IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

Vague Γ Generalized Sets In Topological Spaces

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Abstract: The aim of this paper is to explore the notion of vague sets and to define a new class of vague generalized γ closed sets in topological spaces and investigate their properties.

Keywords: Vague set, Vague topology, Vague generalized γ closed set, Vague point

1.Introduction:

Lotfi A.Zadeh [17]a professor of electrical engineering with the University of California at Berkeley, published the first papers on his new theory of Fuzzy sets and Systems in the year 1965. Zadeh [17] is widely known as the father of a mathematical framework called fuzzy logic which was an early approach to artificial intelligence. In early 90's Gau's and Buehrer [7] introduced the notion of vague sets. Intuitionistic fuzzy sets have been introduced by Krassimir Atanassov [2] (1983) as an extension of Lotfi Zadeh's notion of fuzzy set, which itself extends the classical notion of a set. In topology, the concept of vague γ generalized closed sets extends the traditional notion of closed sets in a topological space. These sets are characterized by a specific closure operator, denoted by γ, which introduces a broader perspective on the closure operation. The term "vague" implies a more flexible and generalized closure property, allowing for a nuanced understanding of closed sets beyond the classical definition. Exploring vague γ generalized closed sets contributes to a richer comprehension of topological structures and their properties, offering a valuable perspective for researchers and practitioners in the field.

2. Preliminaries

Definition 2.1[4]: A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function t_A:U → [0,1] and
(ii) A false membership function f_A:U → [0,1]

Where t_A(x) is a lower bound on the grade of membership of x derived from the "evidence for x", f_A(x) is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x)+ f_A(x)\leq 1$. Thus, the grade of membership of u in the vague set A is bounded by a subinterval [t_A(x),1-f_A(x)] of [0,1]. This indicates that if the actual grade of membership of x is μ(x), then, $t_A(x)\leq\mu(x)\leq 1-f_A(x)$. The vague set A is written as A= {<x, [t_A(x),1-f_A(x)]>/u∈U} where the interval [t_A(x),1-f_A(x)] is called the vague value of x in A, denoted by V_A(x).

Definition 2.2[7]: Let A and B be vague sets of the form A= {<x, [t_A(x),1-f_A(x)]>/xϵX} and B= {<x, [t_B(x),1-f_B(x)]>/xϵX} then

(i)A⊆B if and only if t_A(x)≤ t_B(x) and 1-f_A(x)≤ 1-f_B(x) for all xϵX

(ii)A=B if and only if A⊆B and B⊆A

(iii)A^c={<x,f_A(x),1-t_A(x)>/xϵX}

(iv)A∩B= {<x, min(t_A(x), t_B(x)), min(1-f_A(x),1-f_B(x))>/xϵX}

(v)A∪B= {<x, (t_A(x)∨t_B(x), (1-f_A(x)∨1-f_B(x))>/xϵX}

For the sake of simplicity, we shall use the notation A= {<x,[t_A,1-$f_{A]}$>} instead of A= {<x,[t_A(x),1-f_A(x)]>/xϵX}.

Definition 2.3: A subset A of a topological space (X,τ) is called

(i) A preclosed set [13] if cl(int(A)) ⊆A

(ii) A semi-closed set [9] if int(cl(A)) ⊆A

(iii) A regular closed set [18] if A =cl(int(A))

(iv) A α-closed set [14] if cl(int(cl(A)))⊆A

(V) A closed set if cl(A)=A

Definition 2.4: A subset A of a topological space (X,τ) is called

(i) A generalized closed set (briefly g- closed) [8] if cl(A)⊆U whenever A⊆U and U is an openset in X

(ii) A generalized closed set (briefly sg- closed) [3] if scl(A) ⊆ U whenever A⊆U and U is semi-open set in X

(iii) A generalized semi-closed set (briefly gs-closed) [1] if scl(A) ⊆U whenever A⊆U and U is open set in X

(iv) A generalized semi pre closed set (briefly gsp-closed) [6] if spcl(A) ⊆U whenever A⊆U and U is open set in X

(v) A generalized pre closed set (briefly qp-closed) [4] if pcl(A) ⊆U whenever A⊆U and U is open set in X

(vi) A generalized α-closed set (briefly gα-closed) [11] if αcl(A) ⊆U whenever A⊆U and U is α-open set in X

(vii) A α-generalized closed set (briefly αg-closed) [10] αcl(A) ⊆U whenever A⊆U and U is open set in X.

