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GENERALIZED NEUTROSPHIC PYTHAGOREAN PRE - OPEN SETS IN GENERALIZEDNEUTROSOPHIC PYTHAGOREAN TOPOLOGICAL SPACES

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ABSTRACT: In this paper a Generalized Neutrosophic Pythagorean Generalized pre – open sets and a Generalized Neutrosophic Pythagorean Generalized pre - closed sets are introduced .Some of its properties are also studied. Also we have provided some applications of Generalized Neutrosophic Pythagorean in generalized pre - closed sets namely Generalized Neutrosophic Pythagorean p $T_{1/2}$ space and Generalized Neutrosophic Pythagorean gp $T_{1/2}$ space .

KEYWORDS: Generalized Neutrosophic Topology, Generalized Neutrosophic Pythagorean Generalized pre-closed sets ,Generalized Neutrosophic Pythagorean Generalized pre- open sets, Generalized Neutrosophic Pythagorean p $T_{1/2}$ space and Generalized Neutrosophic Pythagorean gp $T_{1/2}$ space .

I.INTRODUCTION:

The Concept of fuzzy set was introduced by Zadeh in 1965. After the introduction of Intuitionistic Fuzzy set by Atanassov in 1986,R.R.Yager Generalized Intuitionistic Fuzzy set and presented a new set called Pythagorean fuzzy set. In 1991,A.S. Binshahan introduced and investigated the notions of Fuzzy pre-open and Fuzzy pre- closed sets. In 2003,T. Fukutake,R.K. Saraf,M. Caldas and S.Mishra introduced Generalized pre-closed Fuzzy sets in Fuzzy Topological Space. P.Rajarajeswari and L.Senthil Kumar introduced Generalized pre-closed sets in Intuitionistic Fuzzy Topological Spaces . Wang introduced the idea of Single Valued Neutrosophic Sets(SVNS). In 2012,A.A. Salama and Albowi introduced the concept of Generalized Neutrosophic Sets and Generalized Neutrosophic Topological Spaces.The idea of Neutrosophic Pythagorean sets with Dependent Neutrosophic Pythagorean Sets was introduced by R.Radha and A.Stanis Arul Mary. R.Prema ,R.Radha introduce the concept of Generalized Neutrosophic Pythagorean Sets and its Properties.In this paper we have introduced Generalized Neutrosophic Pythagorean pre-open sets and studied some of their properties.

II. PRELIMINARIES:

Definition:2.1 A Generalized Neutrosophic Pythagorean set A of X is an object having the form A = {<a, T_A,I_A, F_A>/a∈ X} where the function $(T_A)^2+(F_A)^2 \leq 1$ and $(T_A)^2+(F_A)^2+(I_A)^2 \leq 2$ and $T_A \wedge F_A \wedge I_A \leq 0.5$. Here$T_A$ (a) is the truth membership, I_A (a) is the indeterminancy membership and F_A (a) is the false membership.

Definition:2.2 Let A and B be Generalized Neutrosophic Pythagorean set of the form A = {<a, T_A,I_A, F_A>/a∈ X}, B = {<a, T_B,I_B, F_B>/a∈ X}.Then

1. A ⊆ B if and only if $T_A \leq, T_B$ and $I_A \geq I_B$ and $F_A \geq F_B$ for all a∈ X.
2. A = B if and only if A ⊆ B and B ⊆ A.
3. $\overline{A}$ = {<a, $F_A(a), 1 - I_A(a), T_A(a)$>/a∈ X}
4. A ∪ B = {<a, $T_A(a) \vee T_B(a)$, $I_A(a) \wedge I_B(a)$, $F_A(a) \wedge F_B(a)$,>/a∈ X}
5. A ∩ B = {<a, $T_A(a) \wedge T_B(a)$, $I_A(a) \vee I_B(a)$, $F_A(a) \vee F_B(a)$,>/a∈ X}

For the sake of simplicity ,we shall use the notation A = {<a, T_A, I_A, F_A>/a∈ X} instead of A = {<a, $T_A(a), I_{A(a)}, F_A(a)$>/a∈ X} . The GNPs

0 ={<a,0,1,1>/a ∈ X} and 1 ={<a,1,0,0>/ a ∈ X} are respectively the empty set and the universe of X.

