



A Mathematical Method Of Magnitude Ranking Technique For Fuzzy Assignment Problem

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Abstract

Assignment problems have various applications in the real world because of their wide applicability in industry, commerce, management science, etc. Traditional classical assignment problems cannot be successfully used for real life problem, hence the use of fuzzy assignment problems is more appropriate. The assignment cost has been considered as imprecise numbers that can be described by Triangular Fuzzy Numbers which are more realistic and general in nature. In this Paper, the fuzzy assignment problem has been converted into crisp assignment problem using Method of Magnitude and Ranking Function for Robust Ranking Technique for fuzzy costs matrix and solving it by Hungarian Method has been applied to find an optimal solution.

Keywords: Assignment Problem, Triangular Fuzzy Number, Fuzzy Magnitude Ranking Method, Hungarian Method.

1. Introduction

An Assignment Problem is a special type of Linear Programming Problem which deals with assign n number of persons to n jobs (or) assigning various activities for example (jobs or task or sources) to an equal number of service facilities (men, machine, labours, etc.) on one to one basis in such a way so that the total time or total cost involved is minimized and total sale or total profit is maximized or the total satisfaction of the group is maximized. The mathematical formulation of the problem suggests that this is 0-1 programming problem and is highly degenerate all the algorithms developed to find optimal solution of transportation problem are applicable to assignment problem. However, due to the highly degeneracy nature a specially designed algorithm, widely known as Hungarian method proposed by Kuhn [13], is used for its solution.

In this Paper, the fuzzy assignment has been converted into crisp assignment problem using method of Magnitude and Hungarian Assignment has been applied to find an optimal solution. We investigate two realistic problem namely the assignment problem, with fuzzy costs \tilde{C}_{ij} . Since the objectives are to minimize the total cost or to maximize the total profit, subject to some crisp constraints, the objective function is considered also as a fuzzy number. The method is to rank the fuzzy objective values of the objective function by some ranking method for fuzzy number to find the best alternative, On the basis of idea the Robust's ranking method [11] has been adopted to transform the fuzzy assignment problem to a crisp one so that the conventional solution methods may be applied to solve assignment problem. The idea is to transform a problem with fuzzy parameters to a crisp version in the LPP form and solve it by the simplex method. Other than the fuzzy assignment problem other applications and this method can be tried in project scheduling, maximal flow, transportation problem etc.

2: Preliminaries

Definition: 2.1

The **Assignment Problem** is a particular case of the Transportation problem in which the objective is to assign a number of tasks (Jobs or origins or sources) to an equal number of facilities (machines or persons or destinations) at a minimum cost (or maximum profit)

Definition: 2.2

A fuzzy number \hat{A} is said to be **Triangular Fuzzy Numbers** (T.F.N) if it has the following membership function:

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} a_1 \leq x \leq a_2 \\ \frac{x-a_3}{a_2-a_3} a_2 \leq x \leq a_3 \\ 0 \quad \text{otherwise} \end{cases}$$

Then we say that \hat{A} is triangular fuzzy number, Written as:

$\hat{A} = (a_1, a_2, a_3)$ where $a_1, a_2, a_3 \in R$. The interval of confidence for the triangular fuzzy number

$\hat{A} = (a_1, a_2, a_3)$ at α – level set is defined as:

$$A_\alpha = [a_\alpha^L, a_\alpha^U]$$

$$A_\alpha = [a_1 + (a_2 - a_1)\alpha, a_3 - (a_3 - a_2)\alpha], \forall \alpha \in [0,1].$$

Definition: 2.3

Defuzzification is the process of finding singleton value (crisp value) which represents the average value of the TFNs. Here Method of magnitude is used to defuzzify the TFNs because of its simplicity and accuracy

2.3.1: Ranking function:

A ranking function $R: F(R) \rightarrow R$, Where $F(R)$ set of fuzzy numbers defined on set of real number, maps each fuzzy number into real number, where a natural order exists.

2.3.2: Ranking function for triangular fuzzy numbers

The ranking function for $\hat{A} = (a_1, a_2, a_3)$ denoted $R(\hat{A})$ proposed by F. Reuben's is defined by

$$R(\hat{A}) = \frac{1}{2} \int_0^1 (a_\alpha^L + a_\alpha^U) d\alpha$$

Where

$$\begin{cases} a_\alpha^L = a_1 + (a_2 - a_1)\alpha \\ a_\alpha^U = a_3 + (a_3 - a_2)\alpha, \forall \alpha \in [0,1]. \end{cases}$$

2.3.3: Robust Ranking Technique:

For a convex fuzzy number \tilde{a} , the Robust's Ranking index is defined by,

$$R(\tilde{a}) = \int_0^1 (0.5) (a_\alpha^L, a_\alpha^U) d\alpha$$

Where

$$(a_\alpha^L, a_\alpha^U) = \{(b - a)\alpha + a, c - (c - b)\alpha\}$$

which is the α – level cut of the fuzzy number \tilde{a} .