Definition 2.5: A vague topology (VT in short) on X is a family τ of vague sets in X satisfying the following axioms.

(i) $0,1 \in \tau$
(ii) $G_1 \cap G_2 \in \tau$, for any $G_{1\,,}G_2 \in \tau$
(iii) $\cup\, G_i \in \tau$ for any family $\{G_i/$ i∈J$\} \subseteq \tau$

In this case the pair (X,τ) is called a Vague topological space (VTS in short) and any vague set in τ is known as a Vague open set (VOS in short) in X.

The complement A^c of a vague open set A in a Vague topological space (X,τ) is called a vague closed set (VCS in short) in X.

Definition 2.6: Let (X,τ) be a VTS and A= {<x, [t_A,1-f_A]>} be vague set in X. Then the vague interior and a vague closure are defined by

Vint(A) = ∪{G/G is an VOS in X and G⊆A}

Vcl(A)= ∩{K/K is an VCS in X and A⊆K}

Note that for any vague set A in (X,τ), we have Vcl (A^c)=(V int(A))^c and Vint (A^c)=(Vcl(A)).^{c}

Example 2.7: We consider the vague topology. Let X={a,b} and let τ={0,G,1} is an vague topology on X where G ={<x,[0.1,0.5],[0.1,0.6]>}. Here the only open set are 0,1 and G. If A= {<x, [0.1,0.6] [0.1,0.9]>} is a vague topology on X then,

Vint(A)= ∪{G/G is an VOS in X and G⊆A} =G

Vcl(A)= ∩{K/K is an VCS in X and A⊆K} =G^c

Definition 2.8: A vague set A of (X,τ), is said to be a,

(i) A vague pre-closed set if Vcl(Vint(A)) ⊆A
(ii) A vague semi-closed set if Vint (Vcl(A)) ⊆A
(iii) A vague regular- closed set if A=Vcl(Vint(A))
(iv) A vague α closed set Vcl(Vint(Vcl(A))) ⊆A
(v) A vague closed set if Vcl(A)=A

Definition 2.9: An vague set A in (X,τ), is said to be a,

(i) Vague generalized closed set (briefly VGC) if Vcl(A)⊆U whenever A⊆U and U is an vague open set in X
(ii) Vague generalized semi-closed set (briefly VGSC) if Vscl(A) ⊆U whenever A⊆U and U is vague open set in X
(iii) Vague generalized pre closed set (briefly VGPC) if Vpcl(A) ⊆U whenever A⊆U and U is vague open set in X

Properties 2.10: Let A be any Vague set in (X,τ), then

(i) Vint(1-A) =1-(Vcl(A)) and
(ii) Vcl(1-A) =1-(Vint(A))

Proof: (i) By definition Vcl(A)= ∩{K/K is an VCS in X and A⊆K}

1-(Vcl(A)) = 1 −∩{K/K is an VCS in X and A⊆K}

= ∪{{1-K/K is an VCS in X and A⊆K}

= ∪{G/G is an VOS in X and G⊆ 1 −A}

=Vint(1-A)

(ii)The proof is similar to (i)

Theorem 2.11: Let (X,τ), be a VS and let A ∈V(X). Then the following properties hold

(i) Vint(A) ⊂ A
(ii) A⊂B ⇒ Vint(A) ⊂ Vint(B)
(iii) Vint(A) ∈ τ
(iv) A is a vague open set ⟺ Vint(A)=A
(v) Vint (Vint(A)) =Vint(A)
(vi) Vint (0) =0, Vint (1) =1
(vii) Vint(A∩B) = Vint(A)∩ Vint(B)
(viii) (Vint(A))^c=Vcl (A^c)

Proof: The proof is obvious.

