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Dust acoustic solitary waves in a dusty plasma with degenerate electrons and kappa distributed ions

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Abstract
The non-linear propagation of dust-acoustic (DA) waves in a unmagnetized dusty plasma with finite temperature degeneracy of inertialess electrons and kappa distribution ions is studied. Using the standard reductive perturbation technique, a nonlinear Korteweg-de-Vries (KdV) like equation is derived. The effects of kappa-distributed ions and the parameter $T = \frac{T_d}{T_e}$ on the profiles of the amplitudes and widths of the solitary structures are examined numerically. It is found that k plays a significant role in the formation of bright solitons and dark solitons.

Keywords: Dust-acoustic (DA) waves, Degenerate plasmas, Solitary waves, KdV equation

I.INTRODUCTION

The dynamics of dusty plasma has been studied for several decades both theoretically and experimentally. The dusty plasma occur naturally in outer space environments such as asteroids zones, planetary ring system, cometary tails, interstellar clouds and lower parts of Earth's ionosphere etc. as well as in laboratory and technological studies [1-4].

Recently, physicists are observed in the dusty plasma system [5-7] using the properties of wave dynamics, namely, dust-acoustic (DA) waves in understanding electrostatic density perturbations and potential structure. The non-linear properties of DAWs arise due to the inertia of the dust mass and restoring force which are provided by the thermal pressure of ions and electron.

In this paper, we investigate non-linear propagation of dust-acoustic (DA) waves in a unmagnetized dusty plasma with finite temperature degeneracy of inertialess electrons and kappa distribution ions. By the standard reductive perturbation technique, a nonlinear Korteweg-de-Vries (KdV) like equation is derived. The effects of kappa-distributed ions and the parameter $T = \frac{T_d}{T_e}$ on the proles of the amplitudes and widths of the solitary structures are examined numerically. It is found that k plays a significant role in the formation of bright solitons and dark solitons.

II. BASIC EQUATIONS AND DERIVATION OF THE KdV

A. Basic equations

We consider the non-linear propagation of dust-acoustic (DA) waves in a unmag-netized dusty plasma with finite temperature degeneracy of inertia less electrons and ions obeying Kappa velocity distribution. In degenerate regime, the electron density n_e and the scalar pressure p_e can be obtained using the Fermi-Dirac distribution [8-9]

$$n_e = \frac{Li_{3/2}[-\exp(\xi_{\mu e})]}{Li_{3/2}[-\exp(\xi_{\mu e0})]}, \tag{1}$$

$$p_e = \frac{Li_{5/2}[-\exp(\xi_{\mu e})]}{Li_{3/2}[-\exp(\xi_{\mu e0})]}, \tag{2}$$

Where $Li_\nu(-z)$ is the polylogarithm function with index ν. Also $\xi_{\mu e} = \frac{\mu_e}{K_B T_e}$ and $\xi_{\mu e0} = \frac{\mu_{e0}}{K_B T_e}$ are the degenerate parameters corresponding to the perturbed and unperturbed chemical potential μ_e and μ_{e0} respectively.

The basic set of normalized equations governing the dynamics of DA waves consist of the dust continuity and momentum balance equation, the momentum equations for inertia less electrons and the Poisson equation. These are

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \tag{3}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{\partial \phi}{\partial x} - \frac{5}{3} T n_d^{-1/3} \frac{\partial n_d}{\partial x}, \tag{4}$$

$$0 = \frac{\partial \phi}{\partial x} - \nu \frac{Li_{3/2}[-\exp(\xi_{\mu e0})]}{Li_{3/2}[-\exp(\xi_{\mu e})]} \frac{\partial n_e}{\partial x}, \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \delta n_e - \mu n_i + n_d, \tag{6}$$

Here n_d is the dust number density normalized by its unperturbed value n_{d0}, u_d is the dust velocity normalized by the DA speed $c_d = \sqrt{\frac{Z_d k_B T_e}{m_d}}$ with Te denoting the electron temperature, k_B the Boltzmann constant and Z_d the charge dust state i.e., the number of electrons/ions residing on the dust grain surface. Also ϕ is the electrostatic wave potential normalized by $\frac{m_d c_d^2}{e Z_d}$ where e is the magnitude of the electron charge, $T = \frac{T_d}{T_e}$ with T_d denoting the dust temperature. The time and space variable are respectively, in the units of the dust plasma period $\omega_{pd}^{-1} = \sqrt{\frac{m_d}{4\pi\, n_{do}\, Z^2 e^2}}$ and Debye length $\lambda_D = \sqrt{\frac{k_B T_e}{4\pi\, n_{do}\, Z^2 e^2}}$, At equilibrium the charge neutrality condition reads

