



Maximal Certified Dominating Set In Graph

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Abstract: A certified Dominating set D of a Graph $G = (V, D)$ is a maximal certified dominating set. If $V - D$ is not a certified dominating set G . The maximal certified domination $\gamma_{mcer}(G)$ of G is the minimum domination set. In this refer we and observe some results. Maximum Dominating set consist maximum number of possible non-touching vertices in the given graph. Maximal Domination Set consist minimum number of possible non-touching vertices which is adjacent with all the edges in the graph. Various types of Graphs, Structure and their properties also have to be discussed. In this paper, we initiate for study of this parameter.

Keyword: Domination set, certified domination, maximal certified dominations.

1 Introduction

All graph considered here are finite, undirected without loops are multiple edges. A set D of vertices in a graph $G = (V, E)$ is a dominating set of G if every vertex in $V - D$ is adjacent to some vertex D . The domination number $\gamma(G)$ of G is minimum cardinality of a dominating set. For an early survey on $\gamma(G)$, [1]. We follow the notation and terminology of Harary [2]. Let G be a graph with p vertices and q edges.

A dominating set D of a graph $G = (V, E)$ maximal dominating set if $V - D$ is not a dominating set of G . The maximal domination number $\gamma_m(G)$ of G is the minimum cardinality of a maximal dominating set, this concept introduced by Kullay and Janakiram [5]. A dominating set D of G is called certified if every vertex $v \in D$ has either zero or at least two neighbors in $V_G - D$. The cardinality of a minimum certified dominating set in G is called the certified dominating number of G and denoted by $\gamma_{cer}(G)$. Motivation of this parameter we introduced maximal certified domination number of a

graph and defined has follows..

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observe some results. Maximum Dominating set consist maximum number of possible non-touching vertices in the given graph. Maximal Domination Set consists minimum number of possible non-touching vertices which is adjacent with all the edges in the graph. Various types of Graphs, Structure and their properties also have to be discussed. In this paper, we initiate for study of this parameter.

Definition 1.1. Let $G = (V, E)$, where V represents the vertex set of G and E represents the edge set of G . The elements of $V(G)$ are called vertices and the cardinality $|V(G)|$ of V is the order of G . The elements of $E(G)$ are called edges and the cardinality $|E(G)|$ of E is the size of G . The degree of a vertex v , denoted as $deg(v)$, refers to the number of edges incident with v . The maximum degree among all vertices in G is denoted as $\Delta(G)$. The open neighborhood of a vertex u in G is the set of its neighboring vertices.

Definition 1.2. A set $S \subseteq V(G)$ is called dominating set if $N_G[S] = V(G)$. A

dominating set S is a minimal dominating set if no proper subset $S' \subseteq S$ is a dominating set. A minimum cardinality of a dominating set of G is called domination number of G , and is denoted by $\gamma(G)$. A dominating set S with $|S| = \gamma(G)$ is called a γ -set.

Definition 1.3. A dominating set $S \subseteq V(G)$ is called certified dominating set of G if every vertex $v \in S$ has either zero or at least two neighbors in $V(G) - S$. A minimum cardinality of a certified dominating set of G is called certified domination number of G and denoted by $\gamma_{cer}(G)$. A certified dominating set of S with $|S| = \gamma_{cer}(G)$ is called a γ_{cer} -set [7].

1.1 Result

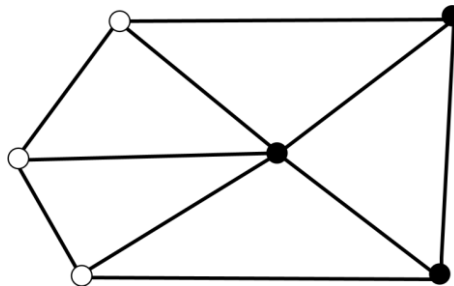
Proposition 1.4. If P_n is path on N vertices, then

$$\gamma_{mcr}(P_n) = \begin{cases} 1 & \text{If } n = 1 \text{ or } n = 3 \\ 2 & \text{if } n = 2 \\ 4 & \text{if } n = 4 \\ \frac{n}{3} & \text{Otherwise} \end{cases}$$

Proposition 1.5. If C_p is a Cycle on p vertices, then

$$\gamma_{mcr}(C_p) = \begin{cases} \frac{p}{3} + 1 & \text{If } p \neq 3 \\ \frac{p}{3} + 2 & \text{If } p = 3n \text{ for } n > 2 \end{cases}$$

Proposition 1.6. For a Wheel W_p of order $p \geq 3$,

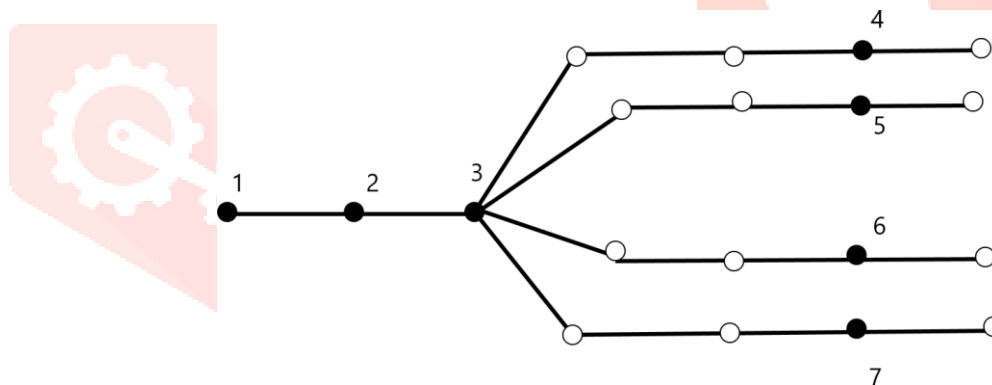


$$\gamma_{mcr}(W_p) = 3$$

Proposition 1.7. For any Complete bipartiate graph $K_{m,n}$, then

$$\gamma_{mcr}(K_{m,n}) = m + 1, \text{ if } m \leq n.$$

Proposition 1.8. For any graph G , $\gamma_{mcr}(G) = p$, iff $G = K_p$ or K_p .



$$\gamma_{mcr}(G) = 7$$

Observation: Every vertex of a graph G belongs to every maximal certifi- domi-nating set of G .

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