IJCRT.ORG ISSN : 2320-2882
INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

AN INTRODUCTION TO MICRO BINARY TOPOLOGICAL SPACES

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Abstract: The purpose of this paper, is to introduce and study the micro binary topological spaces. The notions of Micro Binary open sets, Micro Binary closed sets, Micro Binary interior, and Micro Binary closure are introduced and their basic properties are discussed with suitable examples.

Index terms: Micro Binary Topology, Micro Binary open sets, Micro Binary closed sets, Micro Binary interior, Micro Binary closure, and Micro Binary Continuity.

1. INTRODUCTION

M. Lellis Thivagar [1] introduced the concept of Nano topological space concerning a subset X of a universe U. Nano topology is based on the concept of lower approximation, upper approximation, and boundary region.

Simple extension concept in Nano topology we can extend some more open sets that topology is called Micro Topology. Later in 2019 S. Chandrasekar [2] introduced the concept of Micro Topology by applying the simple extension concept in Nano Topology.

S. Nithyanantha Jothi and P. Thangavelu [3] introduced the concept of binary topological spaces. In this paper, we introduce Micro Binary topology, Micro Binary open sets, and Micro Binary closed sets, and some of their properties are investigated.

2. PRELIMINARIES

Definition 2.1[4]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U.

1. The lower approximation of X for R is the set of all objects, which can be for certain classified as X for R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup \{R(x) : R(x) \subseteq X\}\ x{\in}U$ where R(x) denotes the equivalence class determined by x ∈ U.
2. The upper approximation of X for R is the set of all objects, which can be possibly classified as X for R and it is denoted by $U_R(X)$.

   That is, $U_R(X) = \bigcup \{R(x) : R(x) \cap X = \phi\}$.

3. The boundary region of X for R is the set of all objects, which can be classified neither as X nor as $B_R(X)$ denotes not-X concerning R and it. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 [4]

Let (U, R) be the approximation space and let $X, Y \subseteq U$. Then the following conditions hold:

1. $L_R(X) \subseteq X \subseteq UR(X)$
2. $L_R(\phi) = U_R(\phi) = \phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9. $U_R(X^C) = [L_R(X)]^C$
10. $L_R(X^C) = [U_R(X)]^C$
11. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
12. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3 [2]

Let U be a universe, R be an equivalence relation on U, and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$

where X ⊆ U satisfies the following axioms:

1. U, $\phi \in \tau_R(X)$
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
3. The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$. Then $\tau_R(X)$ is called the Nano topology on U for X. The space $(U, \tau_R(X))$ is the Nano topological space. The elements of $\tau_R(X)$ are called Nano open sets.

Definition 2.4 [1]

$(U, \tau_R(X))$ is a Nano topological space here $\mu_R(X) = \{L \cup (L' \cap \mu)\}$: L, L′ ∈ $\tau_R(X)$ and is called Micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau R(X)$. The Micro topology $\mu_R(X)$ satisfies the following axioms:

1. U, ∅ ∈ $\mu_R(X)$.
2. The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.
3. The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$. Then $\mu_R(X)$ is called the Micro Topology on U for X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological spaces and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set are called a Micro closed set.

Definition 2.5 [1]

- Mic-int(A) = ∪ {G / G is a Mic-OS in X and G⊆A}.
- Mic-cl(A) = ∩ {K / K is a Mic-CS in X and A⊆K}.

Definition 2.6 [3]

Let X and Y be any two nonempty sets. A binary topology is a binary structure

$M \subseteq P(X) \times P(Y)$ from X to Y which satisfies the following axioms:

1. $(\emptyset, \emptyset) \in M$; $(X, Y) \in M$.
2. $(A_1 \cap A_2, B_1 \cap B_2) \in M$ where $A_1, A_2, B_1, B_2 \in M$
3. If $(A\alpha, B\alpha: \alpha \in A)$ is a family of members of M, then $(u_\alpha \in AA_\alpha, \cup\alpha \in AB_\alpha) \in M$. If M is a binary topology from X to Y, then the triplet (X, Y, M) is called binary topological space and the members of M are called the binary open sets of the binary topological space (X, Y, M).

3. MICRO BINARY TOPOLOGICAL SPACES

In this paper, I introduced and studied the concept of Micro Binary Topological Spaces

$((U_1, U_2), (\emptyset, \emptyset), \tau_R(X), \mu_R(X))$

Definition 3.1

Throughout this paper, $(U_1, U_2, \tau_R(X_1, X_2))$ represent Nano binary topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset (H_1, H_2) of a space $(U_1, U_2, \tau_R(X_1, X_2))$, $NB\circ(H_1, H_2)$ and $NB(H_1, H_2)$ denote the Nano binary interior of (H_1,H_2) and the Nano binary closure of (H_1,H_2) respectively.

