Star Generalized Alpha Closed Sets In Pythagorean Fuzzy Topological Spaces

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Abstract: In this paper a Pythagorean Fuzzy star generalized \( \alpha \)-closed sets and a Pythagorean Fuzzy star generalized \( \alpha \)-open sets are introduced. Some of its properties are also analyzed. Also we have provided some applications of Pythagorean Fuzzy star generalized \( \alpha \)-closed sets namely Pythagorean Fuzzy \( T_{1/2} \) space and Pythagorean Fuzzy \( \ast_{\alpha}T_{1/2} \) space.

Key Words: Pythagorean Fuzzy topology, Pythagorean Fuzzy star generalized alpha closed sets, Pythagorean Fuzzy star generalized alpha open sets, Pythagorean Fuzzy \( T_{1/2} \) space and Pythagorean Fuzzy \( \ast_{\alpha}T_{1/2} \) space.

1. INTRODUCTION

The concept of Fuzzy sets was introduced by Zadeh in 1965. After the introduction of Intuitionistic Fuzzy set by Atanassov in 1986, R.R. Yager initiated Intuitionistic Fuzzy set and presented a new set called Pythagorean Fuzzy set. In 1991, A.S. Binsahan introduced and investigated the notions of Fuzzy pre-open and Fuzzy pre-closed sets, In 2003, T. Fukutake, R.K. Saraf, M. Caldas and S. Mishra introduced generalized pre-closed Fuzzy sets in Fuzzy topological space, P. Rajarajeswari and L. Senthil Kumar introduced generalized pre-closed sets and Intuitionistic Fuzzy topological spaces. In this paper we have introduced Pythagorean Fuzzy star generalized \( \alpha \)-closed sets and some of its characterizations are discussed.

2. PRELIMINARIES

**Definition 2.1:** A Pythagorean Fuzzy set (PFS in short) \( A \) in \( X \) is an object having the form \( A = \langle a, \lambda_A(a), \mu_A(a) \rangle / a \in X \) where the functions \( \lambda_A(a) : X \rightarrow [0,1] \) and \( \mu_A(a) : X \rightarrow [0,1] \) denote the degree of membership (namely \( \lambda_A(a) \)) and the degree of non-membership (namely \( \mu_A(a) \)) of each element \( a \in X \) to set \( A \) respectively,

\[
0 \leq \lambda_A(a)^2 + \mu_A(a)^2 \leq 1 \text{ for each } a \in X.
\]

**Definition 2.2:** Let \( A \) and \( B \) be PFSs of the form \( A = \langle a, \lambda_A(a), \mu_A(a) \rangle / a \in X \) and \( B = \langle a, \lambda_B(a), \mu_B(a) \rangle / a \in X \). Then

1) \( A \subseteq B \) if and only if \( \lambda_A(a) \leq \lambda_B(a) \) and \( \mu_A(a) \geq \mu_B(a) \) for all \( a \in X \)
2) \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \)
3) \( A^c = \{ a, \mu_A(a), \lambda_A(a) \} / a \in X \}
4) \( A \cap B = \{ a, \lambda_A(a) \wedge \lambda_B(a), \mu_A(a) \vee \mu_B(a) \} / a \in X \}
5) \[ A \cup B = \{ a, \lambda_A(a) \lor \lambda_B(a), \mu_A(a) \land \mu_B(a) \} / a \in X \]  

For the sake of simplicity, we shall use the notation \( A = \{ a, \lambda_A(a), \mu_A(a) > / a \in X \} \). The Pythagorean Fuzzy sets \( 0 = \{ a, 0,1 > / a \in X \} \) and \( 1 = \{ a,0,1 > / a \in X \} \) are respectively the empty set and the whole set of \( X \).

**Definition 2.3:** A Pythagorean Fuzzy topology (PFTS in short) by subsets of a non-empty set \( X \) is a family of Pythagorean Fuzzy sets satisfying the following axioms.