Theorem 2.12: Let (X,τ) be a VS and let A∈ V(X). Then the following properties holds.

(i) (A) ⊂ Vcl(A)
(ii) A⊂B ⇒ Vcl(A) ⊂ Vcl(B)
(iii) Vcl(A)^c ∈ τ
(iv) A is a vague closed set ⟺ Vcl(A)=A
(v) Vcl(Vcl(A))=Vcl(A)
(vi) Vcl(0)=0, Vcl(1)=1
(vii) Vcl(A∪B) = Vcl(A)∪ Vcl(B)
(viii) (Vcl(A))^c=Vint (A^c)

Proof: The proof is obvious

3. VAGUE γ GENERALIZED CLOSED SETS

In this section we have introduced vague γ generalized closed sets and studied some of their properties.

Definition 3.1: An vague set A in an vague topological spaces (X,τ) is said to be an vague γ generalized closed set (VγGCS for short) Vγcl(A)⊆U whenever A⊆U and U is an VγOS in (X,τ). The complement A^c of an VγGCS A in an VTS (X,τ) is called vague γ generalized open set (VγGOS in short) in X.

The family of all VγGCSs of an vague topological spaces (X,τ) is denoted by VγGC(X).

Example 3.2: Let X={a,b} and let τ ={0,G,1} is an VT on X where G={<x,[0.5,0.8][0.3,0.7]>}. Here the only γ open sets are 0, X, and G. Then the VS A= {<x,[0.4,0.9] [0.4,0.8]>} is an VγGCS in (X,τ).

Theorem 3.3: Every VCS is an VγGCS in (X,τ) but not conversely in general.

Proof: Let A be an VCS in X and let A⊆U where U is an VγOS in X. As γcl(A)⊆cl(A)=A⊆U, by hypothesis, we have γcl(A)⊆U. Hence A is an VγGCS in
(X,τ).

Example 3.4: Let the vague set A = {<x,[0.3,0.7],[0.4,0.5]>} and G={<[0.4,0.7],[0.4,0.6]>} is an VγGCS but not an VCS in (X, τ) as Vcl(A) = 1≠ A.

Theorem 3.5: Every VRCS is an VγGCS in (X,τ) but not conversely in general.

Proof: Let A be an VRCS. Since every VRCS is an VCS, by theorem 3.3, A is an VγGCS in (X,τ).

Example 3.6: Let VS A = {<x,[0.3,0.6], [0.4,0.5]>} and G= {<x, [0.4,0.7][0.4,0.6]>} is an VγGCS but not an VRCS in X as Vcl(int(A)) = 0 ≠ A.

Theorem 3.7: Every VSCS is an VγGCS in (X,τ) but not conversely in general.

Proof: Let A be an VSCS in X and let A ⊆ U where U is an VγOS in X. Since γcl(A)⊆scl(A)=A⊆ U, by hypothesis, we have γcl(A) ⊆ U. Hence A is an VγGCS in (X,τ).

Example 3.8: Let the vague set A = {<x,[0.4,0.8], [0.5,0.6]>} and G={<x,[0.5,0.8][0.5,0.7]>} is an VγGCS but not an VSCS in X as int(cl(A)) = 1⊈ A.

Theorem 3.9: Every VPCS is an VγGCS in (X,τ) but not conversely in general.

Proof: Let A be an VPCS in X and let A ⊆ U where U is an VγOS in X. As γcl(A)⊆pcl(A)=A⊆ U, by hypothesis, we have γcl(A) ⊆ U. Hence A is an VγGCS in
(X,τ).

