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A Study On Distance - Similarity Measures Of Intuitionistic Fuzzy Sets And Its Applications

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Abstract: Intuitionistic fuzzy sets (IFS) provide a mathematical framework for dealing with the uncertainty and ambiguity associated with decision-making. The study meticulously scrutinizes existing similarity measures, offering a detailed overview before introducing and discussing several innovative measures. According to our numerical simulation results, the measures that have been proposed are well suited for the management of logistics and supply chains. The findings attest to their efficacy in fostering swift and accurate decision-making, thereby mitigating the risk of economic losses and instability. Through numerical comparisons, it is possible to determine the effectiveness of the proposed distance and similarity measures over the existing measures in the intuitive fuzzy environment as compared to the suggested measures. This comparative insight serves as a compass for decision-makers seeking optimal solutions amid uncertain conditions. Finally, we demonstrate the application of the suggested measures in patterns recognition, medical diagnosis, and the formulation of multicriteria decision making based on the results obtained.

Keywords: Intuitionistic fuzzy sets, Distance & Similarity measures, Pattern recognition and Multi Criteria Decision Making

1. Introduction

The incorporation of vagueness within the framework of fuzzy sets facilitates a more authentic and nuanced portrayal of the uncertainty that is inherently embedded in a myriad of real-world scenarios.

Zadeh (1965) developed the concept of fuzzy set (FS), which addressed the membership degree (MD) and Atanassov (1986) generalized the concept of FS by including non-membership degree (NMD). Intuitionistic fuzzy is a significant generalization of fuzzy sets useful for dealing with uncertainty and imprecision. For an Atanassov's intuitionistic fuzzy set (AIFS), its membership degree and non-membership degree are both real numbers in [0, 1], and their summation is less than 1. The difference between 1 and their summation leads to another parameter of AIFS, namely, the hesitancy degree.

De et al. (2001) explored Sanchez's approach for medical diagnosis, extending this concept by incorporating the principles of intuitionistic fuzzy set theory, which serves as a generalization of fuzzy set theory. Chan et al. (2007) discovered that supplier selection is a multiple criteria decision-making (MCDM) problem affected by qualitative and quantitative criteria. Wang and Xin (2005) introduced several distance measures between IFSs and put those measures to pattern recognitions.

Park et al. (2009) introduced an innovative approach for computing the distance between intuitionistic fuzzy sets (IFSs) by leveraging a three-dimensional representation of IFSs and also applied it in pattern recognition. Song et al. (2014) did investigation of the new measure's classification capability which was carried out based on two numerical examples and medical diagnosis.

Boran and Akay (2014) presented a biparametric similarity measure, utilizing this technique for pattern recognition. Baccour et al (2015) put forward two semi-metric distance measures for Intuitionistic Fuzzy Sets (IFSs). Davvaz and Sadrabadi (2016) modified established distance techniques and employed them in the diagnostic process. Liu and Chen (2017) suggested that group decision-making can be performed using Heronian aggregation operators within Intuitionistic Fuzzy Systems (IFSs). Iqbal and Rizwan (2019) explored the applicability of Intuitionistic Fuzzy Systems (IFSs) in the realms of medicine and pattern recognition, employing a novel similarity technique. Various IFS similarity techniques, as investigated and applied by Hong and Kim (1999), Li et al. (2007), Shi and Ye (2013), and Ye (2011), have been considered in tasks such as diagnostic analysis and pattern recognition. Xuan Wu (2021) showcases the utility of the proposed distance-based knowledge measure by employing it to create an innovative solution for addressing MAGDM problems with intuitionistic fuzzy information. The effectiveness and rationality of the method are elucidated through application examples and comparative analysis. Using pattern recognition of building materials, marital choice-making, and election results, Ejegwa (2021) presented how decision-making could be processed. In their work, Zeng and Cui (2022) introduce an innovative distance measure for intuitionistic fuzzy sets, demonstrating its adherence to the axiomatic definition of a distance measure.

Specifically, this paper

i. revisits certain existing similarity-distance techniques between IFSs,

ii. introduce an improved similarity-distance technique between IFSs,

iii. show comparison analysis of the new similarity-distance technique in intuitionistic fuzzy domain, and

iv. implement the new similarity-distance technique to determine some decision-making situations.

2. Preliminaries:

Definition 2.1 (Zadeh,1965). Let a non-empty set Y= $\{y_1, y_2, \ldots y_n\}$be the universe of discourse. Then a fuzzy set E in Y is defined as follows:
$E = \{ < y, \alpha_E(y) > \}| : y € Y \}$ where $\alpha_E(y) : Y \rightarrow [0,1]$ is the membership degree.