Definition 3.2

Let (U_1, U_2) be the universe, R be an equivalence on (U_1, U_2) and

$\tau_R(X_1, X_2) = \{(U_1,U_2), (\phi,\phi), L_R(X_1, X_2), U_R(X_1, X_2), B_R(X_1, X_2)\}$ where $(X_1, X_2) \subseteq (U1,U2)$.

Then by the property R (X_1, X_2) satisfies the following axioms:

1. (U_1, U_2) and $(\phi, \phi) \epsilon (X_1, X_2)$
2. The union of the elements of any sub-collection of $\tau_R(X_1, X_2)$ is in $\tau_R(X_1, X_2)$
3. The intersection of the elements of any finite sub-collection of $\tau_R(X_1, X_2)$ is in $\tau R(X_1, X_2)$.

That is, $\tau_R(X_1, X_2)$ is a topology on (U_1, U_2) called the Nano binary topology (Shortly Topology τ_R (X_1,X_2)) on

(U_1, U_2) with respect to (X_1, X_2). We call $(U_1, U_2, \tau_R(X_1, X_2))$ as the Nano binary topological spaces. The elements of $\tau_R(X_1, X_2)$ are called Nano binary open sets.

Definition 3.3

$((U_1, U_2), (\emptyset, \emptyset), \tau_R(X_1, X_2))$ is a Nano Binary topological space here

$\mu_R(X_1, X_2) = \{L \cup (L' \cap \mu)\}: L, L' \in \tau_R(X_1, X_2)$ and is called Micro Binary topology

(Shortly Topology $\mu_R(X_1, X_2)$) of $\tau_R(X_1, X_2)$ by μ where $\mu \notin \tau_R(X_1, X_2)$. The Micro binary topology $\mu_R(X_1, X_2)$ satisfies the following axioms:

1. $(U_1, U_2), (\emptyset, \emptyset) \in \mu_R(X_1, X_2)$.
2. The union of the elements of any sub-collection of $\mu_R(X_1, X_2)$ is in $\mu_R(X_1, X_2)$.
3. The intersection of the elements of any finite sub-collection of $\mu_R(X_1, X_2)$ is in $\mu_R(X_1, X_2)$. Then $\mu_R(X_1, X_2)$ is called the Micro Binary topology on (U_1, U_2) with respect to (X_1, X_2).

Then $((U_1, U_2), (\emptyset, \emptyset), \tau_R(X_1, X_2), \mu_R(X_1, X_2))$ are called Micro Binary topological spaces, and the elements of

$\mu_R(X_1, X_2)$ are called Micro Binary open sets and the complement of a Micro Binary open set is called a Micro Binary closed set.

EXAMPLE

Let $U_1 = \{a, b, c\}$ and $U_2 = \{1,2\}$ with $(U_1, U_2)\ R/ = \{(\{a, b\},\{1\}), (\{c\},\{2\})\}$ and $(X_1, X_2) = \{(\{a,c\},\{2\})\}$.

Then the Nano binary topology $\tau_R(X1, X2) = \{(\emptyset, \emptyset),(U_1, U_2), (\{c\},\{2\})\}$. Then $\mu = (\{a\},\{1\})$.

Then Micro Binary open set = $\mu_R(X_1, X_2) = \{(\emptyset, \emptyset), (U_1, U_2), (\{c\},\{2\}), (\{a\},\{1\}), (\{a, c\},\{1, 2\})\}$ is called Micro

Binary topology on (U_1, U_2) with respect to (X_1, X_2) and the elements in the Micro Binary Topology are called the

Micro Binary Open sets. Then $((U_1, U_2), (\emptyset, \emptyset), \tau_R(X_1, X_2), \mu_R(X_1, X_2))$ is called the Micro Binary Topological Space.

PROPOSITION 3.4

Suppose $(A, B) \subseteq (C, D) \subseteq (U_1, U_2)$ and $((U_1, U_2), (\emptyset, \emptyset), \tau_R(X1, X2), \mu_R(X_1, X_2))$ is a Micro Binary space. Then

(i) MB-cl$(\emptyset, \emptyset) = (\emptyset, \emptyset)$ and MB-cl$(U_1,U_2) = (U_1, U_2)$.

(ii) $(A, B) \subseteq$ MB-cl(A, B).

(iii) $(A, B)1^* \subseteq (C, D)1^*$.

(iv) $(A, B)2^* \subseteq (C, D)2^*$.

(v) MB-cl(A , B) $\subseteq$ MB-cl(C , D).

(vi) MB-cl (MB-cl(A , B)) = MB-cl(A , B).

Proof.

The properties (i) and (ii) follow easily.

Now $(A, B)1^* = \cap \{A\alpha: (A\alpha, B\alpha)$ is binary closed

and. $(A, B) \subseteq (A\alpha, B\alpha)\} \subseteq \cap \{A\alpha: (A\alpha, B\alpha)$ is binary closed and

$(C, D) \subseteq (A\alpha, B\alpha)\}$

$= (C, D)1^*$. This proves (iii).