Definition 2.3 A Generalized neutrosophic Pythagorean topology(GNPT in short) by subset of a non empty X is a family of Generalized neutrosophic Pythagorean sets satisfying the following axioms .

1. 0,1 ∈ τ
2. $P_1 \cap P_2 \in \tau$, for every $P_1, P_2 \in \tau$
3. $\cup P_i \in \tau$ for any arbitrary family $\{P_i / i \in \tau\}$

In this case the pair (X,) is called a (GNPTS) and any GNPS P in τ is called GNPOS in X. The complement $\overline{A}$ of a Generalized Neutrosophic Pythagorean open set A in a GNPTS (X, τ) is called a Generalized Neutrosophic Pythagorean closed set (GNPCS in short).

Definition 2.4 Let (X, τ) be a GNPTS and A = {<a, T_A, I_A, F_A>/a∈ X} be a GNPS in X. Then the interior and the closure of A are denoted by GNP int (A) and GNP cl(A) and are defined a s follows

1. GNPint(A) = ∪{P|P is a GNPOS in X and P ⊆ A}
2. GNPcl(A) = ∩{K|K is a GNPCS in X and A ⊆ K}

Note that for any GNPS A in (X, τ), we have GNPcl(A^C)= (GNPint(A))^C)ant GNPint(A^C) =(GNPcl(A))^C)

Definition 2.5 Let (X, τ) be a GNP subset of X. Then the following holds

1. GNP cl(∅) = ∅ and GNP cl (X) = X
2. A is a GNP cs if and only if A = GNP cl(A)
3. GNP cl(GNP cl(A)) = GNP cl(A)
4. A ⊆ B implies that GNP cl(A) ⊆ GNP cl(B)
5. GNP cl(A ∩ B) ⊆ GNP cl(A) ∩ GNP cl(B)
6. GNP cl(A ∪ B) =GNP cl(A) ∪ GNP cl(B)

Definition 2.6 Let (X, τ) be a GNPTS and let A and B be Generalized Neutrosophic Pythagorean sets Then the following holds

1. GNP int(∅) = ∅ and GNP int (X) = X
2. A is a GNPOS if and only if A = GNP int (A)
3. GNP int GNP int(A)) = GNP int(A)
4. A ⊆ B implies that GNP int(A) ⊆ GNP int(B)
5. GNP int(A ∩ B) =GNP int(A) ∩ GNP int(B)
6. GNP int(A ∪ B) ⊇ GNP int(A) ∪ GNP int(B)

Definition 2.7 Let (X, τ) be a GNPTS and let A and B be two GNPS in (X, τ). If B is GNPOS,
then GNP cl(A) ∩ B ⊆GNP cl(A ∩ B). So if G is a GNPCS and H is any GNPS then GNP int (H∪G) ⊂GNP int (H) ∪ G

Definition 2.8 A GNPS A = {<a, T_A, I_A, F_A>} in GNPTS (X, τ) is said to be an

1. Generalized Neutrosophic Pythagorean semi closed set (GNPSCS in short) if GNPint(GNPcl(A)) ⊆ A,
2. Generalized Neutrosophic Pythagorean semi open set (GNPSOS in short) if A ⊆ GNPcl(GNPint(A))
3. Generalized Neutrosophic Pythagorean pre-closed set (GNPPCS in short) if GNPcl(GNPint(A)) ⊆ A
4. Generalized Neutrosophic Pythagorean pre-open set (GNPPOS in short) if A ⊆ GNPint(GNPcl(A))
5. Generalized Neutrosophic Pythagorean α-closed set (GNPαCS in short) if GNPcl(GNPint(GNPcl(A))⊆ A

6. Generalized Neutrosophic Pythagorean α-open set (GNPαOS in short) if A ⊆ GNPint(GNPcl(GNPint(A)))

Definition 2.9 Let A be a GNPS of a GNPTS (X, τ). Then the GNPSint(A) and the GNPScl(A) are defined as

