_0, I(0^+) = I_0, \end{cases} \quad S \geq S_E = EL,$$

Where S_E=EL denotes the economic threshold and δ, m ($0 \leq \delta < 1$, $m \geq 0$) denotes the capturing rates and the number of the released infective inditvidutals, respectively.

We note that system (4) is an impulsive semi - dynamical system. where M = {(S, I) ∈ R_+^2 | S = S_E,0 ≤ I ≤ $\frac{\alpha}{\beta}$} is a closed subset of R_+^2 and continous function φ : (S_E,I ∈ M $\longrightarrow$ (S^+, I^+)= (1- δ) S_E, I + m) ∈ R_+^2. It follows that N = φ (M) {(S,I) ∈ R_+^2 | S = (1- δ) S_E, m ≤ I ≤ $\frac{\alpha}{\beta}$ + m}. Without loss of generality we assume the initial point (S_0^+, I_0^+) ∈ N.

For convenience, we consider two cases, $S_0 \leq S_E$, and $S_0 > S_E$ respectively. In Secs. 3 and 4, we will mostly investigate the first case ($S_0 \leq S_E$). Then we will consider the second case ($S_0 > S_E$) in the last paragraph of Sec. 4.

3. Optimal Impulsive Control Strategies.

In the following, we always assume that the initial value of the susceptible S_0 is not larger is not larger than S_E. Now we investigate how to control pest (the susceptibles) such that the maximum number of the susceptible is not larger than EI by one impulse of releasing and capturing.

For system (4). we will consider two cases $S_E > \frac{\alpha}{\beta}$ and $S_E \leq \frac{\alpha}{\beta}$, respectively. Case I (see Fig. 1) : If $S_E > \frac{\alpha}{\beta}$, then there exists a closed trajectory Γ_i tangent to the line S = S_E, and we have the function for Γ_i f (S, I) = Γ_i where

$$\tilde{r} = \theta \left[\frac{\beta S_E}{\theta} - In\ \frac{\beta S_E}{\theta} - 1\right] > 0. \quad (5)$$

Let

D_r = {(S,I) ϵ R_+^2 : $f\ (S, I) \leq \tilde{r}$ }

Γ_i = $\partial\ D_{\tilde{r}}$ = {(S, I) ϵ R_+^2 : f (S,I) = $\tilde{r}$ }

∩ = {(S,I) ϵ R_+^2 : 0 < S ≤ S_E, 0 < I < ∞ }

and Ω1 = Ω/Dr.

If follows from (4) that any solution with initial state (S_0, I_0) ϵ D$\tilde{r}$ will be free from impulse effects, which implies that D$\tilde{r}$ is a secure region. Therefore, we only need to consider the trajectories from region ∩1 for system (4), and our purpose

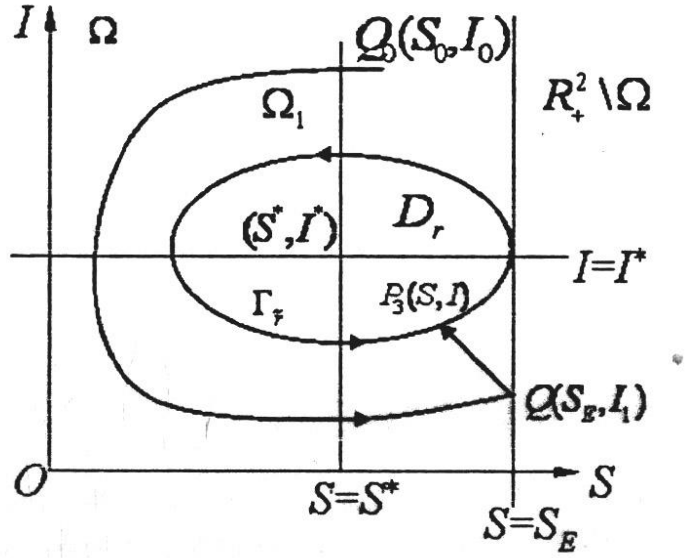

Fig. 1. Diagram for a one pulse strategy : the trajectory of any solution with initial state Q_0 (S_0,I_0) ϵ $\cap_1$ intersects with the line S=S_E at the point Q (S_E, I_1) jumps to point P_3 (S, 1) ϵ Γ, after a one time impulsive effect.

is to let the point is $\cap_1$ jump into the secure region D$\tilde{r}$ by one time pulse. Any trajectory with initial value (So,lo) ϵ $\cap_1$ will intersect with line S=S_E at the point Q (S_E,I_1), where

$$l_1 = -\frac{\alpha}{\beta}\text{ Lambert W}\left(-\frac{\beta I_0}{\alpha}\exp\left(-\frac{\beta I_0+\theta lin\,(S_E S_o)-\beta(S_{E-}S_0)}{\alpha}\right)\right) \qquad (6)$$

which depends on the initial conditions and S_E.

