



A Study On Linear Programming Problems In Real Life Applications

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Abstract:

Linear Programming is an important branch of Operational Research and it is a Mathematical Method to assist the people to carry out scientific Management. Linear Programming is a Mathematical method for determining a way to handle certain types of optimization problems. The main objective of this paper is to understand how Graphical method and Simplex method can be applied in a simple way for a furniture manufacturer and gold smith as to optimize their profit.

Keywords: Linear programming, Graphical Method and Simplex method, Optimum solution

1: Introduction

Operations Research is a branch of Statistics in Applied Mathematics. Operations Research is a discipline that provides scientific methods used to solving real life problems which helps us in determining the optimal decision making. A Linear Programming Problem is a special kind of Operations Research and it consists of three parts. First, the objective function which is to be either maximized or minimized. Second, there is a set of linear constraints which contains the technical specifications of the problems in relation to the given resources or requirements. Third, there is a set of non-negativity constraints. Over past 70 years, Linear Programming has been applied in various fields such as Military, Financial, Marketing, Accounting and Agricultural Problems. The most common methods are used to solve Linear Programming Problems are Graphical Method and Simplex Method. Graphical method is one of the most basic methods to solve Linear Programming Problems by finding the optimal solution. Through this method, we can formulate a real-world problem into a mathematical model. It is used to optimize the two variable linear programming problems. It is a simplest method to solve Linear Programming Problems.

2. Preliminaries

Definition: 2.1

Linear programming problem (LPP) is a technique with achieving the most effective optimal value of a linear functions with several variables. Linear Programming Problems has some restrictions like the variables are non-negative and satisfy a collection of linear inequalities. Linear Programming Problems mostly include manufacturing problems, diet problems and transportation problems.

Definition: 2.2

The linear function which is to be optimized (maximized or minimized) is called an **objective function**.

Definition: 2.3

The conditions expressing the relations between the variables are called **constraints**.

These constraints usually appear in the form of inequalities.

Definition: 2.4

The variables that appear in the objective function are called **Decision variables**. The variables which has to be determined, $s = 1, 2, \dots, n$, to optimize the objective function.

Definition: 2.5

The decision variables should always take non-negative values which is they should be greater than or equal to zero. The value of decision variables can be zero or positive but can't be negative.

Definition: 2.6

A Set of values of the decision variables s_1, s_2, \dots, s_n satisfying all the constraints of a linear programming problem is called a **Solution**.

Definition: 2.7

A Set of values of the decision variables s_1, s_2, \dots, s_n satisfying all the constraints and non-negative conditions of a linear programming problem is called a **Feasible Solution**.

Definition: 2.8

Any feasible solution of a linear programming problem which is optimize the objective function is called an **Optimal Solution**.

Definition: 2.9

Given a system of "m" linear equations with "p" variables ($m < p$). Any solution which is obtained by solving for "m" variables keeping the remaining ($p - m$) variables zero is called a **Basic Solution**.

Definition: 2.10

A **Basic Feasible Solution** is a basic solution which also satisfies the non-negative condition, that is $S_1 \geq 0, S_2 \geq 0, \dots, S_n \geq 0$.

Definition: 2.11

A Basic Feasible Solution is said to be **Degenerate** if the value of at least one basic variable is zero.

Definition: 2.12

A Basic Feasible Solution is said to be **Non-Degenerate** if the value of all basic variables are non-zero and positive.

Definition: 2.13

A Basic Feasible Solution that optimizes the objective function value of linear programming problem is called an **Optimum Basic Feasible Solution**.

Definition: 2.14

The Common region determined by all the constraints $S_1, S_2 \geq 0$ of a linear programming problem is called the **Feasible Region**.

Definition: 2.15

A non-negative variable which is added to the left hand side of the constraints to convert the **inequalities to equalities are called Slack variables**.

It is added when the inequalities are of the type, (\leq) .

Definition: 2.16

A non-negative variable which is subtracted from the left hand side of the constraints to convert the inequalities to equalities are called **Surplus Variables**.

It is added when the inequalities are of the type, (\geq) .

3:Mathematical Formulation of Linear Programming Problems

Step 1: Identify the decision variables.

Step 2 :Express the objective function in terms of decision variables.

Step-3: Write down the constraints with the help of which the objective function is to be optimized.

Step-4 : Write down the non- negative constraints.

