Volume Between Regular Polyhedron And Inscribed Sphere

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Abstract:

In this paper, the volume between a regular polyhedron and inscribed sphere has been obtained for all the five platonic solids.

Key words: Volume, regular polyhedral, platonic solids and sphere.

Introduction:

A polyhedron with faces of identical regular polygon is called a regular polyhedron. A regular polyhedron has all side lengths equal as well as all angles equal. There are five regular polyhedra, namely; tetrahedron, cube, octahedron, dodecahedron and icosahedrons. The regular polyhedra are also called platonic solids.

A tetrahedron has four faces (equilateral triangles), four vertices and six edges. The four vertices of a tetrahedron are at equal distance from each other. A cube has six faces (identical squares), eight vertices and 12 edges. An octahedron has eight faces (identical triangles), six vertices and 12 edges. A dodecahedron has twelve faces (identical pentagons), twenty vertices
and thirty edges. An icosahedron has twenty faces (equilateral triangles), twelve vertices and thirty edges.

In this paper, the volume between a regular polyhedron and inscribed sphere has been obtained for all the five platonic solids.

**Analysis:**

The volume between inscribed regular polyhedron and inscribed sphere has been discussed for all the five platonic solids as follows:

**Case 1: Volume between a regular tetrahedron and inscribed sphere:** This is shown in figure 1 given below:

Let the radius of the sphere be \( r \) units and length of circumscribed regular tetrahedron be \( x \) units. Then

\[
x = \frac{12}{\sqrt{6}} r.
\]

...(1)

Now volume of sphere is given by

\[
A_1 = \frac{4}{3} \pi r^3 \text{units},
\]

and volume of circumscribed regular tetrahedron is given by

\[
A_2 = \frac{x^3}{6\sqrt{2}} = \frac{24}{\sqrt{3}} r^3 \text{units}.
\]
Therefore required volume is given by

\[ A = A_2 - A_1 = \frac{24}{\sqrt{3}} r^3 - \frac{4}{3} \pi r^3 \]

\[ = \frac{4}{3} (6\sqrt{3} - \pi) r^3 \text{ units.} \]

**Case 2: Volume between a cube and inscribed sphere:** This is shown in figure 2 given below:

Let the radius of the sphere be \( r \) units and length of circumscribed cube be \( x \) units. Then

\[ x = 2r. \quad \ldots(2) \]

Now volume of sphere is given by

\[ A_1 = \frac{4}{3} \pi r^3 \text{ units}, \]

and volume of circumscribed cube is given by

\[ A_2 = x^3 = 8r^3 \text{ units}. \]

Therefore required volume is given by

\[ A = A_2 - A_1 = 8r^3 - \frac{4}{3} \pi r^3 \]

\[ = \frac{4}{3} (24 - \pi) r^3 \text{ units.} \]
Case 3: Volume between a regular octahedron and inscribed sphere: This is shown in figure 3 given below:

Let the radius of the sphere be $r$ units and length of circumscribed regular octahedron be $x$ units. Then

$$x = \sqrt{6}r.$$ ...(3)

Now volume of sphere is given by

$$A_1 = \frac{4}{3} \pi r^3 \text{ units.}$$

and volume of circumscribed octahedron is given by

$$A_2 = \frac{\sqrt{2}}{3} x^3 = 4\sqrt{3}r^3 \text{ units.}$$

Therefore required volume is given by

$$A = A_2 - A_1 = 4\sqrt{3}r^3 - \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} (3\sqrt{3} - \pi) r^3 \text{ units.}$$
Case 4: Volume between a regular dodecahedron and inscribed sphere: This is shown in figure 4 given below:

Let the radius of the sphere be \( r \) units and length of circumscribed regular dodecahedron be \( x \) units. Then obviously

\[
x = \frac{4r}{\sqrt{10 + \frac{22}{\sqrt{5}}}}
\]

...(4)

Now volume of sphere is given by

\[
A_1 = \frac{4}{3} \pi r^3 \text{ units},
\]

and volume of circumscribed dodecahedron is given by

\[
A_2 = \frac{(15 + 7\sqrt{5})}{4} x^3 = \frac{16(15+7\sqrt{5})}{(10+\frac{22}{\sqrt{5}})\sqrt{10+\frac{22}{\sqrt{5}}}} r^3 \text{ units.}
\]

Therefore the required volume is given by

\[
A = A_2 - A_1 = \frac{16(15+7\sqrt{5})}{(10+\frac{22}{\sqrt{5}})\sqrt{10+\frac{22}{\sqrt{5}}}} r^3 - \frac{4}{3} \pi r^3
\]

\[
= \frac{4}{3} \left( \frac{12(15+7\sqrt{5})}{(10+\frac{22}{\sqrt{5}})\sqrt{10+\frac{22}{\sqrt{5}}} - \pi} \right) r^3 \text{ units.}
\]
Case 5: Volume between a regular icosahedron and inscribed sphere: This is shown in figure 5 given below:

![Figure 5](image_url)

Let the radius of the sphere be \( r \) units and length of circumscribed regular icosahedron be \( x \) units. Then obviously

\[
x = \sqrt{3}(3 - \sqrt{5})r.
\]

...(5)

Now volume of sphere is given by

\[
A_1 = \frac{4}{3}\pi r^3 \text{ units},
\]

and volume of circumscribed icosahedron is given by

\[
A_2 = \frac{5(3+\sqrt{5})}{12}x^3 = 10\sqrt{3}(7 - 3\sqrt{5})r^3 \text{ units}.
\]

Therefore required volume is given by

\[
A = A_2 - A_1 = 10\sqrt{3}(7 - 3\sqrt{5})r^3 - \frac{4}{3}\pi r^3
\]

\[
= \frac{2}{3}(15\sqrt{3}(7 - 3\sqrt{5}) - 2\pi)r^3 \text{ units}.
\]

Concluding remarks:

Here the volume between sphere and circumscribed all the five platonic solids have been obtained.
References:


Mathematical properties of Platonic solids. Polyhedr.com