On the one hand, we must take control actions such that the number of susceptible individuals is never larger than S_E. On the other hand, we need to consider the cost affectivity of pest control strategies. We assume that the cost capturing is P_1 per susceptible pest and the cost of releasing is P_2 per infected pest.

Now we consider integrated control tactics for (S_0, I_0) ϵ $\cap$ without loss of generality, we assume that point Q (S_E, I_1) jumps to point P_3 (S,I) ϵ $\Gamma_{\tilde{r}}$ after a one time impulsive effect. Hence, the point P_3 satisfies.

$$\begin{cases} \beta S + \beta I - \theta \ln S - \alpha \ln I = h, \\ S - S_E = -\delta S_E \\ I - I_1 = m, \end{cases} \qquad (7)$$

where $\bar{h} = \beta S_E + \alpha - \theta \ln S_{E-} \alpha \ln\frac{\alpha}{\beta}$

In the following, we need to find appropriate values (δ*, m*) which satisfy (7) and also make the total cost function C= p_1 δ*S_E+p_2m* reach its minimum. Now we give the optimal pulse measure in terms of cost as follows.

Theorem 3.1. For any (S_0,I_0) ∈ $\cap_1$ and $\lambda* > \max\left\{\frac{p_1 s_E}{\beta s_E-\theta}, \frac{p_2 I_1}{\alpha-\beta i_1}\right\}$, there erists a pair.

$(\delta^* \text{ m}^*) = \left(1 + \frac{\lambda * \theta}{S_E\,(p_{1-\lambda*\beta})},\ \frac{\lambda * \alpha}{p_{2+\lambda*\beta)}} - I_1\right)$

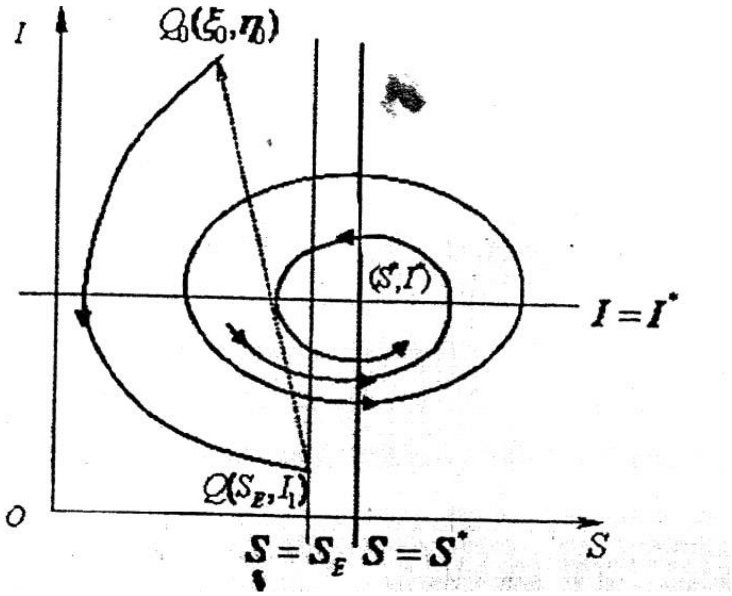

Fig. 2 Diagram for periodic pulse strategy : the trajectory of a solution with initial state $Q_0(\varepsilon_0\eta_0)$ ϵ ∩ intersects with the line S=SE at the point Q (S_E, I_1), then point Q (S_E, I_1), jumps to point Q_0 ($\varepsilon_0\eta_0$.

Which minimizes the cost of integrated control tactics, where λ^* is the unique solution of equation

$$g(\lambda) = \beta \frac{\lambda\theta}{p_1 - \beta\lambda} - \beta \frac{\lambda\theta}{p_2 + \beta\lambda} + \alpha \ln + \frac{\lambda\alpha}{p_2 + \beta\lambda} + \theta \ln\left(-\frac{\lambda\theta}{p_1 - \beta\lambda}\right) + \bar{h} = 0$$

Case I (see Fig. 1) : If $S_E > \theta/\beta$, then the above arguments show that we can manipulate the number of capturing pests and releasing infected pests such that the size of the susceptible is no larger than ET and the total cost reaches minimum.