1. GNPS int (A) = ∪{K|K is a GNPOS in X and K ⊆ A}
2. GNPS cl(A) = ∩{K|K is a GNPCS in X and A ⊆ K}

Definition 2.10 Let A be a GNPS in (X, τ), Then

1. GNPS cl(A) = A∪GNP int(GNP cl(A))
2. GNPS int(A) = A ∩GNP cl(GNP int(A)

Definition 2.11 A GNPS A ={<a, T_A,I_A, F_A>} in a GNPTS (X, τ) is said to be an

1.Generalized Neutrosophic Pythagorean regular open set (GNPROS) if A = GNPint(GNPcl(A))

2. Generalized Neutrosophic Pythagorean regular closed set (GNPRCS) if A = GNPcl(GNPint(A))

Definition 2.12 A GNPS A of a GNPTS (X, τ) is a Generalized Neutrosophic Pythagoraen generalized closed set (GNPGCS) if GNPcl(A) ⊆ U whenever A ⊆ U and U is a GNPOS in X.

Definition 2.13 Let a GNPS A of a GNPTS (X, τ) . Then the alpha closure of A (GNPαcl(A)) is defined as GNPαcl(A) = ∩{K|K is a GNPCS in X and A ⊆ K}and the alpha interior of A (GNPαint(A)) is defined as GNPαint(A) = ∩{K|K is a GNPCS in X and K ⊆ A }.

Definition 2.14 Let A be a GNPS in (X, τ), then

1. GNPαcl(A) = A ∪ GNPcl(GNPint(GNPcl(A)))
2. GNPαint(A) = A ∩ GNPint(GNPcl(GNPint(A)))

Definition 2.15 A GNPS A of a GNPTS (X, τ) is said to be a Generalized Neutrosophic Pythagorean alpha generalized closed sets (GNPαGCS in short) if GNPαcl(A) ⊆ U whenever A ⊆ U and U is a GNPOS in X.

Definition 2.16 Let (X, τ) be a GNPTS and A ={<a, T_A,I_A, F_A>} be a GNPS in X. The pre interior of A is denoted by GNPpint(A) and is defined by the union of all GNP pre-open sets of X which are contained in A. The intersection of all GNP pre-closed sets containing A is called the pre closure of A and is denoted by GNPpcl(A).

1. GNPpint(A) = ∪{G|G is a GNPPOS in X and G ⊆ A}
2. GNPpcl(A) = ∩{K|K is a GNPPCS in X and A ⊆ K}

Definition 2.17 If A is a GNPS in X, then GNPpcl(A) = A ∪ GNPcl(GNPint(A)).

Definition 2.18 For any GNPS A in (X, τ),We have

1. X – GNP int(A) = GNP cl(X-A)
2. X – GNP cl(A) = GNP int(X-A)

III.GENERALIZED NEUTROSOPHIC PYTHAGOREAN GENERALIZED PRE -CLOSED SETS

Definition 3.1 A GNPS A is said to be Generalized Neutrosophic Pythagorean generalized pre-closed set (GNPGPCS) in (X, τ) if GNPpcl(A) ⊆ U whenever A ⊆ U and U is a GNPOS in X. The family of all GNPGPCSs of a GNPTS (X, τ) is denoted by GNPGPC(X).

Example 3.2 Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U= {< a, 0.2, 0.6,0.6 >, < b, 0.3,0.6, 0.7 >}. Then the GNPS A = {< a, 0.1,0.6, 0.7>, < b, 0.2,0.6, 0.8 >} is a GNPPCS in X.

Theorem3.3 Every PFCS is a PFGPCS but not conversely.

Proof: Let A be a GNPCS in X and let A ⊆ U and U is a GNPOS in (X, τ). Since GNPpcl(A) ⊆ GNPcl(A) and A is a GNPCS in X, GNPpcl(A) ⊆ GNPcl(A) = A ⊆ U. Therefore A is a GNPGPCS in X.

Example 3.4 Let X = {a, b} and let τ = 0, U, 1} be a PFT on X, where U= {< a, 0.2, 0.6,0.6 >, < b, 0.3,0.6, 0.7 >}. Then the GNS A = {< a, 0.1,0.6, 0.7 >, < b, 0.2,0.6, 0.8 >} is a GNPGPCS in X but not a GNPCS in X.

Theorem 3.5: Every GNPαCS is a GNPGPCS but not conversely.
Proof: Let A be a GNPαCS in X and let A ⊆ U and U is a GNPOS in (X, τ). By hypothesis, GNPcl(GNPint(GNPcl(A))) ⊆ A. Since A ⊆ GNPcl(A), GNPcl(GNPint(A)) ⊆ GNPcl(GNPint(GNPcl(A))) ⊆ A. Hence GNPpcl(A) ⊆ A ⊆ U. Therefore A is a GNPGPCS in X.