$$\mu \equiv \frac{n_{i0}}{Z_d n_{d0}} = 1 + \delta \equiv 1 + \frac{n_{e0}}{Z_d n_{d0}}$$

and

$$\nu = \frac{Li_{1/2}[-\exp(\xi_{\mu e0})]}{Li_{3/2}[-\exp(\xi_{\mu e0})]}.$$

The number density for the ion can be written in dimensionless form as [10]

$$n_i = \left(1 - \frac{\phi}{k - 3/2}\right)^{-k+1/2}, \qquad (7)$$

B. Derivation of the KdV equations

To derive the KdV equations we use the stretched coordinates

$$\xi = \epsilon^{\frac{1}{2}}(x - v_0 t), \tau = \epsilon^{\frac{3}{2}} t, \qquad (8)$$

The dependent variables are expanded as follows:

$$n_\alpha = 1 + \epsilon n_{\alpha 1} + \epsilon^2 n_{\alpha 2} + \ldots\ldots\ldots, \quad \alpha = e, i, d$$
$$u_d = \epsilon u_{d1} + \epsilon^2 u_{d2} + \ldots\ldots\ldots\ldots, \qquad (9)$$
$$\phi = 1 + \epsilon\phi_1 + \epsilon^2\phi_2 + \epsilon^3\phi_3 + \ldots\ldots\ldots$$

and

$$\xi_{\mu e} = \xi_{\mu e0} + \epsilon\xi_{\mu e1} + \epsilon^2\xi_{\mu e2} + \ldots\ldots\ldots\ldots, \quad ,$$

Where ϵ is a small parameter measuring the weakness of the dispersion and v_0 is the phase velocity of the DAWs.
Substituting Eqs. (9) into Eqs. (3)- (7), we obtained from the lowest order in ϵ,

$$n_{d1} = \frac{3}{5T - 3v_0^2}\phi_1, n_{e1} = \phi_1, n_{i1} = \beta_1\phi_1$$

$$u_{d1} = \frac{3v_0}{5T - 3v_0^2}\phi_1$$

where $\beta_1 = \frac{k-1/2}{k-3/2}$ and $v_0 = \pm\sqrt{\frac{5T}{3} + \frac{1}{\delta-\mu\beta_1}}$

To next higher order in ϵ, we obtain a set of equations

$$\frac{\partial n_{d1}}{\partial \tau} - v_0 \frac{\partial n_{d2}}{\partial \xi} + \frac{\partial v_{i2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{i1} v_{i1}) = 0, \tag{10}$$

$$\frac{\partial u_{d1}}{\partial \tau} - v_0 \frac{\partial u_{d2}}{\partial \xi} + u_{d1}\frac{\partial u_{d1}}{\partial \xi} = \frac{\partial \phi_2}{\partial \xi} - \frac{5T}{3}\left(\frac{\partial n_{d2}}{\partial \xi} - \frac{n_{i1}}{2}\frac{\partial n_{d1}}{\partial \xi}\right), \tag{11}$$

$$\frac{\partial n_{e2}}{\partial \xi} = \frac{\partial \phi_2}{\partial \xi} + \xi_{\mu e1}\frac{Li_{-1/2}[-\exp(\xi_{\mu e})]}{Li_{1/2}[-\exp(\xi_{\mu e0})]}\frac{\partial \phi_1}{\partial \xi}, \tag{12}$$

$$n_{i2} = \beta_1\phi_2 + \beta_2\phi_1^2, \tag{13}$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \delta n_{e2} - \mu n_{i2} + n_{d2}, \tag{14}$$

where

$$\beta_2 = \frac{k^2 - 1/4}{2(k-3/2)^2}$$

Combining Eqs. (10)- (14), we get a KdV equation

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0, \tag{15}$$

Where

$$A = \frac{(3v_0^2 - 5T)^2}{18v_0}\left[\frac{3(18v_0 + 9v_0^2 - 5T)}{(5T - 3v_0^2)^3} - \delta\alpha_1 + 2\mu\beta_2\right]$$

$$B = \frac{(3v_0^2 - 5T)^2}{18v_0}$$

and

$$\alpha_1 = \frac{Li_{3/2}[-\exp(\xi_{\mu e0})]Li_{-1/2}[-\exp(\xi_{\mu e0})]}{Li_{1/2}[-\exp(\xi_{\mu e0})]^2}$$

The equation (15) has a solitary wave solution for a moving frame with a speed u_0

$$\phi_1 = \frac{3u_0}{A} sech^2 \left[\sqrt{\frac{u_0}{4B}} (\xi - u_0\tau) \right], \qquad (16)$$

where u_0 is the speed of the soliton and $\phi_m = \frac{3u_0}{A}$is the amplitude and $w = \sqrt{\frac{4B_0}{u_0}}$ is the width of the soliton. So, the DASWs are bright or dark soliton according to A/B > 0 or A/B < 0.