Definition 2.2. (Atanassov,1986). An IFS signified
An intuitionistic fuzzy set denoted as $E = \{ < y, \alpha_E(y), \beta_E(y) > : y € Y \}$, where $\alpha_E, \beta_E : Y \rightarrow [0,1]$ are the membership and non-membership degrees for all y € Y and $0 \leq \alpha_E(y) + \beta_E(y) \leq 1$, $\gamma_E(y) € [0,1] = 1- \alpha_E(y) -\beta_E(y)$ is called the hesitation degree of E.

In the context of Intuitionistic Fuzzy Sets (IFS), both distance and similarity measures play crucial roles in assessing the relationships between sets.

Definition 2.3: For E and F as IFSs in Y, the similarity measure of E and F signified by S(E,F) is a mapping S : IFS x IFS $\rightarrow$ [0,1] satisfying

i) $0 \leq S(E,F) \leq 1$
ii) S(E,F) = 1 iff E=F
iii) S(E,F) = S(F,E)
iv) $S(E,G) \leq S(E,F) + S(F,G)$, where G is an IFS in Y

When S(E,F) approaches 1, it implies E and F are more close (i.e., high similarity rate), and if S(E,F) approaches 0, then E and F are not close, i.e., the similarity/resemblance rate is low.

Definition 2.4: For E and F as IFSs in Y, the similarity measure of E and F signified by D(E,F) is a mapping D : IFS x IFS $\rightarrow$ [0,1] satisfying

i) $0 \leq D(E,F) \leq 1$

ii) D(E,F) = 0 iff E=F

iii) D(E,F) = D(F,E)

iv) $D(E,G) \leq D(E,F) + D(F,G)$, where G is an IFS in Y

When D(E,F) approaches 0, it implies E and F are more close and if D(E,F) approaches 1, then E and F are not close.

From the ongoing, we see that

S(E,F) = 1- D(E,F) and D(E,F) = 1- S(E,F)

Let Y= $\{y_1, y_2, \ldots, y_n\}$, $n < \infty$, then for the IFSs E and F in Y, we present the following similarity-distance measures:

Hong and Kim (1999)

$S(E,F)= 1- 1/2n\sum_{i=1}^{n} [\,|\alpha_E(y)- \alpha_F(y)| +|\beta_E(y)- \beta_F(y)|]$

$D(E,F)= 1/2n\sum_{i=1}^{n} [\,|\alpha_E(y)- \alpha_F(y)| +|\beta_E(y)- \beta_F(y)|]$

The similarity-distance measure is not reliable because the hesitation parameter is not captured in the technique.

Shi and Ye (2013)

$S(E,F)= 1/n\sum_{i=1}^{n} \alpha_E(y)\alpha_F(y) + \beta_E(y)- \beta_F(y)| + \gamma_E(y)\ \gamma_F(y)\ \ |/\surd\ \alpha^2{}_E\ )+ \beta^2{}_E(y) + \gamma^2{}_E\ \surd\ \alpha^2{}_E + \gamma^2{}_F + \beta^2{}_F$

$D(E,F)= 1 - 1/n\sum_{i=1}^{n} \alpha_E(y)\alpha_F(y) + \beta_E(y)\beta_F(y) + \gamma_E(y)\gamma_F(y)\ \ /\surd\ \alpha^2{}_E\ )+ \beta^2{}_E(y) + \gamma^2{}_E\ \surd\ \alpha^2{}_E + \gamma^2{}_F + \beta^2{}_F$

This similarity-distance measure incorporates the three parameters of IFS and as such avoids information leakage and loss.

Li et al (2007)

S(E,F)= 1-(1/2n$\sum_{i=1}^{n}$ [(α_E(y)- α_F(y))^2 +(β_E(y)- β_F(y))^2])$^{1/2}$

D(E,F)= (1/2n$\sum_{i=1}^{n}$ [(α_E(y)- α_F(y))^2 +(β_E(y)- β_F(y))^2])$^{1/2}$

Similarly, this similarity-distance measure is not reliable because the hesitation parameter is not captured in the technique.

