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NON-LINEAR DUAL PROGRAMMING UNDER THE CONCEPT OF B-CONVEXITY

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Abstract: This paper consists of various duality theorems for nonlinear programming problems under B-convexity assumptions.

1. Introduction: Recently Bector and Singh [2] have introduced B - vex functions which are weaker than convex functions and more recently Bector, Suneja and Lalitha [8] have introduced Pseudo b - vex and quasi b - vex functions which are weaker than pseudo convex and quasi convex functions respectively. P. Kanniappan & P. Pandian [7] introduced b-vexity is non-linear programming duality. R.B. Patel introduced duality for non-linear fractional programming involving generalized semilocally B-vex functions. Vasile Preda and Anton Bata to rescu [10] introduced duality for minimax generalized Bvex programming involving n-set functions. But they have not considered the recently developed concepts like B-convex duality. Hence in this paper an attempt is made to fill the gap in the aim of research. This paper consists various duality theorem for non-linear programming problem under B-convexity assumptions.

2.1 Definition : The function f is said to be B - convex at u □ x. w.r.t.

b (□, u) and (x - u) if □ x □ X.

b (x, u) [f(x) - f(u)] $(x - u)^t$ □ f(u)

2.2 Definition : The function f is said to be quasi b-convex at u □ x with respect to b (x, u) and (x - u) if □ x □ X.

f (x) □ f (u) □ b (x, u) $□^+$ (x, u) □ f (u) □ 0

2.3 Definition : The function f is said to be strictly quasi b - convex at u ⍰ X with respect to b (x, u) and (x - u) if ⍰ x ⍰ X, and x = u.

f (x) ⍰ f (u) ⍰ b (x, u) $(x - u)^t$ ⍰ f (u) < 0

2.4 Definition : The function f is said to be semistrictly quasi b-convex at u ⍰ X with respect to b (x, u) and (x - u) if ⍰ x ⍰ X.

f (x) < f (u) ⍰ b (x, u) $(x - u)^t$ ⍰ f (u) < 0

The connection b/u b-convex and quasi b-convex function is that every

b-convex function is quasi b-convex but the converse is not true. We can easily see that every b-convex function with respect to b (x, u), with b (x, u) > 0 is semi strictly quasi b-convex with respect to the same b (x, u). However the converse is not true.

Example : Let x = {-1, 1} define f : x ⍰ R by

f (x) = x + x^3 and b : X x X ⍰ R+ by

(x - u) = 0

b (x, u) = -1 x u ⍰ 0

= -xu, xu < 0

Then f is semi strictly quasi b-convex with respect to b (x, u) but not b-convex with respect to b (x, u) because for

x = $\frac{-1}{7}$, u = $\frac{-1}{2}$, we can see that

b (x, u) [f (x) - f (u)] < $(x - u)^t$ ⍰ f (u)

Every semi strictly quasi b-convex with respect to b (x, u) is quasi bconvex with respect to same b (x, u) but the converse is not true.

This is demonstrated by the following example.

Example :

Let x = (-1, 1). Define f : x R by f (x) = x^3 and

define b : X x X R+ by
 1 xu 0

b (x, u) = 0, xu 0 (x - u) = 0

Then f is quasi b-convex with respect to b (x, u) but it is not semi strictly quasi b-convex with respect to b (x, u) because for x = 0 and u = $\frac{1}{2}$.

b (x, u) $(x - u)^t$ f (u) = 0 and f (x) < f (u)

3 Formulation :

3.1 Primal Formulation :

Let us assume that the function f, g & h are differentiable on X.

Consider the following non-linear programming problem
(P) minimize f (x)

x X

Subject to g (x) 0

3.2 Dual Formulation :

(D) Maximize f (u)

U X

Subject to f (u) + y^tg (u) = 0 (1)

y^tg (u) = 0 (2)

x = 0 (3)

If f is convex with respect to b_o (x, u) and (x - u) and each component g, that is g_j, j = 1, 2, . . . m is b-convex w.r.t. bj (x, u) and (x - u) on x with b_o (x, u) > 0, then (D) is dual to P.

4. Feasibility :

The following feasible terminology is used in duality theorems.

(i) A point x ⍰ X is said to be (P) - feasible optional if x is a feasible (optimal) º º solution of the primal problem (P).

(ii) The value of the objective function for the problem (P) at a point x is called as º (P) - objective value at x . º

5 Duality Theorems :

5.1 : (Weak duality theorem) : Let x be (P) - feaisble and (u, y) be D - feasible. If f is semi strictly quasi b-convex at u with respect to (x, u) and ytg is strictly quasi b-convex at u with respect to b (x, u) ⍰ feasible (x, u, y) then f(x) ⍰ f (u).