III. Results and Discussion

In this section, we numerically investigate the parametric dependence of DASWs, given by Eq. (15). The effects of k and T on the profile of amplitudes and widths of the solution (16) are shown in Figs. 1, 2 and 3 respectively. Fig. 1 and Fig. 2 show that the amplitude and width of the solitary waves increases as the value of k increases. Fig. 3 shows that the amplitude and width of the solitary waves decreases as the value of T increases. Numerical simulation reveals that there exist a critical value of k (~ 0.2) at which the soliton formation collapses and below or above which the bright soliton (Fig. 4) or dark (Fig. 5) soliton appears. Note that in Fig. 4 and Fig. 5 , we have shown that the plot of the solution (16) against ξ and τ for different values of the parameter k and T.

IV.Conclusion

In this paper, we have studied the non-linear propagation of dust-acoustic (DA) waves in a unmagnetized dusty plasma with finite temperature degeneracy of inertialess electrons and kappa distribution ions. By the standard reductive perturbation technique, a nonlinear Korteweg-de-Vries (KdV) like equation is derived. The influence of k distributed ions on the properties of DASWs has investigated. It is found that amplitude of soliton increases with increasing k. The influence of k distributed ions play a dominant role in the formation of both bright and dark solitons. We believe that the novel solitary structures and the properties of the solitons would be helpful for better understanding the nonlinear features and propagation characteristics of dust acoustic (DA)waves in astrophysical setting (e. g. superdense neutron stars and white dwarfs) as well as in laboratory plasmas.