Szmidt and Kacprzyk (2000)

S(E,F)= 1- 1/2$\sum_{i=1}^{n}$ [|α_E(y)- α_F(y)| +|β_E(y)- β_F(y)| + | γ_E(y)- γ_F(y) |]

D(E,F)= 1/2$\sum_{i=1}^{n}$ [|α_E(y)- α_F(y)| +|β_E(y)- β_F(y)| + | γ_E(y)- γ_F(y) |]

S(E,F)= 1- 1/2n$\sum_{i=1}^{n}$ [|α_E(y)- α_F(y)| +|β_E(y)- β_F(y)| + | γ_E(y)- γ_F(y) |]

D(E,F)= 1/2n $\sum_{i=1}^{n}$ [|α_E(y)- α_F(y)| +|β_E(y)- β_F(y)| + | γ_E(y)- γ_F(y) |]

S(E,F)= 1-(1/2n$\sum_{i=1}^{n}$ [(α_E(y)- α_F(y))^2 +(β_E(y)- β_F(y))^2 + (γ_E(y)- γ_F(y))^2])$^{1/2}$

D(E,F)= (1/2n$\sum_{i=1}^{n}$ [(α_E(y)- α_F(y))^2 +(β_E(y)- β_F(y))^2 + (γ_E(y)- γ_F(y))^2])$^{1/2}$

This similarity-distance measure incorporates the three parameters of IFS and as such avoid information leakage and loss.

3. New measure of Intuitionistic Fuzzy sets

A new similarity-distance technique for IFSs that incorporates the three characteristic features of IFS to avoid information loss is introduced. Suppose we have IFSs E and F in

Y={ y_1,y_2 …y_n }; n < ∞, the new similarity-distance measure is

S(E,F) = 1 - 1/n $\sum_{i=1}^{n}$[cos[π/2 - π/6 { | α_E(y) – α_F(y) | + | β_E(y) – β_F(y) | + | γ_E(y) – γ_F(y) | }]

D(E,F) = 1/n$\sum_{i=1}^{n}$ [cos[π/2 - π/6 { | α_E(y) – α_F(y) | + | β_E(y) – β_F(y) | + | γ_E(y) – γ_F(y) | }]

i) $0 \le S(E,F) \le 1$

Since, $0 \le | \alpha_E - \alpha_F | \le 1$

$0 \le | \beta_E - \beta_F | \le 1$

$0 \le | \gamma_E - \gamma_F | \le 1$

Therefore we obtain,

$0\le$ { | α_E(y) – α_F(y) | + | β_E(y) – β_F(y) | + | γ_E(y) – γ_F(y) | }]≤ 1

ii) S(E,F) = 1 iff E=F

If S(E,F) =1

Then,

$\Rightarrow 1 - 1/n\sum_{i=1}^{n}$ [cos[π/2 - π/6 { $|\alpha_E(y)-\alpha_F(y)| + |\beta_E(y)-\beta_F(y)| + |\gamma_E(y)-\gamma_F(y)|$ }]]

=1

$\Rightarrow\sum_{i=1}^{n}$ [cos[π/2 - π/6 { $|\alpha_E(y)-\alpha_F(y)| + |\beta_E(y)-\beta_F(y)| + |\gamma_E(y)-\gamma_F(y)|$ }]]=0

$\Rightarrow$ cos[π/2 - π/6 { $|\alpha_E(y)-\alpha_F(y)| + |\beta_E(y)-\beta_F(y)| + |\gamma_E(y)-\gamma_F(y)|$ }]]=0

$\Rightarrow\{ |\alpha_E(y)-\alpha_F(y)| + |\beta_E(y)-\beta_F(y)| + |\gamma_E(y)-\gamma_F(y)| \}=0$

$\Rightarrow\{ |\alpha_E(y)-\alpha_F(y)| =0, |\beta_E(y)-\beta_F(y)| =0, |\gamma_E(y)-\gamma_F(y)| \}=0$

$\alpha_E(y) = \alpha_F(y)$, $\beta_E(y) = \beta_F(y)$, $\gamma_E(y) = \gamma_F(y)$

Hence, E=F

Conversely if E=F, then

$\alpha_E(y) = \alpha_F(y)$, $\beta_E(y) = \beta_F(y)$, $\gamma_E(y) = \gamma_F(y)$

$\Rightarrow |\alpha_E(y)-\alpha_F(y)| =0$

$\Rightarrow |\beta_E(y)-\beta_F(y)| =0$

$\Rightarrow |\gamma_E(y)-\gamma_F(y)| = 0$

S(E,F) $= 1 - 1/n\sum_{i=1}^{n}$ [cos[π/2 - π/6 { $|\alpha_E(y)-\alpha_F(y)| + |\beta_E(y)-\beta_F(y)| + |\gamma_E(y)-\gamma_F(y)|$ }]]

$= 1 - 1/n\sum_{i=1}^{n}$ [cos[π/2 - π/6(0)]