Example 3.10: Let the vague set A = {<x,[0.2,0.7], [0.3,0.5]>} and G={<[0.2,0.7][0.3,0.5]>} is an VγGCS but not an VPCS in X, as Vcl(int(A)) = G^c⊈A.

Theorem 3.11: Every VαCS is an VγGCS in (X,τ) but not conversely in general.

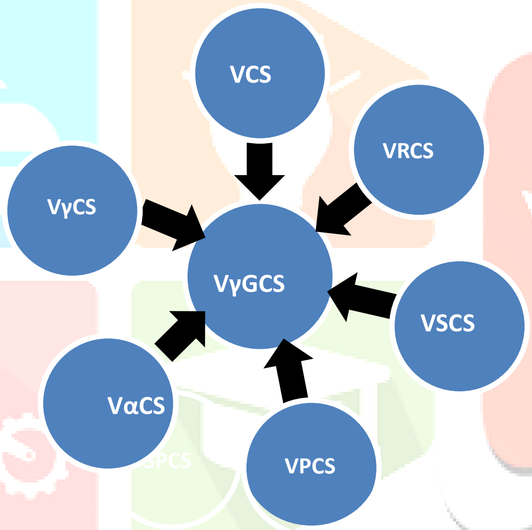
Proof: Let A be an VαCS in X and let A⊆U where U is an VγOS in (X,τ). As γcl(A)⊆αcl(A)=A⊆U, by hypothesis, we have γcl(A) ⊆ U. Hence A is an VγGCS in (X,τ).

Example 3.12: Let the VS A = {<x,[0.3,0.6],[0.1,0.5]>}and G={<x,[0.3,0.6],[0.2,0.8]>} is an VγGCS but not an VαCS in X as Vcl(Vint(Vcl(A)))= G^c⊈A.

Theorem 3.13: Every VγCS is an VγGCS in (X,τ) but not conversely in general.

Proof: Let A be an VγCS and let A⊆ U where U is an VγOS in (X,τ). Then γcl(A)=A⊆U, by hypothesis, we have γcl(A)⊆U. Hence A is an VγGCS in
(X,τ).

Example 3.14: Let the vague set A={<x,[0.2,0.7],[0.3,0.5]>} and G = {<x,[0.2,0.7][0.3,0.5]>} is an VγCS but not an VγGCS in X , as cl(int(A)) ∩ int(cl(A)) = 1⊈A.



The reverse implications are not true in general in the above diagram.

Remark 3.15: The union of any two VγGCS is not an VγGCS in general as seen in following example.

Example 3.16: Let X = {a,b} and G_1 = {<x,[0.4,0.7],[0.4,0.5]>}, then τ_1 ={0,G_1,1} is a vague topological space on X and let G_2={<x,[0.3,0.8],[0.5,0.6]>}, then τ_2={0,G_2,1} is a vague topological space on X and let the vague set A_1={<x,[0.3,0.7],[0.4,0.5]>} and A_2={<x,[0.2,0.7],[0.4,0.5]>} are vague γ generalized closed set but the union of these two set is not vague γ generalized closed set in X.

Remark 3.17: The intersection of any two VγGCS is not an VγGCS in general as seen in the following example.

Example 3.18: Let X = {a,b} and G_1 = {<x,[0.4,0.7],[0.4,0.5]>} then τ_1 ={0,G_1,1} is a vague topological space on X and let G_2={<x,[0.3,0.8],[0.5,0.6]>}, then τ_2={0,G_2,1} is a vague topological space on X and let the vague set A_1={<x,[0.3,0.7],[0.4,0.5]>} and A_2={<x,[0.2,0.7],[0.4,0.5]>} are vague γ generalized closed set but the intersection of these two set is not vague γ generalized closed set in X.

Theorem 3.19: Let (X,τ) be an VTS.Then for every A∈VϒGC(X) and for every
B∈VS(X), A⊆B⊆γcl(A)⟹B∈VϒGC(X).