1) \( 0, 1 \in \tau \)
2) \( G_1 \cap G_2 \in \tau \) for every \( G_1, G_2 \)
3) \( \cup G_i \) for any arbitrary family \( \{ G_i \} \in J \)

In this case the pair \((X, \tau)\) is called a Pythagorean Fuzzy topological space (PFTS in short) and any Pythagorean Fuzzy set \( G \) in \( \tau \) is called a Pythagorean Fuzzy open set (PFOS in short) in \( X \). The complement \( A^C \) of a Pythagorean Fuzzy open set \( A \) in a Pythagorean Fuzzy topological space \((X, \tau)\) is called a Pythagorean Fuzzy closed set (PFCS in short).

**Definition 2.4:** Let \((X, \tau)\) be a PFTS and \( A = \{ a, \lambda_A(a), \mu_A(a) > / a \in X \} \) be Pythagorean Fuzzy set in \( X \). Then the interior and the closure of \( A \) are denoted by PFint\((A)\) and PFcl\((A)\) and are defined as follows.

\[ \text{PFint}(A) = \bigcup \{ G | G \text{ is a PFOS in } X \text{ and } G \subseteq A \} \]
\[ \text{PFcl}(A) = \bigcap \{ K | K \text{ is a PFCS in } X \text{ and } A \subseteq K \} \]

Also, it can be established that PFcl\((A)\) is a PFCS if and only if PFcl\((A)\) = \( A \) and PFint\((A)\) is a PFOS if and only if PFint\((A)\) = \( A \). We say that \( A \) is PF-dense if PFcl\((A)\) = \( X \).

**Definition 2.5:** A Pythagorean Fuzzy set \( A = \{ a, \lambda_A(a), \mu_A(a) > / a \in X \} \) in a Pythagorean Fuzzy topological space \((X, \tau)\) is said to be a

- Pythagorean Fuzzy semi closed set (PFSCS in short) if PFint\((PFcl(A)) \subseteq A\)
- Pythagorean Fuzzy semi open set (PFOS in short) if \( A \subseteq \text{PFcl}(\text{PFint}(A))\)
- Pythagorean Fuzzy a-closed set (PFαCS in short) if PFcl\((PFαOS(A)) \subseteq A\)
- Pythagorean Fuzzy a-open set (PFαOS in short) if \( A \subseteq \text{PFcl}(\text{PFαint}(A))\)
- Pythagorean Fuzzy β-closed set (PFβCS in short) if PFcl\((PFβOS(A)) \subseteq A\)
- Pythagorean Fuzzy β-open set (PFβOS in short) if \( A \subseteq \text{PFcl}(\text{PFβint}(A))\)

**Definition 2.6:** Let \( A \) be a PFS of a PFTS \((X, \tau)\). Then the Pythagorean Fuzzy semi-interior of \( A \) (PFsint\((A)\) in short) and the Pythagorean Fuzzy semi-closure of \( A \) (PFsc\((A)\) in short) is defined as

\[ \text{PFsint}(A) = \bigcup \{ K | K \text{ is an PFOS in } X \text{ and } K \subseteq A \} \]
\[ \text{PFsc}(A) = \bigcap \{ K | K \text{ is an PFCS in } X \text{ and } A \subseteq K \} \]

**Definition 2.7:** Let \( A \) be a PFS in \((X, \tau)\), then

1) \( \text{PFsc}(A) = A \cup \text{PFcl}(\text{PFint}(A)) \)
2) \( \text{PFsint}(A) = A \cap \text{PFcl}(\text{PFint}(A)) \)

**Definition 2.8:** A PFS \( A = \{ a, \lambda_A(a), \mu_A(a) > / a \in X \} \) is said to be a

- Pythagorean Fuzzy regular open set (PFROS) if \( A = \text{PFint}(\text{PFcl}(A)) \)
- Pythagorean Fuzzy regular closed set (PFRCS) if \( A = \text{PFcl}(\text{PFint}(A)) \)

**Definition 2.9:** A PFS \( A \) of a PFTS \((X, \tau)\) is a Pythagorean Fuzzy generalized closed set (PFGCS in short) if \( \text{PFcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is a PFOS in \( X \).