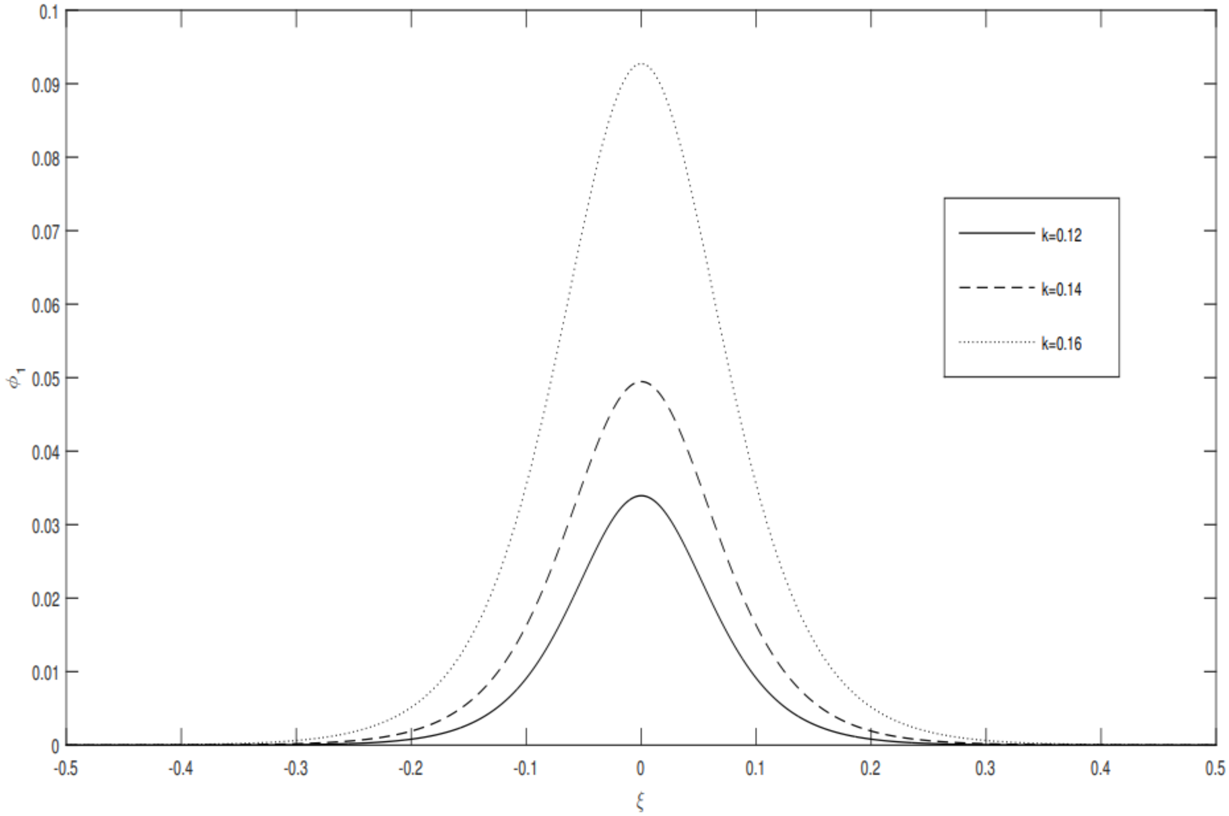

Figure 1: Plot of ϕ_1 against ξ for different values of k for the solution (16).

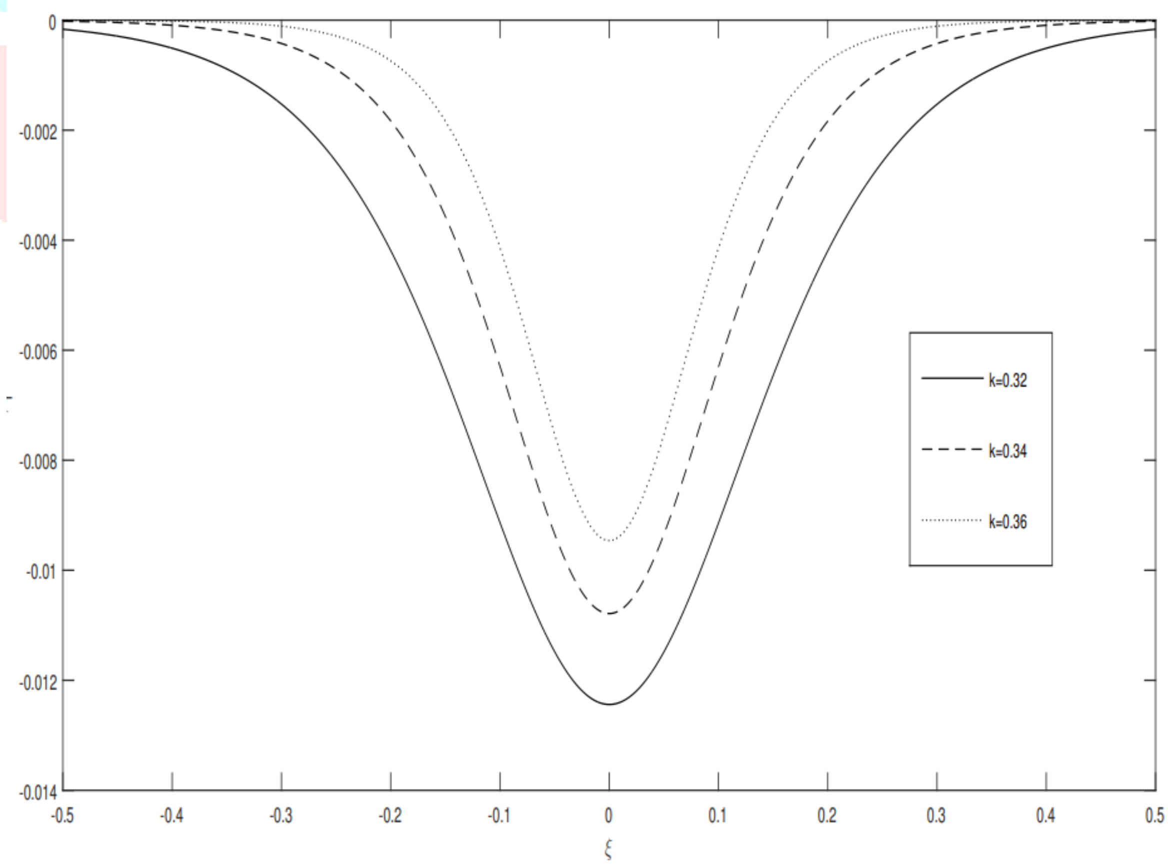

Figure 2: Plot of ϕ_1 against ξ for different values of k for the solution (16).