Example 3.6: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.6, 0.5,0.4 >, < b, 0.3,0.5, 0.7 >}. Then the GNPS A = {< a, 0.3,0.5, 0.7 >, < b, 0.1,0.5, 0.9 >} is a GNPGPCS in X but not a GNPαCS in X since GNPcl(GNPint(GNPcl(A))) = {< a, 0.4, 0.5,0.6 >, < b, 0.7,0.5, 0.3 >} ⊈ A.

Theorem 3.7: Every GNPGCS is a GNPGPCS but not conversely.
Proof: Let A be a GNPGCS in X and let A ⊆ U and U is a GNPOS in (X, τ). Since GNPpcl(A) ⊆ GNPcl(A) and by hypothesis, GNPpcl(A) ⊆ U. Therefore A is a GNPGPCS in X.

Example 3.8: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.6,0.5, 0.4>, < b, 0.3, 0.5,0.7> Then the GNPS A = {< a, 0.3,0.5, 0.7 >, < b, 0.1,0.5, 0.9 >} is a GNPGPCS in X but not a GNPGCS in X since A ⊆ U but GNPcl(A) = {< a, 0.4,0.5, 0.6 >, < B, 0.7,0.5, 0.3 >} ⊈ U.

Theorem 3.9: Every GNPRCS is a GNPGPCS but not conversely.
Proof: Let A be a GNPRCS in X. By Definition 2.11, A = GNPcl(GNPint(A)). This implies GNPcl(A) = GNPcl(GNPint(A)). Therefore GNPcl(A) = A. That is A is a GNPCS in X. By Theorem 3.3, A is a GNPGPCS in X.

Example 3.10: Let X = {a, b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.4, 0.4,0.6 >, < b, 0.7, 0.4,0.3 >}. Then the GNPS A = {< a, 0.3,0.5, 0.7 >, < b, 0.2,0.5, 0.8 >} is a GNPGPCS but not a GNPRCS in X since GNPcl(GNPint(A)) = 0 ≠ A.

Theorem 3.11: Every GNPPCS is a GNPGPCS but not conversely.
Proof: Let A be a GNPPCS in X and let A ⊆ U and U is a GNPOS in (X, τ). By Definition 2.8, GNPcl(GNPint(A)) ⊆ A. This implies that GNPpcl(A) = A ∪ GNPcl(GNPint(A)) ⊆ A. Therefore GNPpcl(A) ⊆ U. Hence A is a GNPGPCS in X.

Example 3.12: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.2,0.3, 0.8 >, < b, 0.3,0.5, 0.7 >}. Then the GNPS A = {< a, 0.4, 0.3,0.7 >, < b, 0.4,0.6, 0.6 >} is a GNPGPCS but not a GNPPCS in X since GNPcl(GNPint(A)) = 1 ⊈ A.

Theorem 3.13: Every GNPαGCS is a GNPGPCS but not conversely.
Proof: Let A be a GNPαGCS in X and let A ⊆ U and U is a GNPOS in (X, τ). By Definition 2.14, A ∪ GNPcl(GNPint(GNPcl(A))) ⊆ U. This implies GNPcl(GNPint(GNPcl(A))) ⊆ U and GNPcl(GNPint(A)) ⊆ U. Therefore GNPpcl(A) = A ∪ GNPcl(GNPint(A)) ⊆ U. Hence A is a GNPGPCS in X.

Example 3.14: Let X = {a, b} and let τ = {0, U, 1} be a PFT on X, where T = {< a, 0.4,0.4, 0.6 >, < b, 0.6,0.3, 0.4 >}. Then the GNPS A = {< a, 0.5,0.4, 0.5 >, < b, 0.7, 0.3,0.3 >} is a GNPGPCS but not a GNPαGCS in X since GNPαcl(A) = 1 ⊈ U.

Theorem 3.15: GNPSCS and GNPGPCS are independent to each other.