3.1:General Linear Programming Problem

The linear programming problem involving more than two variables may be expressed as

Maximized or Minimized $z = g_1 S_1 + g_2 S_2 + \dots + g_n S_n$

Subject to the constraints,

$$C_{11} S_1 + C_{12} S_2 + \dots + C_{1n} S_n \leq \geq C_{21} S_1 + C_{22} S_2 + \dots + C_{2n} S_n$$

$$\leq \geq 2$$

.....

$$C_{m1} S_1 + C_{m2} S_2 + \dots + C_{mn} S_n \leq \geq a_m$$

and the non-negativity restrictions

$$S_1, S_2, \dots, S_n \geq 0.$$

3.2:Canonical Form of Linear Programming Problem

The Objective function is to be , Maximized $z = g_1 S_1 + g_2 S_2 + \dots + g_n S_n$

Subject to the constraints, $C_{11} S_1 + C_{12} S_2 + \dots + C_{1n} S_n \leq a_1$

$$C_{21} S_1 + C_{22} S_2 + \dots + C_{2n} S_n \leq a_2$$

.....

$$C_{m1} S_1 + C_{m2} S_2 + \dots + C_{mn} S_n \leq a_m$$

and the non-negativity restrictions $s_1, s_2, \dots, s_n \geq 0$

This form is called as **Canonical Form** of linear programming problem.

4: Procedure for Solving Linear programming problem

4.1: Algorithm for Solving a Graphical Method

The graphical method can be started as follows.

Step 1: Formulate given problem into a linear programming problem

Step 2:

Consider the inequality constraint as equality all are straight line in the XoY plane.

Corresponding to each equality and non-negativity restriction.

Step 3 :

Identify the region which satisfies all constraints bounded by the lines drawn in step 2. If the optimal solution exists, this region will be the convex set. This is because of the theorem, "The set F_S of all feasible solution, if non-empty, is convex".

Step 4 :

Find the corner points of the solution region, which is a convex set and find the point of intersection of the bounded lines by solving the equation of the corresponding lines,

Step 5 :

Calculate the values of the objective function z corresponds to each vertices. This is because of the theorem, "If F_S is non-empty, the objective function (z) has either an unbounded minimum or it is minimum at a vertex of F_S ".

Step 6 :

The solution of the problem is given by the co-ordinates of that vertices which optimize the objective function.

4.2: Algorithm for Solving a Simplex Method :

The simplex algorithm can be started as follows.

Step 1 : Formulate the given problem into a linear programming problem.

Step 2:

Check whether the objective function of the given problem is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximization by, Minimum of $z = -$ Maximum of z

Step 3 :

Check whether all $a_i (i = 1, 2, \dots, m)$ are positive. If not, convert it into positive by multiplying the equation by -1.

Step 4 :

Express the problem in the standard form by introducing slack or surplus variables to convert the inequality constraints into equations.

g_j (Row) is the coefficient of the variable in the objective function.

Where g_B (column)-coefficient of basic variables.

Where y_B (column)-basic variable.

Step 5 :

Compute the net evaluation $z_j - g_j$ that is $z_j - g_j = g_B (c_j - g_j)$.

$$z_j = \sum g_B i s_{ij}$$

If all $z_j - g_j \geq 0$, then the initial basic feasible solution is an optimum basic feasible solution.

If at least one $z_j - g_j < 0$, then the basic feasible solution is not optimal, go to next step.

Step 6 :

To find the entering variable, that is, key column.

If there are more than one negative, $z_j - g_j$ choose the most negative of them.

If there are more than one variable having the same most negative $z_j - g_j$, then any one of the variables can be selected arbitrarily as the entering variable.

If all $s_{ir} \leq 0$, then there is an unbounded solution.

If at least one $s_{ir} > 0$, then the corresponding variable enters the basis.

Step 7 :

To find the leaving variable or key row:

Compute the ratio $\theta = y^{S_{B_i k}},_{ir} > 0$

If the minimum of these ratios be , then choose the variable S_k to leave the basis called the key row and the element at the intersection of the key row and the key column is called the key element.

Step 8 :

Form a new table by dropping the leaving variable and introducing the entering variable along with associated value under g_B column. The leaving element is converted to unity by dividing the key equation by key element and all other elements in its column to zero by using the formula:

$$i. \quad \text{New key row} = \frac{\text{old key row}}{\text{pivot}}$$

$$ii. \quad \text{New equation} = \text{old equation} - [\text{corresponding column coefficient} \times \text{new pivot row}]$$

Step 9 :

Go to step 5 and repeat all the procedure until either an optimum solution exists or there is an indication of unbounded solution.

