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Construction of Group-Divisible Fourth-Order Slope Rotatable Designs using BIBD and its Applications

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ABSTRACT

The study sheds light on the "Fourth-order rotatable slope design" to evaluate the overall methodology and statistical process to determine the connection between response variables and independent factors. It has been identified that FORSD with BIBD optimises the statistical poutine and constructive insights byminimising the number of experimental runs. The methodology section evaluates the general consequential steps of analysing relevant data through BIBD including statistical analysis and "response surface analysis". The statistical method of obtaining "group-divisible fourth-order rotatable designs" has been evaluated with mathematical expression. The outcome shows an in- depth understanding of the effect of influencing factors on the specific responsevariable. This study is extremely significant as "fourth-order slope design" givesbetter insights into the relationship between DV and IV and can be effectively applied for industrial and research purposes in various sectors.

KEYWORDS: "Fourth-order rotational slope design", BIBD, TORD, response variables, influential factors, "fourth-orderslope design", and "response surface analysis".

1. Introduction

"Fourth-order rotatable slope design" is a complex mathematical and statistical task that involves determining the multiple factors and the interaction between them by designing experiments efficiently. This study aims to give an overview of the fundamental concept, procedures, and methods of "fourth-order rotatable slope design" constructed with "Balanced Incomplete Block Design" (BIBD). As opined by Victorbabu and Jyostna (2022) [2] , "fourth-order rotatable slope designs" are commonly used in experimental designs to analyse response surfaces. It is usually used to identify the relationship between various independent factors or variables to optimise the process in terms of determining an output response. "Fourth-order" reflects the degree of polynomial which is used in the process to model the "response surface". Alternatively, "rotatable slope designs" are those statistical designs that follow specific statistical properties including orthogonality and rotatability relying on the rotation of the design space. "Balanced Incomplete Block Design" plays a pivotal role in combinational designs by developing efficient experimental designs based on tube relationships of various independent factors. BIBD is generally characterised by its different parameters which are described in the belowseduction.

- "v" represents the number of experimental runs (factor levels)
- "r" denotes the treatment number used in every block
- "b" reflects the number of experimental trials or blocks
- "k" represents the block number where every treatment has taken place
- "λ" denotes the positive integer which is the measure of the "degree of balance"

It is important to study these abovementioned parameters to find out the relationship between different independent variables of studies to design a "fourth-order rotatable slope". BIBD assures that each treatment takes place across every block an equal number of times to offer a structured path to analyse the effect of variousfactors on the study [3].

Generally, the "third-order rotatable designs" can be categorised into two segments such as designs in terms of non-sequential experimentation and designs that are useful for sequential experimentation. As per the studies of Hemavathi *et al.* (2022) [1], the approach of sequential experimentation includes the procedure of conducting trials step by step. Contrarily, the approach of non-sequential experimentation runs the overall procedure in one go. Henceforth, responses generated from the trials of non-sequential experimentation become irrelevant if the model lacks fit, which leads to a loss of time and huge resources. The existing TORDs ("third-order rotatable design") are applicable only for non-sequential experiments with systematic factor levels. Maintaining the same number of factors at all levels in sequential experiments in "third-order designs" is difficult. Therefore, the study will shed light on the "fourth-level rotational slope design" through BIBD.

It is essential to determine the levels and factors first to construct a "fourth-order slope" with rotational designs. Factors to be studied are recognised first with their respective levels according to different conditions or settings of independent factors. The selection of suitable BIBD plays a crucial role in satisfying the experimental requirements concerning the number of levels or factors, desired balance level and block size [4]. The extension of the design needs to be taken into serious consideration in a "fourth-order rotatable slope design" so that the BIBD can effectively complete the design by analysing multiple factors in the fourth-level design space. Every treatment level in a BIBD in terms of "fourth-order rotatable slope design" corresponds to a particular combination of independent factors [1]. Here, assigning every factor to suitable treatment or treatments is crucial to ensure the orthogonality and balance of the design. Afterwards, the design matrix is created by using factor assignment and the selected BIBD structure to represent the experimental layout. Each row involves a design matrix associated with experimental runs. Accordingly, columns in the BIBD reflect different independent factors and their rotational levels. It is crucial to conduct additional runs to convert the selected BIBD into a "fourth-order rotatable slope design". Additional points are generated in this stage by rotating all design points by not influencing the desirable properties of the design. Data is collected related to response variables in every experimental run according to the design matrix. This method optimises the outcomes by modelling the relationship between various response factors or variables.