**Definition 2.10:** Let a PFS \( A \) of a PFTS \((X, \tau)\). Then the Pythagorean Fuzzy β closure of \( A \) (PFβcl in short) and the Pythagorean Fuzzy β interior of \( A \) (PFβint in short) is defined as

\[ \text{PFβcl}(A) = \{ K | K \text{ is a Pythagorean Fuzzy β closed set in } X \text{ and } A \subseteq K \} \]
\[ \text{PFβint}(A) = \{ K | K \text{ is a Pythagorean Fuzzy β open set in } X \text{ and } K \subseteq A \} \]
Definition 2.11: Let A be a PFS in (X,τ), then
1) PFβcl(A) = A ∪ PFcl(PFint(PFcl(A)))
2) PFβint(A) = A ∩ PFint(PFcl(PFcl(A)))

Definition 2.12: A PFS A of a PFTS (X,τ) is said to be a Pythagorean Fuzzy β generalized closed set (PFβGCS in short) if PFβcl(A) ⊆ U whenever A ⊆ U and U is a PFOS in X.

Definition 2.13: Let (X,τ) be a PFTS and A =< a,λA(a),μA(a) > be a PFS in X. The α-interior of A is denoted by PFαint(A) and is defined by the union of all Fuzzy α-open sets of X which are contained in A. The intersection of all Fuzzy α-closed sets containing A is called the α-closure of A and is denoted by (PFαcl(A)).
PFαint(A) = U {G | G is a Pythagorean Fuzzy α-open set in X and G ⊆ A}
PFAcl(A) = ∩ {K | K is a Pythagorean Fuzzy α-closed set in X and A ⊆ K}

Definition 2.14: If A is a PFS in X, then PFAcl(A) = A ∪ PFAcl(PFαint(A)).
Definition 2.15: If A is a PFS in (X,τ), we have X − PFAcl(A) = PFAcl(X − A) and X − PFAcl(A) = PFAint(X − A).

3. PYTHAGOREAN FUZZY STAR GENERALIZED ALPHA CLOSED SETS

Definition 3.1: A PFS A is said to be Pythagorean Fuzzy star generalized α-closed set (PF*GaCS in short) in (X,τ) if PF*Gacl(A) ⊆ U whenever A ⊆ U and U is a PFαOS in X. The family of all PF*GaCSs of an PFTS (X,τ) is denoted by PF*GaCS(X).

Example 3.2: Let X = {a,b} and let τ = {0,U,1} be a Pythagorean Fuzzy topology on X, where U = {< a,0.3,0.7 >, < b,0.4,0.6 >}. Then the Pythagorean Fuzzy set A={< a,0.3,0.7 >, < b,0.3,0.6 >} is a Pythagorean Fuzzy star generalized α-closed set in X.

Theorem 3.3: Every PFCS is a PF*GaCS but not conversely.
Proof: Let A be a PFCS in X and let A ⊆ U and U is an PFαOS in (X,τ). Since PFαcl(A) ⊆ PFcl(A) and A is a PFCS in X, PFαcl(A) ⊆ PFcl(A) = A ⊆ U. Therefore A is a PF*GaCS in X.

Example 3.4: Let X = {a,b} and let τ = {0, U, 1} be a PFT on X, where U = {< a,0.3,0.7 >, < b, 0.4, 0.6 >}. Then the PFS A = {< a, 0.3, 0.7 >, < b, 0.4, 0.6 >} is a PF*GaCS in X but not an PFCS in X.