=1

Therefore, S(E,F) = 1 iff E=F

iii) If E and F are IFSs in Y, then

S(E,F) $= 1 - 1/n\sum_{i=1}^{n}$ [cos[π/2 - π/6 { $|\alpha_E(y)-\alpha_F(y)| + |\beta_E(y)-\beta_F(y)| + |\gamma_E(y)-\gamma_F(y)|$ }]]

$= 1-1/n\sum_{i=1}^{n}$ [cos[π/2 - π/6 { $|\alpha_F(y)-\alpha_E(y)| + |\beta_F(y)-\beta_E(y)| + |\gamma_F(y)-\gamma_E(y)|$ }]]

= S(F,E)

Similarly , for distance measures, the new measure has all the properties.

4. Numerical Comparison

Suppose E and F are IFSs in Y= {a,b,c} defined by

E= {(a, 0.6,0.3), (b,0.1,0.5), (c,0.2,0.5)}

F= {(a,0.7,0.2) (b,0.1,0.5), (c,0.2,0.5)}

Table 1

Results of Similarity-Distance measure

	Similarity	**Distance**
Hong and Kim (1999)	0.956667	0.043333
Shi and Ye (2013)	0.3282	0.6718
Li et al (2007)	0.746667	0.253333
Szmidt and Kacprzyk (2000)	0.9	0.1
	0.929289	0.070711
	0.962667	0.037333
NEW TECHNIQUE	**0.965157**	**0.034843**

The results show that the new technique gives the most reliable index for the relationship between the IFSs. In a nutshell, the new similarity technique has edge over the existing approaches studied in this work because it considers the complete parameters of IFS to avoid error due to omission distinct from the approaches in Hong and Kim (1999), Iqbal and Rizwan (2019), Li et al. (2007) and Ye (2011).

Similarly, the observation is the same for the distance measures, which are the similarity measures counterpart.

5. Model Of Intuitionistic Fuzzy Sets (IFSS) In Medical Diagnosis

Let us consider $\{D_1, D_2, ..., D_m\}$ be a set of number of possible diseases and $\{P_1, P_2, ..., P_n\}$ be a set of n number of patients. Here, symptoms for different diseases as well as symptoms of the patients are expressed in linguistic expressions most of the time. For this reason, uncertainty and fuzziness exist for the expression of the symptoms of the patients and disease. So, to express the symptoms, IFSs has been used. Therefore, the distance between the symptoms the disease D_i and symptoms of patient P_j can be evaluated as follows using the proposed distance measure:

$$D(E,F)= 1/n\sum_{i=1}^{n} [\cos[\pi/2 - \pi/6 \{ | \alpha_E(y) - \alpha_F(y) | + | \beta_E(y) - \beta_F(y) | + | \gamma_E(y) - \gamma_F(y) | \}]$$

In this study, the proposed distance measure has been applied in a decision-making problem of medical diagnosis.

Let P = {Ahan, Lara, Ash, Rahul}, S = {Temperature, Headache, Stomach problem, Cough, Chest pain} and D = {Viral fever, Malaria, Typhoid Stomach problem, Chest pain}.

Table 2

Symptoms-diseases intuitionistic fuzzy relation.

Symptoms	Viral fever (Vf)	Malaria (Ma)	Typhoid (Ty)	Stomach pain (Sp)	Chest pain (Cp)
Temperature (Tm)	(0.4, 0.0)	(0.7, 0.0)	(0. 3,0. 3)	(0.1, 0.7)	(0.1, 0.8)
Headache (He)	(0.3, 0.5)	(0.2, 0.6)	(0.6, 0.1)	(0.2, 0.4)	(0.8, 0.8)
Stomach problem (Sp)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Cough (Co)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Chestpain (Cp)	(0.1, 0.7)	(0.1, 0.8)	(0.1, 0.9)	(0.2, 0.7)	(0.8, 0.1)

Table 3.

Patients- symptoms intuitionistic fuzzy relation

	Temperature	Headache	Stomach problem	Cough	Chest pain
Ahan	(0.8, 0.1)	(0.6, 0.1)	(0.2, 0.8)	(0.6, 0.1)	(0.1, 0.6)
Lara	(0.0, 0.8)	(0.4, 0.4)	(0.6, 0.1)	(0.1, 0.7)	(0.1, 0.8)
Ash	(0.8, 0.1)	(0.8, 0.1)	(0.0, 0.6)	(0.2, 0.7)	(0.0, 0.5)
Rahul	(0.6, 0.1)	(0.5, 0.4)	(0.3, 0.4)	(0.7, 0.2)	(0.3, 0.4)