Significance of "fourth-order rotatable slope design" through BIBD

This section is going to focus on the benefits and significance of "fourth-order rotatable slope design" by using BIBD in determining the relationship of multiple variables in statistical research studies. The following are the specific significance of using "fourth-level design" through BIBD:

- ***Optimisation:*** "Fourth-level designs" are ideal for optimising the statistical processes by recognising the most suitable factor settings and design matrix that can lead to the expected outcomes.
- ***Efficiency:*** A high number of experimental runs in BIBD can make the analysis complicated and challenging. Therefore, it is expected to complete the overall statistical process with a minimum number of experimental runs in terms of measuring the numeric relationship between various factors [5]. These fourth-level designs are efficient in studying a wide range of independent variables in combination with the help of a few experimental runs. It optimises the outcome and accuracy of slope designs and outcomes with minimum effort and time investment.
- ***Robustness:*** This "fourth-level rotational" design maximises the validity of the design matrix. The property of rotatability assures the design retains its validity in case of the true optimum displacement from the exact centre of the design space. The "fourth level design" enhances the validity of the design compared to the second and third-level design of rotational slope through BIBD.
- ***Resource savings:*** It also saves resources such as time and costs by minimising the number of experimental runs in selected BIBD.
- ***Statistical rigour:*** The "fourth-order rotatable slope design" provides a systematic and rigorous method of experimentation. It maximises the reliability and validity of the outcomes through data analysis [6]. Henceforth, it can be stated that "fourth-level designs" provide more accurate and valid hypotheses in terms of the relationship between various factors through BIBD analysis.

The study is going to focus on the overall method of the "fourth-order rotatable slope design" including all mathematical and statistical perspectives. Accordingly, the result will be discussed precisely along with its practical implications.

2.Methodology

This section is going to shed light on the overall method of designing a “fourth-order rotatable design of slope” with BIBD. As opined by Magdalene (2020) [6], BIBD is widely used in the statistical field to determine the relationship of dependent factors with all existing independent factors based on responses and intercepts. The “fourth-order rotatable slope design” (FORSD) refers to an experimental and complex specialised design that incorporates a “balanced incomplete block design”. The principles of BIBD are followed to explore the link between response variables and various factors, especially in those statistical measures where experimentation is time-consuming and costly as well. The fundamental goal of FORSD is to optimise the overall BIBD settingsto obtain accurate outcomes by minimising the number of experimental runs. Experimental design helps researchers follow a systematic approach to study the impact of various existing factors on a particular responsevariable. The FORSD is considered a powerful and effective method in statistical fields in terms of optimising processes and systems. The overall method for “fourth-order rotatable slope design” has been generalised here and described in the below section.

Selection of relevant factors: It is essential to recognise specific factors that can impact response variables significantly. These influencing factors may include variables, settings or parameters which are relevant to the investigated statistical problem. It can be time, pressure or temperature within a manufacturing setting which has the potential to influence the outcomes remarkably.

Determination of factor levels: Once completing the selection of influencing factors the next step is to determine the level of testing each identified factor. The selection of factor levels majorly depends on the a wide-range of theoretical and real-time values that a specific recognised factor can adopt. A higher level of rotational slope design provides more in-depth insights into the influence of the selected influencing factors; although it may require a greater number of experiences run in the overall design process [8]. Therefore, the “fourth-order rotatable slope design” can provide a better understanding of the effect of influencing factors on the response-dependent variable compared to the second and third-order rotational design with more experimental runs and expanded response surface of BIBD.

Developing an experimental matrix: In BIBD, an experimental matrix is utilised as the fundamental element for rotational slope design. In FORSD, the experimental matrix combines influencing factors which are tested during experimental runs. It includes the principles of BIBD in terms of comparing and estimating the impact ofthe factor matrix efficiently.

Application of BIBD in "fourth-order slope design": BIBD is widely used in statistics to conduct variance analysis and experimental design. It is effective in finding out the impact of various factors on response variables. The systematic arrangement of BIBD includes factor levels, treatment combination, blocks, replication and orthogonality by reducing the number of experimental runs. Usually, the structured design of BIBD does not need to test each combination of factors. It has been used in the FORSD method to optimise balance efficiencyalong with statistical validity. BIBD is specifically used in "fourth-level rotational slope design" which may include an exceptionally large number of experimental runs to complete the factorial design efficiently throughthe minimum number of runs.