Example 3.16: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.4, 0.5,0.6 >, < b, 0.2,0.5, 0.8>}. Then the GNPS A = U is a GNPSCS but not a GNPGPCS in X since A ⊆ U but GNPpcl(A) = {< a, 0.6, 0.5,0.4 >, < b, 0.8,0.5, 0.2 >} ⊈ U.

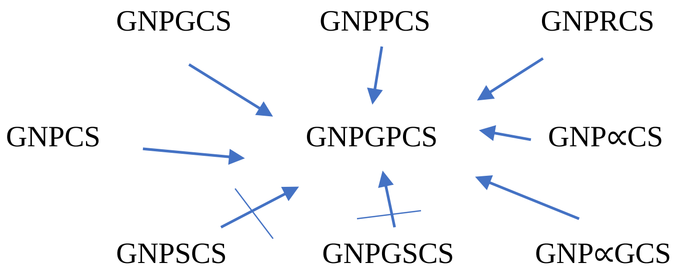
Example 3.17: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.7,0.4, 0.3 >, < b, 0.7,0.4,0.3 >}. Then the GNPS A = {< a, 0.6,0.3, 0.4 >, < b, 0.5, 0.3,0.2 >} is a GNPGPCS but not a GNPSCS in X since GNPint(GNPcl(A)) ⊈ A.

Theorem 3.18: GNPGSCS and GNPGPCS are independent to each other.

Example 3.19: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.4,0.5, 0.6 >, < b, 0.1, 0.5,0.8 >}. Then the GNPS A = U is a GNPSCS but not a GNPGPCS in X since A ⊆ U but GNPpcl(A) = {< a, 0.6, 0.5,0.4 >, < b, 0.8,0.5, 0.1 >} ⊈ U.

Example 3.20: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.3,0.4, 0.7 >, < b, 0.2,0.4, 0.6 >}. Then the GNPS A = {< a, 0.2, 0.4,0.7 >, < b, 0.2, 0.4,0.7 >} is a GNPGPCS but not a GNPGSCS in X since A ⊆ U but GNPscl(A) = 1 ⊈ U.

The following implications are true.



Remark 3.21: The union of any two GNPGPCSs is not a GNPGPCS in general as seen in the following example.

Example 3.22: Let X = {a, b} be a GNPTS and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.2,0.5, 0.8 >, < b, 0.3, 0.3,0.7 >}. Then the GNPSs A = {< a, 0.1,0.7, 0.9 >, < , 0.2,0.7, 0.8 >}, B = {< a, 0.2,0.6, 0.8 >, < b, 0.2,0.6,0.8 >} are GNPGPCSs but A ∪ B is not a GNPGPCS in X.

IV. GENERALIZED NEUTROSOPHIC PYTHAGOREAN GENERALIZED PRE-OPEN SETS

In this section we introduce Generalized Neutrosophic Pythagorean generalized pre-open sets and studied some of its properties.

Definition 4.1: A GNPS A is said to be a Generalized Neutrosophic Pythagorean generalized pre-open set (GNPGPOS) in (X, τ) if the complement $\overline{A}$ is a GNPGPCS in X.

The family of all GNPGPOSs of a GNPTS (X,τ) is denoted by GNPGPO(X).

Example 4.2: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U= {< a, 0.2,0.3, 0.8 >, < b, 0.3, 0.3,0.6 >}. Then the GNPS A = {< a, 0.8,0.3, 0.2 >, < b, 0.7,0.3, 0.2 >} is a GNPGPOS in X.

Theorem 4.3: For any GNPTS (X,τ), we have the following:

1. Every GNPOS is a GNPGPOS
2. Every GNPSOS is a GNPGPOS
3. Every GNPαOS is a GNPGPOS
4. Every GNPPOS is a GNPGPOS.

The converse of the above statements need not be true which can be seen from the following examples.

Example 4.4: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.2,0.4, 0.8 >, < b, 0.2,0.4, 0.6 >}. Then the GNPS A = {< a, 0.8,0.4, 0.2 >, < b, 0.7,0.4, 0.2 >} is a GNPGPOS in (X, τ) but not a GNPOS in X.

Example 4.5: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.4,0.3, 0.6 >, < b, 0.2,0.3,0.7 >}. Then the GNPS A = {< a, 0.7,0.3, 0.3 >, < b, 0.7, 0.3,0.2 >} is a GNPGPOS but not a GNPαOS in X.