The proof for (iv) is analog. Now, MB-cl(A , B) = $((A, B)1^*, (A, B)2^*) \subseteq ((C, D)1^*, (C, D)2^*)$ = MB-cl(C, D) that establishes (v).

4. CONTINUOUS FUNCTIONS IN MICRO BINARY TOPOLOGICAL SPACES

Continuity between Micro topological spaces plays a dominant role in analysis. In this section, the concept of Micro Binary Continuity between a Micro topological space and a Micro Binary topological space is introduced and its basic properties are studied.

Definition 4.1

Let $f: Z \to X \times Y$ be a function. Let $A \subseteq X$ and $B \subseteq Y$. We define

$f^{-1}(A, B) = \{z \in Z: f(z) = (x, y) \in (A, B)\}$.

Definition 4.2

Let $((U_1, U_2), (\emptyset, \emptyset), \tau_R(X1, X2), \mu_R(X_1, X_2))$ be a Micro Binary topological space and let $(Z, \mu_R(X))$ be a Micro topological space. Let f: Z X×Y be a function. Then f is called Micro Binary continuous if $f^{-1}(A, B)$ is open in Z for every Micro Binary open set (A, B) in X × Y.

LEMMA 4.3

Let f: Z → X× Y be a function. For A ⊆ X and B ⊆ Y, we have

$Z\backslash f^{-1}(A, B) = f^{-1}(A, Y\backslash B) \cup f^{-1}(X\backslash A, B) \cup f^{-1}(X\backslash A, Y\backslash B)$

PROOF.

Let (x, y) = f(z). $z \in f^{-1}(X\backslash A, Y\backslash B)$

f(z) ∈ (X\A, Y\B) (x, y) ∈ (X\A, Y\B) x ∈ X\A and y ∈ Y\B x ∉ A and y ∉ B.

(x, y) ∉ (A, B) f(z) ∉ (A, B) z ∉ f-1(A, B)

⇒ z ∈ Z\ f -1(A, B).

Thus, f-1 (X\A, Y\B) ⊆ Z\ f -1(A, B).

z ∈ f-1 (A, Y\B) ⇒ f(z) ∈ (A, Y\B)

⇒ (x, y) ∈ (A, Y\B) where (x, y) = f(z).

⇒ x ∈ A and y ∈ Y\B ⇒ x ∈ A and y ∉ B. (x, y) ∉ (A, B) ⇒f(z) ∉ (A, B)

⇒ z ∉ f -1(A, B) ⇒ z ∈ Z\ f -1(A, B).

Thus, f-1 (A, Y\B) ⊆ Z\ f -1(A, B).

Similarly, we can prove that $f^{-1}(X\backslash A, B) \subseteq Z\backslash f^{-1}(A, B)$.

The above arguments show that,

$f^{-1}(A, Y\backslash B) \cup f^{-1}(X\backslash A, B) \cup f^{-1}(X\backslash A, Y\backslash B) \subseteq Z\backslash f^{-1}(A, B)$. Now $z \in Z\backslash f^{-1}(A, B)$. ⇒ $z \in Z$ and $z \notin f^{-1}(A, B)$

⇒z ∈ Z and f(z) ∉ (A, B)
⇒ z ∈ Z and (x, y) ∉ (A, B) where f(z) = (x, y)
⇒ z ∈ Z and (x, y) ∈ (A, Y\B) or (x, y) ∈ (X\A, B) or (x, y) ∈ (X\A, Y\B)
⇒ z ∈ Z and f(z) ∈ (A, Y\B) or f(z) ∈ (X\A, B) or f(z) ∈ (X\A, Y\B)
⇒ $z \in Z$ and $z \in f^{-1}(A, Y\backslash B)$ or $z \in f^{-1}(X\backslash A, B)$ or $z \in f^{-1}(X\backslash A, Y\backslash B)$
⇒ $z \in f^{-1}(A, Y\backslash B) \cup f^{-1}(X\backslash A, B) \cup f^{-1}(X\backslash A, Y\backslash B)$.
Thus $Z\backslash f^{-1}(A, B) \subseteq f^{-1}(A, Y\backslash B) \cup f^{-1}(X\backslash A, B) \cup f^{-1}(X\backslash A, Y\backslash B)$.
Therefore $Z\backslash f^{-1}(A, B) = f^{-1}(A, Y\backslash B) \cup f^{-1}(X\backslash A, B) \cup f^{-1}(X\backslash A, Y\backslash B)$.

CONCLUSION

In this paper, we have introduced the Micro Binary Topological space, and the properties of Micro Binary open sets, Micro Binary closed sets, Micro Binary interior, and Micro Binary closure and Micro Binary Continuity are discussed. This shall be extended in future research with some applications.

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