Proof: Let B⊆U and U be an VγOS in X. Then since, A⊆B, A⊆U. By hypothesis B⊆γcl(A). Therefore ϒcl(B)⊆γcl(ϒcl(A)) =ϒcl(A)⊆U, since A is an VγGCS. Hence B∈VγGC(X).

Theorem 3.20: An vague set A of an VTS (X,τ) is an VϒGCS V and only if $A_{\bar{q}}$F⟹γcl$(A)_{\bar{q}}$F for every VγCS F of X.

Proof: Necessity: Let F be an VγCS and $A_{\bar{q}}$F, then A⊆F^c,we know that "if two vague sets are said to be q-coincident(A_qB in short) if and only if there exists an element x∈X such that μ_A(x)>$v_B(x)$ or $v_A(x)$<$\mu_B(x)$"where F^c is an VγOS. Then γcl(A)⊆ F^c, by hypothesis. Then, γcl$(A)_{\bar{q}}$F.

Sufficiency: Let U be an VγOS such that A⊆U. Then U^c is an VγCS and A⊆(U^c)^c.By hypothesis, $A_{\bar{q}}$U^c⟹γcl$(A)_{\bar{q}}$U^c. Hence γcl(A)⊆(U^c)^c=U. Therefore γcl(A)⊆U. Hence A is an VγGCS in X.

Theorem 3.21: If A is both an VγOS and an VγGCS in (X,τ) then A is an VγCS in (X,τ).

Proof: Since A⊆A and A is an VγOS, by hypothesis γcl(A)⊆A. But A ⊆γcl(A). Therefore γcl(A)=A. Hence A is an VγCSin (X,τ).

Theorem 3.22: Let A be an VγGCS in (X,τ) and $p_{(\alpha,\beta)}$ be an VP in X such that p(α,β)qγcl(A)then cl(p(α,β))qA.

Proof: Let A be an VγGCS and let $p_{(\alpha,\beta)q}$γcl(A). Vcl(p(α, β)) $_{\bar{q}}$A, then by definition, A⊆[cl($p_{(\alpha,\beta)}$)]^c, where [cl($p_{(\alpha,\beta)}$)]^c is an VOS then it is an VγOS. Then by hypothesis, γcl(A)⊆[cl($p_{(\alpha,\beta)}$)]^c =int($p_{(\alpha,\beta)}$)^c⊆[$p_{(\alpha,\beta)}$]^c. This implies that $p_{(\alpha,\beta)\bar{q}}$γcl(A), which is a contradiction to the hypothesis. Hence cl($p_{(\alpha,\beta)}$)$_q$A.

Theorem 3.23: Let F⊆A⊆X where A is an VγOS and an VγGCS in X. Then F is an VγGCS in A if and only if F is an VγGCS in X.

Proof: Necessity: Let U be an VγOS in X and F⊆U. Also let F be an VγGCS in A. Then F⊆A∩U and A∩U is an VγOS in A. Hence gamma closure of F in A, γcl$_A$(F)⊆A∩U and by Theorem 3.21, A is an VγCS. Therefore γcl(A)=A. Now gamma closure of F in X, γcl(F)⊆γcl(F)∩γcl(A)=γcl(F)∩A=γcl$_A$(F)⊆A∩U⊆U. That is γcl(F)⊆U, whenever F⊆U. Hence F is an VγGCS in X.

Sufficiency: Let V be an VOS in A such that F⊆V. Since A is an VγOS in X, V is an VγOS in X. Therefore γcl(F)⊆V, since F is an VγGCS in X. Thus, γcl$_A$(F)=γcl(F)∩A⊆V∩A⊆V. Hence F is an VγGCS in A.