The representation of the diseases as intuitionistic fuzzy sets for symptoms is as follows:

Vf= {Tm(0.4, 0.0), He(0.3, 0.5), Sp(0.1, 0.7),Co(0.4,0.3),Cp(0.1, 0.7)}

Ma= {Tm (0.7, 0.0) , He (0.2, 0.6), Sp (0.0, 0.9), Co(0.7, 0.0), Cp (0.1, 0.8)}

Ty= {Tm(0.3,0.3) , He (0.6, 0.1), Sp (0.2, 0.7), Co(0.2, 0.6), Cp (0.1, 0.9)}

Sp = {Tm((0.1, 0.7), He(0.2, 0.4),Sp(0.8, 0.0), Co(0.2, 0.7), Cp(0.2, 0.7)}

Cp ={Tm (0.1, 0.8). He (0.8, 0.8), Sp (0.2, 0.8), Co (0.2, 0.8), Cp (0.8, 0.1)

Similarly, the representation of the patients as intuitionistic fuzzy sets is also done.

In short, with the help of the following distance matrix, it can be shown to what extent the symptoms of disease differ by the symptoms of patients.

Table 4.

The intuitionistic distance measure between the patients and the diseases in terms of their symptoms

	Ahan	Lara	Ash	Rahul
Viral fever	0.285418	0.389345	0.384733	0.288796
Malaria	0.245653	0.472666	0.436116	0.305639
Typhoid	0.282709	0.321172	0.324291	0.38541
Stomach pain	0.522942	0.145882	0.481359	0.441164
Chest pain	0.535304	0.411488	0.533337	0.476977

As due to the fact, less distance between patient and disease tells more possibility of having the disease, we can predict which disease is suffered by the four people. From the distance matrix, one can observe that, if the doctor agrees Ahan suffers from Malaria, Lara suffers from stomach problem, Ash suffers from Typhoid and Rahul suffers from viral fever.

Table 5.1

Comparison of existing results with the proposed distance measure.

	Ngana et al. (2018)	De et al (2001)	Szmidt and Kacprzyk (2004)	Maheshwari and Shrivastava (2016)	Dutta and Goala (2018)	Proposed distance measure
Ahan	Malaria	Malaria	Viral fever	Viral fever	Malaria	Malaria
Lara	Stomach problem	Stomach problem	Stomach problem	Stomach problem	Stomach problem	Stomach problem
Ash	Typhoid	Malaria	Typhoid	Typhoid	Typhoid	Typhoid
Rahul	Viral fever	Malaria	Malaria	Viral fever	Viral fever	Viral fever

From Table 5, comparison has been made and it is observed that the results obtained by using our proposed distance measure are similar with results obtained by (Ngan *et al*. (2018) and Dutta and Goala (2018)

Table 5.2

Comparison of existing results with the proposed distance measure.

	Hong and Kim (1999)	Shi and ye (2013)	Li et al (2007)	Szmidt and Kacprzyk (2000)	Proposed distance measure
Ahan	Malaria	Stomach Problem	Viral fever	Malaria	Malaria
Lara	Stomach problem	Stomach problem	Malaria	Stomach problem	Stomach problem
Ash	Typhoid	Malaria	Typhoid	Typhoid	Typhoid
Rahul	Viral fever	Malaria	Viral fever	Viral fever	Viral fever

In this table, comparison has been made and it is observed that the that the results obtained by using our proposed distance measure are similar with results obtained by Hong and Kim (1999) and Szmidt and Kacprzyk (2000).

6. Model Of Intuitionistic Fuzzy Sets (IFSS) In Pattern Recognition

By incorporating hesitation in addition to membership and non-membership degrees, intuitionistic fuzzy sets provide a more expressive framework for representing and dealing with uncertainty in pattern recognition tasks. In this method, a collection of patterns is provided, along with an unidentified pattern referred to as a sample (which is also intuitionistic). Both the pattern set and the sample exist within the similar attributes. The objective is to determine the distance between each pattern and the sample. The least distance between any pattern and the sample indicates that the sample belongs to that particular pattern. This encapsulates the essence of pattern recognition.

In this case study, we consider six patterns be represented by IFSs in

$d(A_1, B) = 0.292702$, $d(A_2, B) = 0.269775$, $d(A_3, B) = 0.143172$, $d(A_4, B) = 0.21911$, $d(A_5, B) = 0.194473$ and $d(A_6, B) = 0.269386$

From these results, we see that, the distance between A_3 and B is the smallest, and the distance between A_1 and B is the greatest.