Implementation of experiments: The procedure of the experiment is outlined and initiated with the existing experimental matrix. Thereafter, relevant data is gathered based on every single factor level combination. In this regard, ensuring the conductance of experiments under consistent and controlled conditions is crucial to reduce variability sources.

Statistical analysis: The process of statistical analysis comes after the completion of data collection and experimental runs. It includes the analysis of variance matrix (ANOVA), regression analysis and advanced modelling. Statistical analysis through BIBD helps determine the exact statistical impact of identified factors on the particular response variable.

Analysing response surface: In FORSD, the focus concentrates on depicting the connection between influential factors and the dependent variable in detail. The process involves the inclusion of a "response surface model" to statistically describe the dependent variable as a functional derivative of the identified factors [3]. In this "fourth-order design", non-linear impact and interaction of factors are considered in statistical design through BIBD to get a deeper understanding of the system.

Optimisation: BIBD is used in “fourth-level designs” to fit the “response surface model” into BIB design in terms of getting the optimal factor settings. It helps in maximising as well as minimising response variables in the slope design. It helps in reducing time consumption and cost by ensuring process and quality optimisation.

Iteration and repetition: Generally, the FORSD includes the minimum number of experimental runs for obtaining statistical outcomes. However, sometimes the complex designing method of the “fourth-order design”may require collecting additional data, refining the rotational design and verifying the outcomes to get more detailed insights based on continuous improvement.

Statistical approach for developing GDFORD

The construction of a “fourth-order rotatable slope design” can be developed depending on known BIBD solutions on many occasions. The “group-divisible fourth-order rotatable design” (GDFORD) depends on four BIBD parameters such as $v = 4$, $k = 2$, $r = 3$, and $\lambda = 1$ where $r \geq 3\lambda$. The constriction of the GDFORD process is divided into two-factor groups; the first one is the p-dimension and the second one is the (p-v) dimension where “$p \geq 2$”, and (V-P) is also greater than equals to 2. As per the studies of Hemavathi *et al.* (2020) [8], it is important to write down the transposition of the incidence matrix to start the BIB design. In BIBD, these above-mentioned four factors of the incidence matrix have been taken with an unknown level “a” and 0; here, “a” replaces the value “1” in the above matrix to develop “b” combinations. In this point of design, every “b” combination contains k.a’s and (v-k) number of zeros [7]. Thereafter, the suitably selected point set of “S (c,0,0…..,0)”, “2S (d,0,0…..,0)” and “2S (b,b,b….,b)” are combined together. The combining level is considered$d^2 = tc^2$ where $t \geq 0$ and a unique solution can be evaluated by an assumption of “t” = $(1+2t^2)^3/ (1+2t^3)^2$. This obtained equation satisfies a specific value of $t = 0.4544$ while constructing a four-dimensional “GDFORD”. Allunknown levels are measured by determining moment conditions for “group-divisible fourth-order rotatable design”.

Let's assume that. $\lambda = f(d)$ is the function associated with the radius of the “fourth-level rotatable design”. Here, “d” represents the radius of the design and λ is a scaling parameter. In addition, $d^2 = \sum x_{iu}^2$ with a “lower limit”of (i =1) and an “upper limit” is 4, $d_1^2 = \sum xi^2$ with an “upper limit” of 2 and a “lower limit” of i = 1, and $d_2^2 = \sum xj^2$ with an “upper limit” 4 and “lower limit” j = 3.

(1) “$\sum xi^2 = N\lambda 2$”, for “i = 1, 2” “$\sum xj^2 = N\lambda 2$”, for “j = 3, 4” “$\sum xi^4 = 3N\lambda 4$”, for “i = 1, 2” “$\sum xj^4$ =

3Nλ4", for "j = 3, 4" "$\sum xi^6$ = 15Nλ6", for "i = 1, 2"

"$\sum xj^6$ = 15Nλ6", for "j = 3, 4"

(2) "$\sum xi^2xj^4$ = 5Nλ6", for i ≠ j and for "i = 1,2"; "j = 3, 4""∑j≠j' $xj^2x_{j'}^4$ = 5Nλ6", for "j, j' = 3,4"

"∑i≠i' $xi^2x_{i'}^4$ = 5Nλ6", for "i, i' = 1,2"

(3) "∑∑∑$xi^2xj^2x_k^2$ = Nλ6". for "i ≠ j ≠ k"

All the parameters in the "fourth-order rotation design" have been evaluated in the above section. In both function groups, the moment of all odd numbers has been considered as zero. Two specific non-singularity conditions have been considered in this study for constructing GDFORD.