Theorem 3.5: Every PFβCS is a PF*GaCS but not conversely.
Proof: Let A be a PFβCS in X and let A ⊆ U and U is a PFαOS in (X,τ). By hypothesis, PFβcl(PFβint(PFcl(A))) ⊆ A. Since A ⊆ PFcl(A), PFβcl(PFβint(PFcl(A))) ⊆ PFcl(PFβint(PFcl(A))) ⊆ A. Hence PFαcl(A) ⊆ A ⊆ U. Therefore A is a PF*GaCS in X.

Example 3.6: Let X = {a,b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.3, 0.7 >, < b, 0.4, 0.6 >}. Then the PFS A = {< a, 0.2, 0.8 >, < b, 0.2, 0.7 >} is a PF*GaCS in X but not a PFβCS in X since P F cl(P F int(P F cl(A))) = {< a, 0.6, 0.4 >, < a, 0.7, 0.2 >} ⊈ A.

Theorem 3.7: Every PFβGS is a PF*GaCS but not conversely.
Proof: Let A be a PFβGS in X and let A ⊆ U and U is a PFαOS in (X,τ). By hypothesis, PFβcl(PFβint(PFcl(A))) ⊆ A. Since A ⊆ PFcl(A), PFβcl(PFβint(PFcl(A))) ⊆ PFcl(PFβint(PFcl(A))) ⊆ A. Hence PFαcl(A) ⊆ A ⊆ U. Therefore A is a PF*GaCS in X.

Example 3.8: Let X = {a,b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.3, 0.7 >, < b, 0.4, 0.6 >}. Then the PFS A = {< a, 0.2, 0.8 >, < b, 0.3, 0.7 >} is a PF*GaCS in X but not a PFβGS in X since A ⊆ U But PFcl(A) = {< a, 0.7, 0.3 >, < b, 0.6, 0.4 >} ⊈ U.
**Theorem 3.9:** Every PFRC is a PF*GaCS but not conversely.

Proof: Let A be a PFRC in X. Then we know that A = PFcl(PFint(A)). This implies PFcl(A) = PFcl(PFint(A)). Then, PFcl(A) = A. Therefore, A is a PFCS in X. Hence we know that, A is a PF*GaCS in X.

**Example 3.10:** Let X = {a,b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.7, 0.3 >,< b, 0.8, 0.1 >}. Then the PFS A = {< a, 0.4, 0.6 >,< b, 0.3, 0.7 >} is a PF*GaCS but not a PFRC in X since PFcl(PFint(A)) = 0 ≠ A.

**Theorem 3.11:** Every PFαCS is a PF*GaCS but not conversely.

Proof: Let A be a PF*GaCS in X and let A ⊆ U and U is a PFαOS in (X, τ). We know that, PFcl(PFint(A)) ⊆ A. This implies PFαcl(A) = A ∪ PFcl(PFint(A)) ⊆ A Therefore, PFαcl(A) ⊆ U. Hence, A is a PF*GaCS in X.

**Example 3.12:** Let X = {a,b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.6, 0.4 >,< b, 0.5, 0.5 >}. Then the PFS A = {< a, 0.5, 0.5 >,< b, 0.4, 0.6 >} is a PF*GaCS but not a PFαCS in X since PFcl(PFint(A)) = 1 ⊄ A.

**Theorem 3.13:** Every PFβGCS is a PF*GaCS but not conversely.

Proof: Let A be a PFβCS in X and let A ⊆ U and U is a PFαOS in (X, τ). We know that, A ∪ PFcl(PFint(PFcl(A))) ⊆ U. This implies PFcl(PFint(PFcl(A))) ⊆ U and PFcl(PFint(A)) ⊆ U. Therefore PFβcl(A) = A ∪ PFcl(PFint(A)) ⊆ U. Hence A is a PF*GaCS in X.

**Example 3.14:** Let X = {a,b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.6, 0.4 >,< b, 0.3, 0.7 >}. Then the PFS A = {< a, 0.6, 0.4 >,< b, 0.3, 0.7 >} is a PF*GaCS but not a PFβGCS in X since (PFβcl(A)) = 1 ⊄ U.