Table 6

Comparative Analysis

	P (A_1, B)	P (A_2, B)	P (A_3, B)	P (A_4, B)	P (A_5, B)	P (A_6, B)
Hong and Kim (1999)	0.2375	0.1875	**0.125**	0.15	0.226	0.234
Shi and ye (2013)	0.567	0.0456	**0.0456**	0.5478	0.289	0.3455
Li et al. (2007)	0.5866	0.278	**0.2242**	0.067	0.4566	0.0576
Iqbal and Rizwan (2019)	0.069	0.05768	**0.032009**	0.04123	0.04567	0.036419
Ejegwa et al (2014)	0.075	0.068	**0.0375**	0.0563	0.05	0.0688
Ejegwa and Agbetayo (2022)	0.085508	0.080435	**0.042888**	0.051558	0.062553	0.051278
NEW PROPOSED MEASURE	0.292702	0.269775	**0.143172**	0.21911	0.194473	0.269386

From the above results shown in the table, our proposed measure A_3 approaches B. Also, results are similar to the existing measures that reveals the uniqueness of the novel measures.

7. Model of Multi-Criteria Decision Making In Supply Chain Management

The core principle of multi-criteria decision-making involves a thorough assessment of each plan within a set of options, considering various attributes as constraints. Ultimately, the goal is to rank or choose the most satisfactory plan from this group. Currently, this approach finds extensive applications in practical scenarios.

Suppose there are m evaluation schemes $S = \{A_1, A_2, \cdots, A_m\}$, and n criteria $C = \{C_1, C_2, \cdots, C_n\}$. The evaluation value of scheme A_i with the attribute C_j is an intuitionistic fuzzy number $s_{ij} = \mu_{ij}, v_{ij}$, where μ_{ij} and v_{ij} represent the membership and non-membership degrees, respectively, and $0 \le \mu_{ij} \le 1$, $0 \le v_{ij} \le 1$, $0 \le \mu_{ij} + v_{ij} \le 1$, $i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, n$.

Steps for the proposed method:

Step 1: Formulate the intuitionistic fuzzy decision matrix.

$D= \{ C_j(A_i)\}_{mn}$

Step 2: Normalize the Decision Matrix

Normalize the decision matrix X by dividing each element by the square root of the sum of the squares of all values in the corresponding column.

$$(\mu_{ij}, v_{ij})_{mn} = \{(\mu_{ij}, v_{ij}) \text{, for benefit criterion } C_j$$
$$(v_{ij}, \mu_{ij}) \text{ for cost criterion } C_j \}$$

Step 3: Identify Ideal and Negative-Ideal Solutions

Determine the positive ideal solution A+ and negative-ideal solution A− for each criterion.

the positive ideal solution $S^+ = s_1^+, s_2^+, s_3^+, s_4^+$ and the negative ideal solution $S^- = s_1^-, s_2^-, , s_3^-, s_4^-$.

$A^+ = \{\max\langle C_j(S_i)\rangle$; j=1,2,…n

If C_J is a benefit criteria

$\min\langle C_j(S_i)\rangle$; j=1,2,…n

If C_J is a benefit criteria }

$A^- = \{\min\langle C_j(S_i)\rangle$; j=1,2,…n

If C_J is a benefit criteria

$\max\langle C_j(S_i)\rangle$; j=1,2,…n

If C_J is a benefit criteria }

Step 4: Calculate the Euclidean Distances

Calculate the Euclidean distance (Di^+ and Di^-) between each alternative and the positive-ideal and negative-ideal solutions using the new measure:

Step 5: Calculate the Closeness index to the Ideal Solution

Calculate the relative closeness (J_i) of each alternative to the ideal solution.

$$J_i = \left| \frac{J_i^-}{J_i^+ + J_i^-} \right|$$

Step 6: Rank the Alternatives

Rank the alternatives based on their relative closeness values in decreasing order. The higher the J_i, the more preferred the alternative.

Supply chain organizations must prudently make precise and sustainable decisions promptly, implementing them with composure. It is evident that conventional time series-dependent approaches to demand and supply planning fall short in addressing contemporary business requirements, given the influence of rapid market fluctuations, evolving commercial dynamics, and the impact of events like pandemics and natural disasters on supply chain management. Looking ahead, there will be a demand for resilient supply chains characterized by adaptable business models, capable of navigating unforeseen shifts and dynamically making sustainable decisions

Example: An automobile manufacturer desires to develop a proactive resiliency strategy for selecting suppliers as its commitment to the global market. Five potential suppliers, named as $S_1, S_2, ..., S_5$, are identified for the analysis. For assessing the suppliers, five decision-makers, i.e., $DM_1, DM_2, . . . , DM_5$, from different departments are invited. The following criteria have been considered in the supplier evaluation and selection: Quality (C_1), Reliability (C_2), Functionality (C_3), and Cost (C_4).