1. "$\lambda 4 / \lambda_2^2 > k / (k+2)$"
2. "$\lambda 2 \lambda 6 / \lambda_4^2 > (k+2) / (k+4)$"

The abovementioned simulation parameters have been taken into the entire design points.

Method of obtaining "group-divisible fourth-order rotatable designs"

The design points in GDFORD exist in four dimensions mentioned in the above section. The variance sum of the study responses is estimated in any direction of the factor axis in every divisible group of the mutually orthogonal space of BIBD. Any point situated on the "p" dimension or (p-v) dimension must function at a specific distance of d_1^2 and d_2^2 respectively from the origin point of the GDFORD. It has been assumed that "D = ((xik))" is the set of "N" number of factor design points [7]. Accordingly, y1, y2, y3 yN are the number of responses taken into account in the rotating slope design to fit the response surface of "fourth-level rotatable design" through BIBD.

$$
\begin{aligned}
Y(X) = b_0 + \sum b_i x_i + \sum b_j x_j \\
+ \sum\sum b_{ij} x_i x_j \\
+ \sum\sum b_{ii'} x_i x_{i'} \\
+ \sum\sum b_{jj'} x_j x_{j'} + \sum b_{ii} x_i^2 + \sum b_{jj} x_j^2 + \sum b_{iii} x_i^3 + \sum b_{jjj} x_j^3 \\
+ \sum\sum b_{ijj} x_i x_j^2 + \sum\sum b_{ii'i'} x_i x_{i'}^2 + \sum\sum b_{jj'j'} x_j x_{j'}^2 + \sum\sum\sum b_{ijk} x_i x_j x_k + e
\end{aligned}
$$

The above equation evaluates the dependent variable (Y(X)) through a linear combination of multiple independent variables (Xs) and all existing coefficients (b's) and b0 is the intercept of the factor matrix which isused in the equation as a constant. MATLAB software is widely used to construct the FORSD efficiently by taking all factors and DV into account. The basic principles of BIBD, matrix and summations are used to analysedata in order to provide accurate outcomes. It has been observed that the "fourth-order" design follows both thelinear and non-linear combination of factor matrices to measure the impact on response variables with a minimum number of experimental runs.

3. Results and discussion

The four-dimensional GDFORD has been generated with respect to 80 design points through BIB designs. It is essential to determine the matrix moment of the four-dimensional GDFORD to calculate the effect on the dependent factor.

Here, moment matrix (M) = xx′. 1/N (where N = 80; as 80 design points have been taken into account).

Two "factor groups" are formed here based on 80 design points. In this context, v = 4, x1, x2 forms the first "factor group" in the "p" direction while x3, x4 constructs another factor group in the (v-p) direction. A complete "fourth-order" design has been constructed on the basis of the following expression:

"$f^t(x) = [f_1^t(x), f_2^t(x), f_3^t(x), f_4^t(x)]$"

The overall design space of the BIBD has been divided into the “p” factor group and the (v-p) factor group. This division of design space eventually satisfies “$d_1^2 = \sum_{i=1}^{2} x_i^2$” and “$d_2^2 = \sum_{j=3}^{4} x_j^2$”. In addition, this division is correspondence to variances “V([ŷ(xi)])” and “V([ŷ(xj)])” consecutively [7].

The “p”-dimensional space has been taken in this study for constructing a “fourth-order design” with BIBD. MATLAB software has been used here to determine the moment matrix M(ξ). The variance of the “p”-factor group is shown in the below figure.

$$M_{11}(\xi) = \begin{bmatrix} 1 & x_1^2 & x_2^2 \\ & x_1^4 & x_1^2x_2^2 \\ (symm) & & x_2^4 \end{bmatrix} = \begin{bmatrix} 1.0000 & 1.0051 & 1.0051 \\ 1.0051 & 2.1524 & 0.7175 \\ 1.0051 & 0.7175 & 2.1524 \end{bmatrix}.$$

$$M_{22}(\xi) = \begin{bmatrix} 0.7175 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M_{44(1)} = \begin{bmatrix} x_1^2 & x_1^4 & x_1^2x_2^2 \\ & x_1^6 & x_1^4x_2^2 \\ symm & & x_1^2x_2^4 \end{bmatrix} = \begin{bmatrix} \lambda_2 & 3\lambda_4 & \lambda_4 \\ & 15\lambda_6 & 3\lambda_6 \\ symm & & 3\lambda_6 \end{bmatrix} = \begin{bmatrix} 1.0051 & 2.1524 & 0.7175 \\ 2.1524 & 6.0000 & 1.2000 \\ 0.7175 & 1.2000 & 1.2000 \end{bmatrix}$$