5: Applications of Linear Programming Problems with Real-World Situations

5.1 : Data Analysis (Graphical Method)

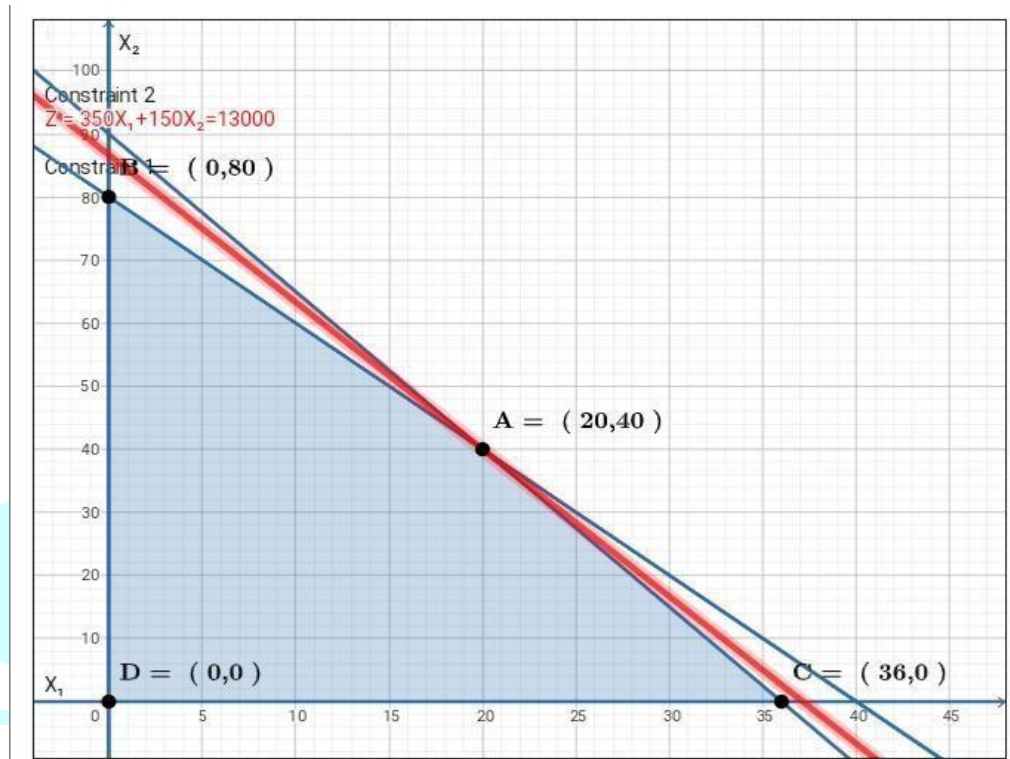
The data were collected from Mr. Murugan owner of Om Murugan Furniture works in Ranipet. The data is given for working of 70 days for making tables and chairs and profit of one table is Rs.350 and one chair is Rs.150.

Description of the problem: A furniture manufacturer produces two products such as table and chair. The production process for both requires a certain hour for carpentry work and a certain hour for painting are given by,

PRODUCT	CARPENTRY	PAINTING	AVAILABLE RESOURCES
TABLE	6	3	240
CHAIR	5	2	180

Each table gives a profit Rs.350 and each chair gives a profit Rs.150. find the maximum profit of furniture manufacturer by using Graphical method.

Solution



Graph: 5.1

By Using an algorithm of Graphical method we obtained the optimal solution of the problem is $Z=13000$ and the value of $X_1 = 20$ and $X_2 = 40$

Therefore, the maximum profit for 70 days of furniture manufacturer is Rs. 13000. He reached the profit by producing 20 tables and 40 chairs.

5.2 Data Analysis (Simplex Method)

The data were collect from Mr. Muthu gold smith of Sri Srinivasan Jewellery in Ranipet. The data is given for working of 60 days for making 8gm of earrings and 2gm of ring. Each earring gives profit of Rs.3600 and each ring gives profit of Rs.900.

Description of the problem:

A gold smiths produces two products such as 8gm of earrings and 2gm of ring. The production of both made by two processes, process 1 include melting, wheeling and dying. Process 2 include fitting, soldering and polishing are given by,

PRODUCT	PROCESS-1	PROCESS-2	AVAILABLE RESOURES
EARRING	2	4	180
RING	1	2	180

Each earring gives profit Rs. 3600 and each ring gives profit Rs.900. find maximum profit of gold smith by using simplex method.

Solution

By using the Procedure of Simplex method ,we obtained the following table:

		g_j	3600	900	0	0
g_B	y_B	S_B	S_1	S_2	y_1	y_2
3600	S_1	90	1	2	1/2	0
0	y_2	0	-1	-2	-1	1
$z_j - g_j = 324000$			0	6300	1800	0

Since all $z_j - g_j \geq 0$, the solution is optimum.

Therefore, the optimum solution is $\max z = 324000$.

It is obtained by $S_1 = 90$ and $S_2 = 0$.

Therefore, the maximum profit of gold smith for 60 days is Rs.324000. He reaches the profit by producing 90 earrings.

Conclusion:

In this paper, We have used Graphical Method and Simplex Method for finding the optimum solution with two case studies. In first case, we have studied and analyzed the profit of the Furniture Manufacturer by using Graphical Method and the profit of the Furniture Manufacturer is Rs.13000 for 70 days. Further, in second case, we have analyzed the profit of the Gold Smith is Rs.324000 for 60 days by using Simplex Method.

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