**Remark 3.15:** Pythagorean Fuzzy semi-closed set and PF*GaCS are independent to each other.

**Example 3.16:** Let X = {a,b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.2, 0.8 >,< b, 0.5, 0.5 >}. Then the PFS A = U is a PFSCS but not a PF*GaCS in X since A ⊆ U but PFαcl(A) = {< a, 0.8, 0.2 >,< b, 0.5, 0.5 >} ⊄ U.

**Example 3.17:** Let X = {a,b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.7, 0.3 >,< b, 0.4, 0.6 >}. Then the PFS A = {< a, 0.6, 0.4 >,< b, 0.3, 0.7 >} is a PF*GaCS but not a PFSCS in X since PFint(PFcl(A)) ⊄ A.

**Remark 3.18:** PFSCS and PF*GaCS are independent to each other.

**Example 3.19:** Let X = {a,b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.1, 0.9 >,< b, 0.5, 0.5 >}. Then the PFS A = U is a PFSCS but not a PF*GaCS in X since A ⊆ U but PFαcl(A) = {< a, 0.9, 0.1 >,< b, 0.5, 0.5 >} ⊄ U.

**Example 3.20:** Let X = {a,b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.8, 0.2 >,< b, 0.6, 0.4 >}. Then the PFS A = {< a, 0.4, 0.6 >,< b, 0.3, 0.7 >} is a PF*GaCS but not a PFSCS in X since PFscl(A) = 1 ⊄ U.
Remark 3.21: From the above theorems and examples we have the following implications.

\[
\begin{array}{ccc}
\text{PFGCS} & \text{PFGCS} & \text{PFGCS} \\
\text{PFSCS} & \text{PFSCS} & \text{PFSCS} \\
\text{PF\alpha CS} & \text{PF\beta CS} & \text{PF\beta CS} \\
\text{PF\beta CS} & \text{PF\beta CS} & \text{PF\beta CS} \\
\text{PFRCS} & \text{PFRCS} & \text{PFRCS}
\end{array}
\]

In this diagram by "A → B" we mean A implies B, "A ↔ B" means B does not imply A and "A ↔ B" means A and B are independent of each other. None of them is reversible.

Remark 3.22: The union of any two PF*G\alpha CSs is not a PF*G\alpha CS in general as seen in the following example.

Example 3.23: Let X = \{a, b\} be a PFTS and let \(\tau = \{0, U, 1\}\) be a PFT on X, where \(U = \{< a, 0.7, 0.3 >, < b, 0.9, 0.1 >\}\). Then the PFSs \(A = \{< a, 0.2, 0.8 >, < b, 0.9, 0.1 >\}\) and \(B = \{< a, 0.7, 0.3 >, < b, 0.8, 0.2 >\}\) are PF*G\alpha CSs but \(A \cup B\) is not a PF*G\alpha CS in X.

4. PYTHAGOREAN FUZZY STAR GENERALIZED ALPHA OPEN SETS

Definition 4.1: A PFS A is said to be a PF*G\alpha OS in \((X, \tau)\) if the complement \(A^c\) is a PF*G\alpha CS in X. The family of all PF*G\alpha OSs of a PFTS \((X, \tau)\) is denoted by PF*G\alpha O(X).

Example 4.2: Let X = \{a, b\} and let \(\tau = \{0, U, 1\}\) be a PFT on X, where \(U = \{< a, 0.8, 0.1 >, < b, 0.7, 0.2 >\}\). Then the PFS \(A = \{< a, 0.9, 0.1 >, < b, 0.8, 0.2 >\}\) is a PF*G\alpha OS in X.

Theorem 4.3: For any PFTS \((X, \tau)\), we have the following:

(i) Every PFOS is a PF*G\alpha OS.
(ii) Every PF\alpha OS is a PF*G\alpha OS.
(iii) Every PF\beta OS is a PF*G\alpha OS.
(iv) Every PF\alpha OS is a PF*G\alpha OS.