Example 4.6: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.6, 0.5,0.4 >, < b, 0.4,0.4, 0.5 >}. Then the GNPS A = {< a, 0.7,0.5, 0.3 >, < b, 0.5, 0.5,0.4 >} is a GNPGPOS but not a GNPPOS in X.

Theorem 4.7: Let (X, τ) be a GNPTS. If A ∈ GNPGPO(X) then V ⊆ GNPint(GNPcl(A)) whenever V ⊆A and V is GNPCS in X.

Proof: Let A ∈ GNPGPO(X). Then $\bar{A}$ is a GNPGPCS in X. Therefore GNPpcl($\bar{A}$) ⊆ U whenever $\bar{A}$ ⊆ U and U is a GNPOS in X. That is GNPcl(GNPint($\bar{A}$)) ⊆ U. This implies $\overline{U}$ ⊆ GNPint(GNPcl(A)) whenever $\overline{U}$ ⊆ A and $\overline{U}$ is GNPCS in X. Replacing $\overline{U}$ by V, we get V ⊆ GNPint(GNPcl(A)) whenever V ⊆ A and V is GNPCS in X.

Theorem 4.8: Let (X, τ) be a GNPTS. Then for every A ∈ GNPGPO(X) and for every A ∈ GNPS(X), GNPpint(A) ⊆ B ⊆ A implies A ∈ GNPGPO(X).

Proof: By hypothesis $\bar{A} \subseteq \bar{B} \subseteq \overline{(GNPpint(A))}$. Let $\bar{B} \subseteq$ U and U be a GNPOS. Since $\bar{A} \subseteq \bar{B}$,implies $\bar{A} \subseteq$ U. But $\bar{A}$ is a GNPGPCS, GNPpcl($\bar{A}$) ⊆ U. Also $\bar{B} \subseteq \overline{(GNPpint(A))}$ = GNPpcl($\bar{A}$). Therefore GNPpcl($\bar{B}$) ⊆ GNPpcl($\bar{A}$) ⊆ U. Hence ($\bar{B}$ is a GNPGPCS. Which implies B is a GNPGPOS of X.

Remark 4.9: The intersection of any two GNPGPOSs is not a GNPGPOS in general.

Theorem 4.10: A GNPS A of a GNPTS (X, τ) is a GNPGPOS if and only if F ⊆ GNPpint(A) whenever F is a GNPCS and F ⊆ A.

Proof:

Necessity: Suppose A is a GNPGPOS in X. Let F be a GNPCS and F ⊆ A. Then $\bar{F}$ is a Pythagorean Fuzzy Open set in X such that $\bar{A} \subseteq \bar{F}$. Since $\bar{A}$ is a GNPGPCS, we have GNPpcl($\bar{A}$) ⊆$\bar{F}$. Hence .$\overline{(GNPpint(A))} \subseteq \bar{F}$. Therefore F ⊆ GNPpint(A).

Sufficiency: Let A be a GNPS of X and let F ⊆ GNPpint(A) whenever F is a GNPCS and F ⊆ A. Then $\bar{A} \subseteq \bar{F}$. and $\bar{F}$ is a GNPOS. By hypothesis, $\overline{(GNPpint(A))} \subseteq \bar{F}$. Which implies GNPpcl($\bar{A}$) ⊆ $\bar{F}$ .Therefore $\bar{A}$ is a GNPGPCS of X. Hence A is a GNPGPOS of X.

Corollary 4.11: A GNPS A of a GNPTS (X, τ) is a GNPGPOS if and only if F ⊆ GNPint(GNPcl(A)) whenever F is a GNPCS and F ⊆ A.

Proof:

Necessity: Suppose A is a GNPGPOS in X. Let F be a GNPCS and F ⊆A.Then $\bar{F}$ is a GNPOS in X such that $\bar{A} \subseteq \bar{F}$. Since $\bar{A}$ is a GNPGPCS, we have GNPpcl(A) ⊆ $\bar{F}$.Therefore GNPCl(GNPint($\bar{A}$) ⊆$\bar{F}$. Hence $\overline{(GNPint(GNPcl(A)))} \subseteq \bar{F}$. Therefore F ⊆ GNPint(GNPcl(A)).