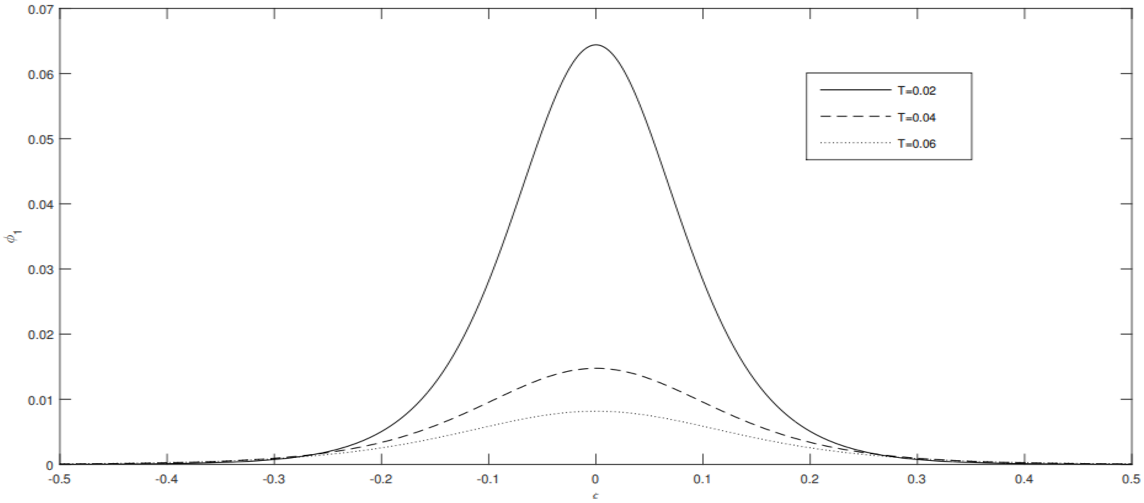

Figure 3: Plot of ϕ_1 against ξ for different values of T for the solution (16).

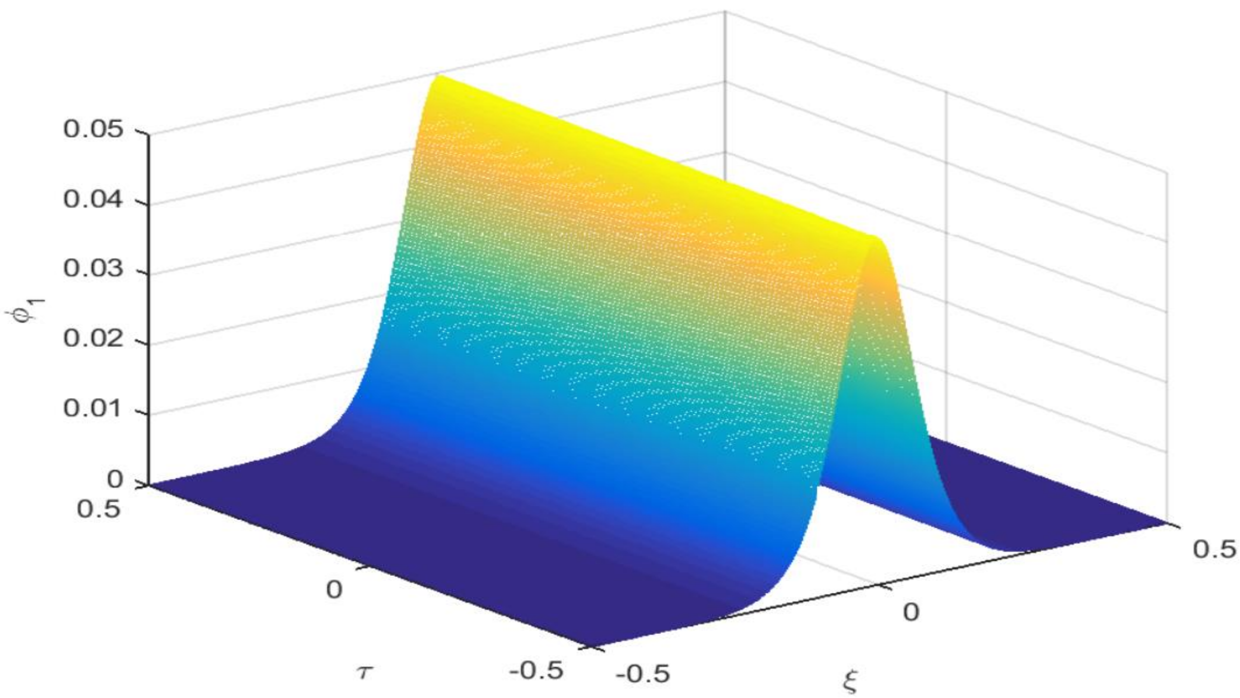

Figure 4: The solitary wave solution for Eq. (15) with parameters k = 0.14 and T = 0.02.