Proof: It is obvious.

Remark 4.4: The converse of the above statements need not be true which can be seen from the following examples.

Example 4.5: Let X = \{a, b\} and let \(\tau = \{0, U, 1\}\) be a PFT on X, where \(U = \{< a, 0.4, 0.6 >, < b, 0.5, 0.5 >\}\). Then the PFS \(A = \{< a, 0.7, 0.3 >, < b, 0.9, 0.1 >\}\) is a PF*G\alpha OS but not a PFOS in X.

Example 4.6: Let X = \{a, b\} and let \(\tau = \{0, U, 1\}\) be a PFT on X, where \(U = \{< a, 0.3, 0.7 >, < b, 0.2, 0.8 >\}\). Then the PFS \(A = \{< a, 0.8, 0.2 >, < b, 0.9, 0.1 >\}\) is a PF*G\alpha OS but not a PF\alpha OS in X.

Example 4.7: Let X = \{a, b\} and let \(\tau = \{0, U, 1\}\) be a PFT on X, where \(U = \{< a, 0.6, 0.4 >, < b, 0.5, 0.5 >\}\). Then the PFS \(A = \{< a, 0.7, 0.3 >, < b, 0.5, 0.5 >\}\) is a PF*G\alpha OS but not a PF\beta OS in X.

Example 4.8: Let X = \{a, b\} and let \(\tau = \{0, U, 1\}\) be a PFT on X, where \(U = \{< a, 0.5, 0.5 >, < b, 0.6, 0.4 >\}\). Then the PFS \(A = \{< a, 0.6, 0.4 >, < b, 0.5, 0.5 >\}\) is a PF*G\alpha OS but not a PF\alpha OS in X.
Remark 4.9: From the above theorem and examples we have the following diagrammatic representation.

![Diagram](image)

In this diagram by "A → B" we mean A implies B and "A ↔ B" means B does not imply A.

**Theorem 4.10:** Let \((X, \tau)\) be a PFTS. If \(A \in PF \ast GaO(X)\) then \(V \subseteq PF_{\text{int}}(PF_{\text{cl}}(A))\) whenever \(V \subseteq A\) and \(V\) is a PFαCS in \(X\).

Proof: Let \(A \in PF \ast GaO(X)\). Then \(A^C\) is a PFαGaCS in \(X\). Therefore \(PF_{\text{cl}}(A^C) \subseteq U\) whenever \(A^C \subseteq U\) and \(U\) is a PFαOS in \(X\). That is \(PF_{\text{cl}}(PF_{\text{int}}(A^C)) \subseteq U\). This implies that, \(U^C \subseteq PF_{\text{int}}(PF_{\text{cl}}(A))\). whenever \(U^C \subseteq A\) and \(U\) is a PFαCS in \(X\). Replacing \(U^C\) by \(V\), we get \(V \subseteq PF_{\text{int}}(PF_{\text{cl}}(A))\) whenever \(V \subseteq A\) and \(V\) is a PFαCS in \(X\).

**Theorem 4.11:** Let \((X, \tau)\) be a PFTS. Then for every \(A \in PF \ast GaO(X)\) and for every \(B \in PFS(X)\), \(PF_{\text{aint}}(A) \subseteq B \subseteq A\) implies \(B \in PF \ast GaO(X)\).

Proof: By hypothesis, \(A^C \subseteq B^C \subseteq (PF_{\text{aint}}(A))^C\). Let \(B^C \subseteq U\) and \(U\) be a PFαOS. Since, \(A^C \subseteq B^C, A^C \subseteq U\). But \(A^C\) is a PFαGaCS, \(PF_{\text{aint}}(A^C) \subseteq U\). Also, \(B^C \subseteq (PF_{\text{aint}}(A))^C = PF_{\text{aint}}(A^C)\). Therefore \(PF_{\text{aint}}(B^C) \subseteq PF_{\text{aint}}(A^C) \subseteq U\). Hence, \(B^C\) is a PFαGaCS, which implies \(B\) is a PFαGaO(X).