Theorem 3.24: For an vague set A, the following conditions are equivalent:

(i) A is an VOS and an VγGCS

(ii) A is an VROS

Proof: (i)⟹(ii)

Let A be an VOS and an VγGCS. Then ϒcl(A)⊆A as A ⊆ A and A is an VϒOS in X, but A⊆γcl(A). This implies that ϒcl(A)=A.Therefore, A is an VϒCS and int(cl(A)) = int(cl(A))∩cl(A)) = int(cl(A))∩cl(int(A)) ⊆A, by hypothesis. Hence int(cl(A))⊆A. Since A is an VOS, it is an VPOS. Hence A⊆int(cl(A)). Therefore A=int(cl(A)). Hence A is an VROS.

(ii)⟹(i)

Let A be an VROS. Therefore A=int(cl(A)). Since every VROS is an VOS we have int(cl(A))∩cl(int(A)) =A ∩cl(int(A)) =A∩cl(A)=A⊆A. Hence A is an VγCS in X and thus A is an VγGCS in X.

Theorem 3.25: For an VOS A in (X,τ), the following conditions are equivalent:

(i) A is an VCS

(ii) A is an VϒGCS and an VQ-set

Proof: (i)⟹(ii) Since A is an VCS, it is an VγGCS by Theorem 3.3. Now int(cl(A))=int(A)=A=cl(A)=cl(int(A)), by hypothesis. Hence A is an VQ-set.

(ii)⟹(i) Since A is an VOS and an VϒGCS, by Theorem 3.24, A is an VROS. Therefore A=int(cl(A)) =cl(int(A)) =cl(A), by hypothesis. Hence A is an VCS in X.

Theorem 3.26: If a subset A of an VTS (X, τ) is nowhere dense, then it is an VγGCS in (X,τ).

Proof: If A is nowhere dense, then int(cl(A)) =0. Let A ⊆ U where U is an VFγOS.
Now ϒcl(A)⊆scl(A) = A∪int(cl(A)) = A ∪ 0= A ⊆ U and hence A is an VγGCS in (X,τ).

Theorem 3.27: Let (X,τ) be an VTS. Then every vague set in (X,τ) is an VγGCS if and only if VγO(X)=VγC(X).

Proof: Necessity: Suppose that every vague set in (X,τ) is an VγGCS in X. Let U∈VγO(X), and by hypothesis, γcl(U)⊆U⊆γcl(U). This implies γcl(U)=U. Therefore
U∈VγC(X). Hence VγO(X)⊆VγC(X)(i). Let A∈VγC(X), then A^c∈VγO(X)⊆VγC(X). That is, A^c∈VγC(X). Therefore A∈VγO(X). Hence VγC(X) ⊆VγO(X). From (i) and (ii) VγO(X)=VγC(X)

Theorem 3.28: If A is an VROS and B is an VαCS, then A ∩ B is an VγGCS in (X, τ).

Proof: Let B be an VαCS and A be an VROS. Then cl(int(cl(B))) ⊆ B and int(cl(A)) = A. Therefore A ∩ B ⊇ A ∩ cl(int(cl(B))) = int(cl(A)) ∩ cl(int(cl(B))) ⊇ int(cl(A)) ∩ int(cl(B)) =int (cl (A ∩B)). We have int (cl(A ∩ B) ⊆ A ∩ B. Hence A ∩ B is an VSCS and by Theorem 3.7, A∩B is an VγGCS in (X, τ).

Theorem 3.29: If A is both an VROS and an VγGCS in (X, τ) then A is an Vγ-clopen set in (X, τ).

Proof: Let A be an VROS and an VγGCS in (X, τ). Then A is an VϒOS and A ⊆ A, ϒcl(A) ⊆ A, by hypothesis. But A ⊆ϒcl(A). Therefore A = ϒcl(A). Hence A is an VγCS in (X, τ). Hence A is an Vγ-clopen set in (X, τ).

Theorem 3.30: If A is both an VαOS and an VγGCS in (X, τ) then A is an VβCS in (X, τ).