Step 1: Construct normalised decision matrix, D

Table 7.1

Construction of decision matrix

	C_1	C_2	C_3	C_4
S_1	(0.8,0.1)	(0.5,0.3)	(0.6,0.2)	(0.8,0.1)
S_2	(0.5,0.2)	(0.6,0.3)	(0.7,0.1)	(0.6,0.2)
S_3	(0.6,0.1)	(0.5,0.4)	(0.4,0.5)	(0.6,0.1)
S_4	(0.4,0.2)	(0.7,0.1)	(0.4,0.3)	(0.7,0.2)
S_5	(0.7,0.1)	(0.5,0.2)	(0.7,0.2)	(0.5,0.4)

According to the decision matrix and step 3, the ideal positive solution $S^+ = s_1^+, s_2^+, s_3^+, s_4^+$ and the ideal negative solution $S^- = s_1^-, s_2^-, , s_3^-, s_4^-$ can be calculated as follows:

$s_1^+ =< 0.3536, 0.5356 >, s_1^- =<0.1293, 0.6464>$

$s_2^+ =< 0.2683, 0.5502 >, s_2^- =< 0.1646, 0.7884$

$s_3^+ =< 0.2609, 0.5608 >, s_3^- = < 0.1204, 0.8402 >$

$s_4^+ =< 0.2962, 0.6050>, s_4^- =< 0.1404, 0.8187$

Step 3: With the help of above step and the new measure, we calculate the Euclidean distance using the new measure:

Table 7.2

The Euclidean distance using the new measure

	$D_i^+ (A_i ,S^+)$	$Di^- (A_i ,S^-)$
S_1	1.2344	2.1234
S_2	1.5678	1.1987
S_3	1.7892	1.2367
S_4	1.7869	1.7897
S_5	1.4667	1.4567

Step 4: Calculate the relative closeness (J_i) of each alternative to the ideal solution.

$$J_i = \left| \frac{J_i^-}{J_i^+ + J_i^-} \right|$$

Table 7.3

Closeness index

	Closeness Index	Ranking
S_1	0.632378	1
S_2	0.433291	4
S_3	0.408705	5
S_4	0.510391	2
S_5	0.49829	3

Step 5: Ranking the Alternatives, we get

$$S_1 > S_4 > S_5 > S_2 > S_3$$

Therefore, S_1 is the best supplier as the maximum among Euclidean distance is selected as the best option.

Conclusion

In this work, we discussed important definitions, several basic operators and properties of Intuitionistic fuzzy sets along with the application based on real world situations. Some properties of the new technique were mathematically presented, and the advantages of the new technique have been discussed. In the theory of IFSs, how to measure the distance accurately and efficiently is still an open issue, which may lead to disruptions in decision-making process. In recent times, several distance measures have been introduced to measure the level of distance between the IFSs. We implemented the new measure for the application of medical diagnosis, pattern recognition and multi-criteria decision making in supply chain management taking various parameters into consideration.

References

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

Boran, F. E., & Akay, D. (2014). A biparametric similarity measure on intuitionistic fuzzy sets with applications to pattern recognition. Information Sciences, 255(10), 45–57. https://doi.org/10.1016/j.ins.2013.08.013

Chan, F.T.S., Kumar, N. (2007). Global supplier development considering risk factors using fuzzy extended AHP-based approach. Omega, volume (35), pp 417–431.August 2007Omega 35(4):417-431

https://www.researchgate.net/publication/222257958_Global_Supplier_Development_Considering_Risk_Factors_Using_Fuzzy_Extended_AHP-Based_Approach

Davvaz, B., & Sadrabadi, E. H. (2016). An application of intuitionistic fuzzy sets in medicine. International Journal of Biomathematics, 9(3), 1650037. https://doi.org/10.1142/S1793524516500376

De, S. K., Biswas, R., & Roy, A. R. (2001). An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy Sets and Systems, 117(2), 209–213.

https://doi.org/10.1016/S0165-0114(98)00235-8

Ejegwa, P. A., & Onyeke, I. C. (2021). Intuitionistic fuzzy statistical correlation algorithm with applications to multi-criteria based decision-making processes. International Journal of Intelligent