**Remark 4.12:** The intersection of any two PFαGaOSs is not a PFαGaOS in general.

**Example 4.13:** Let \(X = \{a, b\}\) and let \(\tau = \{0, U, 1\}\) be a PFTS on \(X\), where \(U = \{< a, 0.6, 0.4 >, < a, 0.8, 0.2 >\}\). Then the PFSs \(A = \{< a, 0.7, 0.3 >, < a, 0.6, 0.4 >\}\) and \(B = \{< a, 0.4, 0.6 >, < a, 0.3, 0.7 >\}\) are PFαGaOSs but \(A \cap B\) is not a PFαOS in \(X\).

**Theorem 4.14:** A PFS \(A\) of a PFTS \((X, \tau)\), is a PFαGaOS if and only if \(F \subseteq PF_{\text{aint}}(A)\) whenever \(F\) is a PFCS and \(F \subseteq A\).

Proof: Necessity: Suppose \(A\) is a PFαGaOS in \(X\). Let \(F\) be a PFCS and \(F \subseteq A\). Then \(F^C\) is a PFOS in \(X\) such that \(A^C \subseteq F^C\). Since \(A^C\) is a PFαGaCS, we have \(PF_{\text{aint}}(A^C) \subseteq F^C\). Hence \((PF_{\text{aint}}(A))^C \subseteq F^C\). Therefore, \(F \subseteq PF_{\text{aint}}(A)\).

Sufficiency: Let \(A\) be a PFS of \(X\) and let \(F \subseteq PF_{\text{aint}}(A)\) whenever \(F\) is a PFCS and \(F \subseteq A\). Then \(A^C \subseteq F^C\) and \(F^C\) is a PFOS. By hypothesis, \((PF_{\text{aint}}(A))^C \subseteq F^C\) which implies \(PF_{\text{aint}}(A^C) \subseteq F^C\). Therefore \(A^C\) is a PFαGaCS of \(X\). Hence \(A\) is a PFαGaO of \(X\).

**Corollary 4.15:** A PFS \(A\) of a PFTS \((X, \tau)\) is a PFαGaOS if and only if \(F \subseteq PF_{\text{aint}}(PF_{\text{cl}}(A))\) whenever \(F\) is a PFCS and \(F \subseteq A\).

Proof: Necessity: Suppose \(A\) is a PFαGaOS in \(X\). Let \(F\) be a PFCS and \(F \subseteq A\). Then \(F^C\) is a PFOS in \(X\) such that \(A^C \subseteq F^C\). Since \(A^C\) is a PFαGaCS, we have \(PF_{\text{aint}}(A^C) \subseteq F^C\). Therefore \(PF_{\text{cl}}(PF_{\text{aint}}(A^C)) \subseteq F^C\). Hence \((PF_{\text{aint}}(PF_{\text{cl}}(A)))^C \subseteq F^C\). Therefore, \(F \subseteq PF_{\text{aint}}(PF_{\text{cl}}(A))\).
Sufficiency: Let A be a PFS of X and let \( F \subseteq \text{PFint}(\text{PFcl}(A)) \) whenever \( F \) is a PFCS and \( F \subseteq A \). Then \( A^C \subseteq F^C \) and \( F^C \) is a PFOS. By hypothesis, \( (\text{PFint}(\text{PFcl}(A)))^C \subseteq F^C \). Hence \( \text{PFcl}(\text{PFint}(A^C)) \subseteq F^C \) which implies, \( \text{PFcl}(A^C) \subseteq F^C \). Hence A is a PF*GαOS of X.