Proof: Let A be an VαOS. Then A is an VϒOS. As A ⊆ A, by hypothesisϒcl(A) ⊆ A. Since βcl(A) ⊆ γcl(A)⊆ A⊆βcl(A), A is an VβCS in (X, τ).

Theorem 3.31: An VS A of X is an VγGCS V γcl(A) ⊆ ker(A).

Proof: Let U be any VϒOS such that A ⊆ U. By hypothesisγcl(A) ⊆ ker(A) and since A ⊆ U, ker(A) ⊆ U. Therefore γcl(A) ⊆ U and hence A is an VγGCS.

Theorem 3.32: If A is both an VγOS and an VγGCS in (X, τ) and suppose that F is an VCS in X. Then A∩ F is an VγGCS in (X, τ).

Proof: Since A is an VγOS and an VγGCS in (X, τ), then by Theorem 3.23 A is an VγCS in X. But F is an VCS in X. Therefore A∩ F is an VγCS in X. Hence A∩ F is an VγGCS in (X, τ).

Theorem 3.33: For an VγGCS A in an VTS (X,τ), the following conditions hold:

(i) If A is an VROS then scl(A) is an VγGCS

(ii) If A is an VRCS then sint(A) is an VγGCS

Proof:(i) Let A be an VROS in (X, τ). Then int(cl(A)) = A. By definition we have scl(A) = A ∪int(cl(A)) = A. Since A is an VγGCS in X, scl(A) is an VγGCS in X.

(ii)Let A be an VRCS in (X, τ). Then cl(int(A)) = A. By definition we have sint(A) = A∩cl(int(A)) = A. Since A is an VγGCS in X, sint(A) is an VγGCS in X.

Remark 3.34: Every VOS, VROS, VSOS, VPOS, VαOS, VγOS, VSPOS in (X, τ) is an VγGOS in (X, τ) but not conversely in general.

Proof: Straightforward.

Example 3.35: Obvious from examples 3.4, 3.6, 3.8, 3.10, 3.12, 3.14 and 3.16, by taking complement of A in the respective examples.

Theorem 3.36: Let (X, τ) be an VTS. Then for every $A \in V\gamma$ GO(X) and for every $B \in$ VS(X), γ int(A) $\subseteq B \subseteq A \Rightarrow B \in V\gamma$ GO(X).

Proof: Let A be any $V\gamma$ GOS of X and B be any VS of X. Let γ int(A) $\subseteq B \subseteq A$. Then A^c is an $V\gamma$ GCS and $A^c \subseteq B^c \subseteq \gamma$ cl(A^c). Therefore, B^c is an $V\gamma$ GCS which implies B is an $V\gamma$ GOS in X. Hence $B \in V\gamma$ GO(X).

Theorem 3.37: An VS A of an VTS (X,τ) is an $V\gamma$ GOS V and only V F $\subseteq \gamma$ int(A) whenever F is an VγCS and F ⊆ A.

Proof: Necessity: Suppose A is an $V\gamma$ GOS in X. Let F be an VγCS such that F ⊆ A. Then F^c is an VγOS and $A^c \subseteq F^c$. By hypothesis A^c is an $V\gamma$ GCS, we have γ cl(A^c) $\subseteq F^c$. Therefore F $\subseteq \gamma$ int(A).

Sufficiency: Let F be an VγCS such that F ⊆ A and F $\subseteq \gamma$ int(A). Then $(\gamma \text{ int}(A))^c \subseteq F^c$ and $A^c \subseteq F^c$. This implies that γ cl(A^c) $\subseteq F^c$, where F^c is an VγOS. Therefore, A^c is an $V\gamma$ GCS. Hence A is an $V\gamma$ GOS in X.

Theorem 3.38: Let (X, τ) be an VTS. Then for every A ∈ VS(X) and for every $B \in V\beta$ O(X), B ⊆ A ⊆ int(cl(int(B))) ⇒ $A \in V\gamma$ GO(X).