The value of any variance can be obtained from the above variances by following the equation below:

“$V_{11}(\xi) = f'_1 [M_{11}(\xi)]^{-1} f_1$”

Further, it can be expressed as “$V_{11}(\xi) = [a + 2bx_1^2 + 2bx_2^2 + cx_1^4 + cx_2^4 + 2dx_1^2x_2^2]$”Hence, “$V_{22}(\xi) = f'_2 [M_{22}(\xi)]^{-1} f_2$”

It can also be expressed as “$V_{22}(\xi) = [nx_1^2 x_2^2]$”

Similarly, it can be determined by following the same way that “V44(1)(ξ) = “V44 (2)(ξ)”.Thus, the response variance “$V([\hat{y}(x_i)]) = \sum_{i=1}^{2} V(x_i)$” $= f(d_1^2)$

Henceforth, it can be said that response variable changes proportionally with the square of influencing factors here. Accordingly, the relationship of any two factors or variables can be convincingly determined by using "fourth-order slope design" with BIBD.

4.Practical application

The "Fourth-order rotational slope design" has been proven extremely effective in optimising the systems or processes by reducing the number of experimental runs and costs. Some practical applications of (FORSD) have been illustrated below.

Manufacturing industry and quality control sector: FORSD is commonly used in the manufacturing industry on the global stage. Material properties, machine and infrastructural settings, processing time and technical improvement are considered the key factors that generally influence the productivity, quality and consistency of products. FORSD with BIBD has become an effective statistical technique to identify the optimal setting with these influential factors to enhance productivity and quality by reducing production time and material waste.

Design and product development: product design and new product development aim to maximise product characteristics according to customer demand [10]. Influential factors such as raw ingredients, sustainable sourcing and consumer preferences are used as factor matrices to design the optimal design with BIBD to produce consumer products with optimal efficiency and features.

Environmental and agricultural fields: The environmental department can take advantage of using the FORSD approach to analyse the impact of multiple environmental factors such as oxygen level, BOD, COD, temperature, and pollution parameters on the ecological setting. It will provide better insights into conservation strategy and effective resource management based on resource surface design and statistical analysis.

Drug and pharmaceutical development: BIBD-based "fourth-level designs" can be beneficial in optimising the research and process of drug formulation. Influential factors such as excipients, the concentration of active drug ingredients and drug manufacturing parameters can influence drug formulation and pharmaceutical development significantly [9]. Therefore, using "fourth-order design" with BIBD can provide the opportunity for researchers to explore ideal formulation and parameter space based on factor matrix.

Social behavioural research: This rotational design can also be utilised in behavioural and social research. This design model can be used to study influential variables like human preference, common behaviour., and decision-making tendencies to articulate effective psychological interventions or guidelines based on accurate statical outcomes.

Energy monitoring sector: In the energy manufacturing and controlling industry, fuel consumption, carbon emissions, and energy production can be taken as specific factors to determine the relationship with response variables such as environmental compliance and operational optimisation [9]. The BIBD-based outcomes can provide the best measures for controlling fuel consumption and environmental pollution by maintaining the same energy production.

Besides this, the food and beverage sector and chemical industries have the potential to use "fourth-order rotational slope design" with BIBD to obtain accurate insights about the impact of influential factors on business through statistical design.

5.Conclusion

The report has shed light on the "fourth-order rotational slope design" with "Balanced Incomplete Block Design". It has been observed that the FORSD can provide better insights into the effect of influential factors onresponsive variables compared to second and third-order designs. However, it comprises a more complex designand a higher number of experimental runs. It becomes comprehensive to determine the relationship of dependentvariables with many moderating factors to determine the most influential factors with accurate statistical value.The fundamental concept of BIBD-based FORSD has been evaluated in the introduction section along with thesignificance of this statistical design. The overall methodology of FORSD has been evaluated in this study by discussing factor levels, experimental matrix, experimental implementation, analysis of response surface, statistical analysis, and optimisation. In addition, the statistical approach of "group-divisible fourth-order rotatable designs" has been evaluated in the methodology section. The statistical outcomes of FORSD based on BIBD have been discussed with relevant mathematical expressions. Finally, the practical implications of this statistical rotational design in various sectors have been illustrated here in this study.

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