**Theorem 4.16:** For a PFS A, A is a PFOS and a PF*GαCS in X if and only if A is a PFαOS in X.

Proof: Necessity: Let A be a PFOS and a PF*GαCS in X. Then PFαcl(A) \( \subseteq A \). This implies PFcl(PFαcl(A)) \( \subseteq A \). Since, A is a PFOS, it is a PFαOS. Hence A \( \subseteq \text{PFint}(\text{PFcl}(A)) \). Therefore A = PFcl(PFαcl(A)) and hence, A is a PFOS in X.

Sufficiency: Let A be a PFOS in X. Therefore A = PFint(PFcl(A)). Let A \( \subseteq U \) and U is a PFαOS in X. This implies PFαcl(A) \( \subseteq A \) and hence A is a PF*GαCS in X.

5. APPLICATIONS OF PYTHAGOREAN FUZZY GENERALIZED ALPHA CLOSED SETS

**Definition 5.1:** A PFTS \((X, \tau)\) is said to be a PF\(\alpha\)T\(1/2\) space space if every PF*GαCS in X is a PFCS in X.

**Definition 5.2:** A PFTS \((X, \tau)\) is said to be a PF*\(\alpha\)gαT\(1/2\) space (PF*\(\alpha\)gαT\(1/2\) space in short) if every PF*GαCS in X is a PFαCS in X.

**Theorem 5.3:** Every PF\(\alpha\)T\(1/2\) space is a PF*\(\alpha\)gαT\(1/2\) space.

Proof: Let X be a PF\(\alpha\)T\(1/2\) space and A be a PF*GαCS in X. By hypothesis A is a PFCS in X. Since every PFCS set is a PFαCS, A is a PFαCS in X. Hence X is a PF*\(\alpha\)gαT\(1/2\) space.

But the converse need not be true which can be seen in the following example.

**Example 5.4:** Let \( X = \{a,b\} \) and let \( \tau = \{0, U, 1\} \) be a PFT on X, where \( U = \{\leq a,0,8,0,2,>\leq b,0,8,0,2,>\} \). Then \((X, \tau)\) is a PF*\(\alpha\)gαT\(1/2\) space. But it is not a PF\(\alpha\)T\(1/2\) space since the PFS A = \(\{\leq a,0,3,0,7,>\leq b,0,6,0,4,>\}\) is a PF*GαCS but not a PFCS in X.

**Theorem 5.5:** Let \((X, \tau)\) be a Pythagorean Fuzzy topological space and X is a PF\(\alpha\)T\(1/2\) space then

1. Any union of PF*GαCSs is a PF*GαCS.
2. Any intersection of PF*GαCSs is a PF*GαOS.

Proof: 1. Let \(\{A_i\}_{i \in I}\) is a collection of PF*GαCSs in a PF\(\alpha\)T\(1/2\) space \((X, \tau)\). Therefore every PF*GαCS is a PFCS. But the union of PFCS is a PFCS. Hence the union of PF*GαCSs is a PF*GαCS in X.

2. Let \(\{A_i\}_{i \in I}\) is a collection of PF*GαOSs in a PF\(\alpha\)T\(1/2\) space \((X, \tau)\). Therefore every PF*GαOS is a PFOS. But the intersection of PFOS is a PFOS. Hence the intersection of PF*GαOSs is a PF*GαOS in X.

**Theorem 5.6:** A PFTS X is a PF*\(\alpha\)gαT\(1/2\) space if and only if PF \(\ast\) GαO(X) = PFαO(X).

Proof: Necessity: Let A be a PF*GαOS in X, then \(A^C\) is a PF*GαCS in X. By hypothesis, \(A^C\) is a PFαCS in X. Therefore A is a PFαOS in X. Hence PF \(\ast\) GαO(X) = PFαO(X).

Sufficiency: Let A be a PF*GαCS in X. Then \(A^C\) is a PF*GαOS in X. By hypothesis, \(A^C\) is a PFαOS in X. Therefore, A is a PFαCS in X. Hence X is a PF*\(\alpha\)gαT\(1/2\) space.
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