Proof: Let B be an $V\beta$ OS. Then B ⊆cl(int(cl(B))). By hypothesis, A ⊆ int(cl(int(B))) ⊆ int(cl(int(cl(int(cl(B)))))) ⊆ int(cl(cl(int(cl(B))))) = int(cl(int(cl(B)))) ⊆ int(cl(cl(A))) ⊆ int(cl(A)) as B ⊆ A. Therefore, A is an VPOS and by Theorem 3.36, A is an $V\gamma$ GOS. Hence $A \in V\gamma$ GO(X).

Theorem 3.39: Let (X,τ) be an VTS. Then for every A ∈ VS(X) and for every B ∈ VRC(X), B ⊆ A ⊆ int(cl(B)) ⇒ A ∈ VϒGO(X).

Proof: Let B be an VRCS. Then B = cl(int(B)). By hypothesis, A ⊆ int(cl(B)) ⊆ int(cl(cl(int(B)))) = int(cl(int(B))) ⊆ int(cl(int(A))) as B ⊆ A. Therefore, A is an VαOS and by Theorem 3.36, A is an VϒGOS. Hence A ∈ VϒGO(X).

Theorem 3.40: Let (X, τ) be an VTS then for every A ∈ VSPO(X) and for every VS B in X, A ⊆ B ⊆ cl(A) ⇒ B ∈ VϒGO(X).

Proof: Let A be an VSPOS in X. Then there exists an VPOS, (say) C such that C ⊆ A ⊆ cl(C). By hypothesis, A⊆ B. Therefore C ⊆ B. Since A ⊆ cl(C), cl(A) ⊆ cl(C) and B ⊆ cl(C), by hypothesis. Thus C ⊆ B ⊆ cl(C). This implies B is an VSPOS. As every VSPOS is an VϒGOS by Theorem 3.36, B ∈ VϒGO(X).

Theorem 3.41: If A is an VϒCS and an VϒGOS in (X, τ) then A is an VϒOS in (X, τ).

Proof: As A ⊇ A, by hypothesis ϒint(A)⊇ A. But we have A ⊇ ϒint(A). This implies A = ϒint(A). Hence A is an VϒOS in X.

Theorem 3.42: Let (X,τ) be an VTS. Then for every A ∈ VS(X) and for every B ∈ VSO(X), B ⊆ A ⊆ int(cl(B)) ⇒ A ∈ VϒGO(X).

Proof: Let B be an VSOS in X. Then B ⊆ cl(int(B)). By hypothesis, A ⊆ int(cl(B)) ⊆ int(cl(cl(int(B)))) = int(cl(int(B))) ⊆ int(cl(int(A))) as B ⊆ A. Therefore, A is an VαOS and by Theorem 3.36, A is an V𝛾GOS. Hence A ∈ V𝛾GO(X).

Theorem 3.43: If A is an VRCS and B is an VαOS, then A ∪ B is an V𝛾GOS in (X, τ).

Proof: Let B be an VαOS and A be an VRCS. Then B ⊆ int(cl(int(B))) and cl(int(A)) = A. Therefore A ∪ B ⊆ A ∪int(cl(int(B))) = cl(int(A))∪int(cl(int(B)))⊆cl(int(A))∪ cl(int(B)) ⊆ cl(int(A ∪ B)). We have A ∪ B ⊆ cl(int(A ∪ B)). Therefore A ∪ B is an VSOS and by Theorem 3.36, A ∪ B is an VγGOS in X.

Theorem 3.44: If an vague set A of an VTS X is both an VCS and an VGOS, then A is an VγGOS in (X, τ).

Proof: Suppose A is both an VCS and an VGOS. Then as A ⊆ A, by hypothesis A ⊆ int(A). But int(A) ⊆ A. Therefore int(A) = A. We have A is an VOS, since every VOS is an VγGOS. Hence A is an VγGOS in X.

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