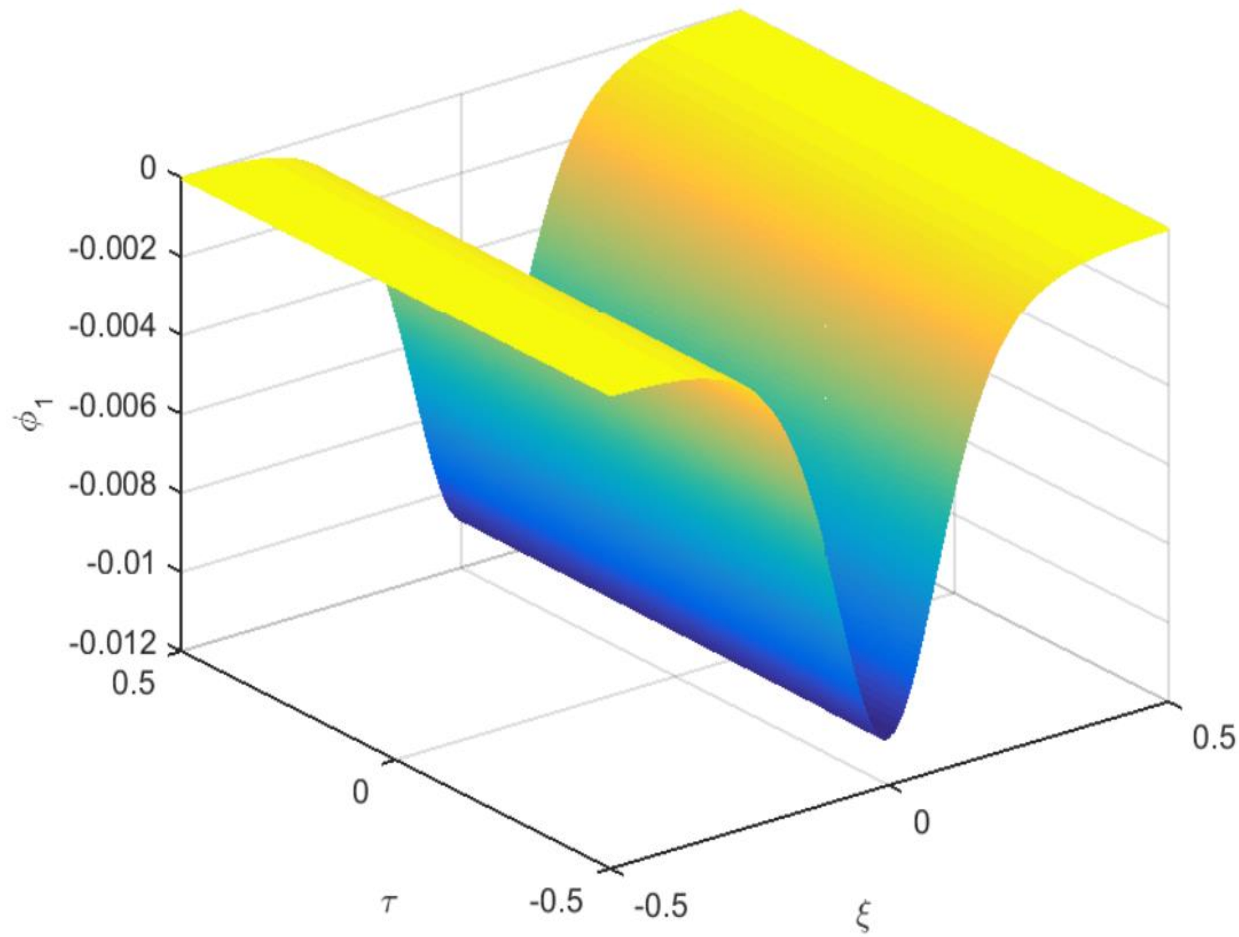

Figure 5: The solitary wave solution for Eq. (15) with parameters k = 0.34 and T = 0.02.

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