Case II (see Fig. 2) : If $S_E \leq \theta/\beta$, then any solution with initial condition (S_0, I_0) ϵ ∩ will intersect with the line S=S_E and infinite number of times, which means that at any one time pulse control cannot keep the susceptible population size smaller than S_E. Therefore, the best way of controlling the pest in such a case is to implement a periodic pulse control.

4. Existance, Stability and Period of Periodic Solution

For the existence of periodic solution of system (4), we have the following main theorem.

Theorem 4.1. If $\frac{1}{\delta}$ (m + θ/β ln 1/1-δ)>S_E, then system (4) has a unique periodic solution with period T.

Remark 4.1 If δ=0 and $I_0 \geq \alpha/\beta$, then in the case there is a unique periodic solution with initinal value (ε_0 η_0)= (S_E, $\frac{m}{1-\exp(-\frac{\alpha m}{\alpha})}$)

Remark 4.2 If m = 0 and $S_E = \frac{\alpha}{\beta s}$ In $\frac{1}{1-\delta}$, then for any $0 < \eta_0 < \alpha/\beta$ there is a unique T-periodic solution with initial value (ε_0 η_0) = $\left(\frac{\theta\,(1-\delta)}{\beta_0} \ln \frac{1}{1-\delta},\ I_0\right)$.

According to the definition[25] of orbitally asymptotically stable system with the property of asymptotic phase, from Lemma 2.1, we have the following theorems.

Theorem 4.2. If α/β_0 In (1- $\frac{2m\beta}{m\beta+\alpha+\sqrt{(m\beta)^2+\alpha^2}}$) + $\frac{m}{\delta}$ + $\frac{\theta}{\beta\delta}$ + In $\frac{1}{1-\delta}$ > S_E, then the T-periodic solution (ε(t), η (t) with initial value ϵ (0), n (0) ϵ Ω_1 $\cap$ N is orbitally asymptotically stable and enjoys the property of asymptotic phase.

Theorem 4.3 If $\alpha/\beta < \eta_0 < \frac{m\beta+\alpha+\sqrt{(m\beta)^2+\alpha^2}}{2\beta}$, then the period T of periodic solution ((ε(t), η (0)) of system (4) satisfies equation.

$$T = \int_{S_{min}}^{S_E} \frac{dS}{S\,(\alpha-\beta I_0(S))} + \int_{S_{(1-\delta)}}^{S_{min}} \frac{dS}{S\,(\alpha-\beta I_1(S))} \qquad (8)$$

If $0 < \eta_0 \leq \alpha/\beta$, then the period T of periodic solution ((ε(t), η (t)) of system satisfies equation.

$$T = \int_{S_{(1-\delta)}}^{S_{min}} \frac{dS}{S\,(\alpha-\beta I_1(S))} \qquad (9)$$

where I = I_k (S) (k=0,1) are two roots of the following equation when S=S,

$$\beta\,(I - \eta_0) - \alpha \text{ In} \frac{I}{\eta 0} = \theta \text{ In} \frac{S}{\varepsilon 0} - \beta\,(S - \varepsilon_0)$$

and S_{min} is the small root of the following equation when I = α/β ,

$$\beta\,(S - (1-\delta)\,S_E) - \theta \ln \frac{S}{(1-\delta)S_E} = \alpha \ln \frac{\alpha}{\beta\eta 0} - \beta\,(\alpha/\beta - \eta_0)$$

If $S_0 \leq S_E$, i.e. (S_0,I_0) ∈ ∩, we seen that we can control pests such that the maximum number of pests is no larger than ET from the above arguments. Now we assume that the initial value of the susceptible S_0 is larger than S_E (S_0>S_E), i.e. (S_0,I_0) ∈ R^2_+/∩. Let δ = 1 - S_E/S_0, then we find that point P0 (S_0, I_0) jumps to point P (S_E, I_0 + m) ∈ ∩, then we can iteratively do the same process as the first case ($S_0 \leq S_E$), i.e. we must rapidly suppress pets because the initial number of susceptible S_0 is too large (S_0>S_E).

Conclusion :

In this paper, we propose and analyze a new state-dependent impulsive dynamical SI model concerning IPM strategy. Both biological and mechanical controls are used together as the pest population reaches the ET. Recently, chemical control is extensively used to suppress and eradicate pests. [9.14] Over use and misuse of chemicals have created many ecological and sociological problems, and the major beneficial effect of SI biological control is the it appears to generate an epidemic and to be subsequently endemic to keep the susceptible pests under ET. Thus biological control may be a better way of ensuring that pest population do not fluctuate widely from one years to the nest.

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