Proof : If x = u, the results is trival

suppose x ⍰ u

Since x is (P) - feasible and (u, y) is D - feasible, we have

ytg (x) - ytg(u) ⍰ 0

By strictly quasi b-convexity of ytg at u

We have

b (x, u) (x - u)t ⍰ yt g (u) <) 0

From (7) we have

$b (x, u) (x - u)^t$ ⍰ f (u) > 0

By semi strict quasi b - convexity of f at u, we have f (x) ⍰ f (u)

Hence the theorem

5.2 Strong Duality Theorem :

Let x be (P) - optimal and let g satisty a constraint qualification at x . Then o o
⍰ y ⍰ R^m such that (x , y) is D - feasible and the p - objective value at x is equal to o o o o
the D - objective value at (x , y). If forever feasible (x, u, y), the function f is semi o o
strictly quasi b - convex at u w.r.t. b (x, u) and ytg is strictly quasi b-convex at u with o
respect to b (x, u) then (x , y) is is (D) - optimal.
o o

Proof :

Since x is (P) optimal and g satisfies a constraint qualification at x by Kuhno o
Tucker condition, ⍰ y ⍰ Rm such that o

⍰ f (x) + ⍰ yt (g (x) = 0 o o o

$y_o t$ g (x^o) = 0

y ⍰ 0
o

⍰ (x , y) is D - feasible and P - objective value at x_o is equal to D - objective o o
value at (x , y). o o

Suppose (x , y) is not D - optimal then ⍰ a D - feasible (u, y) such that o o

f (u, y) > f (x, u) ⍰ (4)

Then ⍰ a (D) - feasible and (u, y) is D - feasible by weak duality

f (x) ⍰ f (u) o

which is a contradiction to (4)

Then (x , y) is D - optimal o o

Hence the theorem.

6 Converse Duality Theorem :

Theorem 6.1 Converse duality :

Let (x , y) be D - optimal and let the n x n Hersian matrix. o o

□^{2} f (x) + □^{2} y^t g (x) o o o

be + ve or -ve definite and the vector □ f (x) = 0 o

If for all feasible x, u, y, f is semi strictly quasi b - convex at u w.r.t. (x, u) and yt

g is strictly quasi b - convex at u w.r.t. b (x, u). Then x is (P) - optimal. o

Proof : Since (x , y) is (D) optimal then by Fritz-John theorem □ p □ R, v □ R^n,
o o

q □ R and S □ R^m such that

□ pf (x) + □ v^t [f (x) + □ y^t g (x)] + q □^{t} y g (x) = 0 □ (4.5) o o o o o o

v^tf (x) = 0 □ (6) o

v^t □ g (x) + qg (x) + S = 0 □ (7) o o

q y t g (x) = 0 □ (8) o o

t □ (9)
V s = 0 o

(p, q, s) □ 0 □ (10)

(p, q, v, s) = 0 □ (11)

Multiplying (7) by $y^{o\ t}$, we have
vt □ y^t g (x) + q y^t g (x) + y^ts = 0 o o o o o

From (8) and (9) we get

v^t □ y^t g (x) = 0 □ (12) o o

Multiplying (5) by v t we have

pv^t ⍰ f (x) + v^t [⍰^{2} f (x) + ⍰^{2} y ^{t} g (x)] v o o o o

+ qv^t ⍰ y ^{t} g (x) = 0 o o

From (6) and (12) we get

v^t [⍰^{2} f (x) + ⍰^{2} y ^{t} g (x)] v = 0 o o o

Since the Hersian Matrix is positive or negative difinite v = 0 since v = 0, (5)

becomes

p ⍰ f(x_o) + q ⍰ y_ot g (xo) = 0 From (5) we have

p ⍰ f (x_o) + q ⍰ (-f (x_o) = 0

which implies (p - q) ⍰ f (x_o) = 0

Since ⍰ f (x_o) = 0, we have p = q

Suppose p = 0 then q = 0 and s = 0 by (7)

⍰ (p, q, v, s) = 0 which is a contradiction to (11)

Thus p ⍰ 0, since p = q, q ⍰ 0 and from (4.10), q > 0

Since v = 0, q > 0 and s ⍰ 0 from (4.7) we have

g (x_o) ⍰ 0

xo is (p) feasible. From the theorem of weak duality xo is (p) - optimal.

Hence the theorem.

6.2 Theorem (Strict converse duality theorem) :

Let x_o be (p) - optimal and let g satisfy a constraint qualification at x_o. If (u_o, y_o) is (D) - optimal f is strictly quasi b - convex at u_o with respect to b_o (x, u) and $y_o^t g$ is strictly quasi b - convex at u_o w.r.t. b (x, u), then $x_o = u_o$ and inf (p) = SUP (D).

Proof : Since x_o is (P) - Primal, g satisfies a constraint qualification at x_o, by KhunTukkar conditions.

▯ a y ▯ R^m such that (x_o, y) is (D) - feasible.

Suppose x_o ▯ u_o

Since x_o is (P) - feasible and (u_o, y_o) is (D) - feasible, we have

$b(x_o, u_o)(x_o - u_o)^t$ ▯ $y_o^t g(u_o) < 0$ (13)

By the feasibility of (u_o, y_o), we have from (13)

$b(x_o, u_o)(x_o - u_o)^t$ ▯ $f(u_o) > 0$

Since $b(x_o, u_o)$ ▯ 0 and $b_o(x_o, u_o)$ ▯ 0, we have

$b_o(x_o, u_o)(x_o - u_o)^t$ ▯ $f(u_o)$ ▯ 0

By strict quasi b - convexity of f at u_o,
We have

$f(x_o) > f(u_o)$

This contradicts that (u_o, y_o) is a D - optimal.

Then $x_o = u_o$ and clearly infimum (P) = Supremum (D).

Hence the theorem

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