Sufficiency: Let A be a GNPS of X and let F ⊆ GNPint(GNPcl(A)) whenever F is a GNPCS and F ⊆ A. Then $\bar{A} \subseteq \bar{F}$. and $\bar{F}$ is a GNPOS. By hypothesis, $\overline{(GNPint(GNPcl(A)))} \subseteq \bar{F}$. Hence GNPCl(GNPint($\bar{A}$)) ⊆$\bar{F}$, which implies GNPpcl($\bar{A}$) ⊆ $\bar{F}$. Hence A is a GNPGPOS of X.

Theorem 4.12: For a GNPS A, A is a GNPOS and a GNPGPCS in X if and only if A is a GNPROS in X.

Proof:

Necessity: Let A be a GNPOS and a GNPGPCS in X. Then GNPpcl(A) ⊆ A. This implies GNPCl(GNPint(A)) ⊆ A. Since A is a GNPOS, it is a GNPPOS. Hence A ⊆ GNPint(GNPcl(A)). Therefore A = GNPint(GNPcl(A)). Hence A is a GNPROS in X.

Sufficiency: Let A be a GNPROS in X. Therefore A = GNPint(GNPcl(A)). Let A ⊆ U and U is a GNPOS in X. This implies GNPpcl(A) ⊆ A. Hence A is a GNPGPCS in X.

V. APPLICATIONS OF PYTHAGOREAN FUZZY GENERALIZED PRE-CLOSED SETS

In this section we provide some applications of Generalized Neutrosophic Pythagorean generalized pre-closed sets.

Definition 5.1: A GNPTS (X, τ) is said to be a Generalized Neutrosophic Pythagorean $pT_{1/2}$ (GNP $pT_{1/2}$) space if every GNPGPCS in X is a GNPCS in X.

Definition 5.2: A GNPTS (X, τ) is said to be a Generalized Neutrosophic Pythagorean gp $T_{1/2}$ (GNPgp$T_{1/2}$) space if every GNPGPCS in X is a GNPPCS in X.

Theorem 5.3: Every GNPp$T_{1/2}$ space is a GNPgp$T_{1/2}$ space. But the converse is not true in general.

Proof: Let X be a GNPp $T_{1/2}$ space and let A be a GNPGPCS in X. By hypothesis A is a GNPCS in X. Since every GNPCS is a GNPPCS, A is a GNPPCS in X. Hence X is a GNPgp $T_{1/2}$ space.

But the converse need not be true which can be seen in the following example.

Example 5.4: Let X = {a, b} and let τ = {0, U, 1} be a GNPT on X, where U = {< a, 0.8,0.4, 0.2 >, < b, 0.8, 0.4,0.2>}.Then (X, τ) is a GNPgp$T_{1/2}$ space. But it is not a GNPp $T_{1/2}$ space since the GNPS A = {< a, 0.2,0.4, 0.6 >, < b, 0.3,0.4,0.6 >} is GNPGPCS but not a GNPCS in X.

Theorem 5.5: Let (X, τ) be a GNPTS and X is a GNPp$T_{1/2}$ space then

(i) Any union of GNPGPCSs is a GNPGPCS.

(ii) Any intersection of GNPGPOSs is a GNPGPOS.

Proof:

(i) Let $\{Ai\}_{i \in J}$ is a collection of GNPGPCSs in a GNPp$T_{1/2}$ space (X, τ). Therefore every GNPGPCS is a GNPCS. But the union of GNPCS is a GNPCS. Hence the union of GNPGPCS is a GNPGPCS in X.

(ii) It can be proved by taking complement in (i).

Theorem 5.6: A GNPTS X is a GNPgp$T_{1/2}$ space if and only if GNPGPO(X) = GNPPO(X).

Proof:

Necessity: Let A be a GNPGPOS in X, then $\overline{A}$ is a GNPGPCS in X. By hypothesis $\overline{A}$ is a GNPPCS in X. Therefore A is a GNPPOS in X. Hence GNPGPO(X) = GNPPO(X).

Sufficiency: Let A be a GNPGPCS in X. Then $\overline{A}$ is a GNPGPOS in X. By hypothesis $\overline{A}$ is a GNPPOS in X. Therefore A is a GNPPCS in X. Hence X is a GNPgp$T_{1/2}$ space..

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