Systems, 36(3), 1386–1407. https://doi.org/10.1002/int.22347

E. Szmidt, and J. Kacprzyk, "A similarity measure for intuitionistic fuzzy sets and its application in supporting medical diagnostic reasoning", Arti_cial Intelligence and Soft Computing - ICAISC 2004, Lect. Notes Comput. Sci., vol. 3070, pp. 388-393, 2004.

http://dx.doi.org/10.1007/978-3-540-24844-6 56

Hong, D. H., & Kim, C. (1999). A note on similarity measures between vague sets and between elements. Information Sciences, 115(1), 83–96

https://doi.org/10.1016/S0020-0255(98)10083-X

Iqbal, M. N., & Rizwan, U. (2019). Some applications of intuitionistic fuzzy sets using new similarity measure. Journal of Ambient Intelligence and Humanized Computing,1–5. https://doi.org/10.1007/s12652-019-01516-7.

Jin Han Park, Ki Moon Lim and Young Chel Kwun. Distance measure between intuitionistic fuzzy sets and its application to pattern recognition.

https://www.researchgate.net/publication/263440646_Distance_measure_between_intuitionistic_fuzzy_sets_and_its_application_to_pattern_recognition

Li, Y., Olson, D. L., & Qin, Z. (2007). Similarity measures between intuitionistic fuzzy (vague) sets: A comparative analysis. Pattern Recognition Letters, 28(2), 278 285. https://doi.org/10.1016/j.patrec.2006.07.009

Liu, P., & Chen, S. M., (2017). Group decision making based onHeronian aggregation operators of intuitionistic fuzzy numbers. IEEE Transaction on Cybernetics, 47(9),2514–2530. https://doi.org/10.1109/TCYB.2016.2634599

Palash Dutta and Soumendra Goala (2018). Fuzzy Decision Making in Medical Diagnosis Using an Advanced Distance Measure on Intuitionistic Fuzzy Sets. https://dx.doi.org/10.2174/1874110X01812010136

PAUL AUGUSTINE EJEGWA. A NOTE ON SOME MODELS OF INTUITIONISTIC FUZZY SETS IN REAL LIFE SITUATIONS, May 2014

https://www.researchgate.net/publication/283120076_A_NOTE_ON_SOME_MODELS_OF_INTUITIONISTIC_FUZZY_SETS_IN_REAL_LIFE_SITUATIONS

Paul Augustine Ejegwa, and Johnson Mobolaji Agbetayo (2022).Similarity-Distance Decision-Making Technique and its Applications via Intuitionistic Fuzzy Pairs

http://dx.doi.org/10.47852/bonviewJCCE512522514

R.T. Ngana, L.H. Sonb, B.C. Cuongc, and M. Alid, "H-max distance measure of intuitionistic fuzzy sets in decision making", applied soft computing, 2018.

http://dx.doi.org/10.1016/j.asoc.2018.04.036

Shi, L. L., & Ye, J. (2013). Study on fault diagnosis of turbine using an improve cosine similarity measure for vague sets. Journal of Applied Sciences, 13(10), 1781–1786/

https://doi.org/10.3923/jas.2013.1781.1786

S.K. De, R. Biswas, and A.R. Roy, "An application of intuitionistic fuzzy sets in medical diagnosis", Fuzzy Sets Syst., vol. 117, pp. 209-213, 2013.

http://dx.doi.org/10.1016/S0165-0114(98)00235-8

S. Maheshwari, and A. Srivastava, "Study on divergence measures for intuitionistic fuzzy sets and its application in medical diagnosis", J. Appl. Anal. Comput., vol. 6, pp. 772-789, 2016. http://dx.doi.org/10.11948/2016050

Szmidt, E., & Kacprzyk, J. (2000). Distances between intuitionistic fuzzy sets. Fuzzy Sets and Systems, 114, 505–518

http://dx.doi.org/10.1016/S0165-0114(98)00244-9

Wang, W., & Xin, X. (2005). Distance measure between intuitionistic fuzzy sets. Pattern Recognition Letters, 26, 2063–2069. https://doi.org/10.1016/j.patrec.2005.03.018

Yafei Song, Xiaodan Wang, Lei Lei, and Aijun Xue. A New Similarity Measure between Intuitionistic Fuzzy Sets and Its Application to Pattern Recognition.

http://dx.doi.org/10.1155/2014/384241

Ye, J. (2011). Cosine similarity measures for intuitionistic fuzzy sets and their applications. Mathematical and Computer Modelling,

53, 91–97. https://doi.org/10.1016/j.mcm.2010.07.022

Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8,338–353. https://doi.org/10.1016/S0019-9958(65)90241-X