2.3.4: Method of Magnitude

A triangular fuzzy number $\check{a} \in F(R)$ can also be represented as a pair $\check{a} = (a, \bar{a})$ of functions $(a(r), \bar{a}(r))$ for $0 \leq r \leq 1$ which satisfies the following requirements:

- $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.
- $\bar{a}(r)$ is a bounded monotonic decreasing left continuous function.

- $\underline{a}(r) \leq \bar{a}(r)$ for $0 \leq r \leq 1$.

Definition: 2.4

For an arbitrary triangular fuzzy number $\tilde{a} = (\underline{a}, \bar{a})$, the number

$a_0 = \left(\frac{\underline{a}(1) + \bar{a}(1)}{2}\right)$ is said to be a location **index number** of \tilde{a} . The two non-decreasing left continuous function $a_* = (a_0 - \underline{a})$ and $a^* = (\bar{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index functions respectively. Hence every triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ can be also be represented by $\tilde{a} = (a_0, a_*, a^*)$.

2.4.1: Ranking of Triangular Fuzzy Number

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari proposed a new ranking method based on the left and the right spreads at some α -levels of fuzzy numbers. For an arbitrary triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$ with parametric form

$\tilde{a} = (\underline{a}(r), \bar{a}(r))$ we define the magnitude of the triangular fuzzy number by \tilde{a} by

$$\begin{aligned} Mag(\tilde{a}) &= \frac{1}{2} \left(\int_0^1 (\bar{a} + \underline{a} + a_0) f(r) dr \right) \\ &= \frac{1}{2} \left(\int_0^1 (a^* + 3a_0 - a_*) f(r) dr \right) \end{aligned}$$

[Since $\bar{a} + \underline{a} + a_0 = a^* + a_0 + a_0 - a_* + a_0 = a^* + 3a_0 - a_*$].

Where the function $f(r)$ is a non-negative and increasing function on $[0,1]$ with $f(0)=0$, $f(1)=1$ and $\int_0^1 f(r) = \frac{1}{2}$. The function $f(r)$ can be considered as a weighting function. In real life application, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r) = r$.

$$\text{Hence } Mag(\tilde{a}) = \left(\frac{a^* + 3a_0 - a_*}{4}\right) = \left(\frac{\bar{a} + \underline{a} + a_0}{4}\right)$$

The magnitude of a triangular fuzzy number \tilde{a} synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. $Mag(\tilde{a})$ is used to rank fuzzy numbers. The larger $Mag(\tilde{a})$, the larger fuzzy number.

2.4.2: Properties of Ranking a Triangular Fuzzy Number

For any two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$

1. $Mag(\tilde{a}) \geq Mag(\tilde{b})$ if and only if $\tilde{a} \geq \tilde{b}$,
2. $Mag(\tilde{a}) \leq Mag(\tilde{b})$ if and only if $\tilde{a} \leq \tilde{b}$,
3. $Mag(\tilde{a}) = Mag(\tilde{b})$ if and only if $\tilde{a} \approx \tilde{b}$.

In this paper, we use this Magnitude Ranking technique for ranking the objective values. $Mag(\tilde{a})$ gives the representative value of the number \tilde{a} . It satisfies the linearity and additive property.

3: Algorithm for solving Fuzzy Assignment Problem

Step 1: First test whether the given fuzzy cost matrix of a fuzzy assignment problem is a balanced one or not. If not change this unbalanced assignment problem into balanced one by adding the number of dummy row(s)/column(s) and the values for the entries are zero. If it is a balanced one (i.e., number of persons are equal to the number of jobs) then go to step2.

Step 2: Defuzzify the fuzzy cost by using Magnitude ranking method.

Step 3: Apply Hungarian Algorithm to determine the best combination to produce the lowest total costs, where each machine should be assigned to only one job and each job requires only one machine.

4. Numerical Example:

Example: 4.1

Here, we are going to solve fuzzy Assignment problem using Magnitude Ranking Technique: To allocate 5 jobs to 5 different machines, the fuzzy assignment cost C_{ij} is given below:

$$\begin{bmatrix} (3,5,7) & (4,6,8) & (1,2,3) & (4,5,9) & (3,8,9) \\ (1,5,9) & (3,5,7) & (2,3,4) & (0,5,5) & (4,5,9) \\ (0,2,4) & (3,7,8) & (0,2,5) & (4,5,8) & (2,3,9) \\ (8,9,10) & (2,7,9) & (2,3,4) & (1,2,3) & (3,5,11) \\ (3,7,23) & (1,3,4) & (2,2,8) & (0,0,2) & (2,16,18) \end{bmatrix}$$

Solution:

In conformation to model the fuzzy assignment problem can be formulation in the following

$$\begin{aligned} &Mag(3,5,7)x_{11} + Mag(4,6,8)x_{12} + Mag(1,2,3)x_{13} + Mag(4,5,9)x_{14} + Mag(3,8,9)x_{15} \\ &Mag(1,5,9)x_{21} + Mag(3,5,7)x_{22} + Mag(2,3,4)x_{23} + Mag(0,5,5)x_{24} + Mag(4,5,9)x_{25} \\ &Mag(0,2,4)x_{31} + Mag(3,7,8)x_{32} + Mag(0,2,5)x_{33} + Mag(4,5,8)x_{34} + Mag(2,3,9)x_{35} \\ &Mag(8,9,10)x_{41} + Mag(2,7,9)x_{42} + Mag(2,3,4)x_{43} + Mag(1,2,3)x_{44} + Mag(3,5,11)x_{45} \\ &Mag(3,7,23)x_{51} + Mag(1,3,4)x_{52} + Mag(2,2,8)x_{53} + Mag(0,0,2)x_{54} + Mag(2,16,18)x_{55} \end{aligned}$$

Subject to

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} + x_{15} &= 1 & x_{11} + x_{21} + x_{31} + x_{41} + x_{51} &= 1 \\ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} &= 1 & x_{12} + x_{22} + x_{32} + x_{42} + x_{52} &= 1 \\ x_{31} + x_{32} + x_{33} + x_{34} + x_{35} &= 1 & x_{13} + x_{23} + x_{33} + x_{43} + x_{53} &= 1 \\ x_{41} + x_{42} + x_{43} + x_{44} + x_{45} &= 1 & x_{14} + x_{24} + x_{34} + x_{44} + x_{54} &= 1 \\ x_{51} + x_{52} + x_{53} + x_{54} + x_{55} &= 1 & x_{15} + x_{25} + x_{35} + x_{45} + x_{55} &= 1 \end{aligned}$$

Where $x_{ij} \in [0,1]$.

$$\mu_{\bar{a}}(x) = f(x) = \begin{cases} \frac{(x - a_1)}{(a_2 - a_1)} & \text{if } a_1 \leq x \leq a_2 \\ \frac{(a_3 - x)}{(a_3 - a_2)} & \text{if } a_1 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

Now we calculate $Mag(3,5,7)$ by applying method of magnitude.

The membership function of the triangular fuzzy number $(3,5,7)$ is

$$\mu_{\bar{a}}(x) = \begin{cases} \frac{(x-3)}{(5-3)}, & 3 \leq x \leq 5 \\ \frac{(7-x)}{(7-5)}, & 5 \leq x \leq 7 \\ 0, & \text{otherwise} \end{cases}$$

$$Mag(\bar{a}) = \frac{1}{2} \int_0^1 (\bar{a} + \underline{a} + a_0) f(r) dr$$

Where $(\bar{a} + \underline{a} + a_0) f(r) = (a^* + 3a_0 - a_*)r$

$$Mag(\bar{a}) = \frac{1}{2} \int_0^1 (a^* + 3a_0 - a_*)r dr$$

$$Mag(3,5,7) = \frac{1}{2} \int_0^1 (7 + 3(5) - 3) r dr = 4.75$$

The membership function of the triangular fuzzy number is $(4, 6, 8)$

$$Mag(4,6,8) = \frac{1}{2} \int_0^1 (8 + 3(6) - 4) r dr = 5.5$$

Similarly,

$$\begin{aligned} \text{Mag}(4,6,8) &= 5.5 \\ \text{Mag}(1,2,3) &= 2 \\ \text{Mag}(4,5,9) &= 5 \\ \text{Mag}(3,8,9) &= 7.5 \\ \text{Mag}(1,5,9) &= 5.75 \\ \text{Mag}(3,7,23) &= 10.25 \end{aligned}$$

$$\begin{aligned} \text{Mag}(3,5,7) &= 4.75 \\ \text{Mag}(2,3,4) &= 2.75 \\ \text{Mag}(0,5,5) &= 5 \\ \text{Mag}(4,5,9) &= 5 \\ \text{Mag}(0,2,4) &= 2.5 \\ \text{Mag}(1,3,4) &= 3 \end{aligned}$$

$$\begin{aligned} \text{Mag}(3,7,8) &= 6.5 \\ \text{Mag}(0,2,5) &= 2.75 \\ \text{Mag}(4,5,8) &= 4.75 \\ \text{Mag}(2,3,9) &= 4 \\ \text{Mag}(8,9,10) &= 7.25 \\ \text{Mag}(2,2,8) &= 3 \end{aligned}$$

$$\begin{aligned} \text{Mag}(2,7,9) &= 7 \\ \text{Mag}(2,3,4) &= 2.75 \\ \text{Mag}(1,2,3) &= 2 \\ \text{Mag}(3,5,11) &= 5.75 \\ \text{Mag}(0,2,2) &= 0.5 \\ \text{Mag}(2,16,18) &= 16 \end{aligned}$$

We replace these values for this corresponding c_{ij} . We get a convenient assignment problem

$$\begin{bmatrix} 4.75 & 5.5 & 2 & 5 & 7.5 \\ 5.75 & 4.75 & 2.75 & 5 & 5 \\ 2.5 & 6.5 & 2.75 & 4.75 & 4 \\ 7.25 & 7 & 2.75 & 9 & 5.75 \\ 10.25 & 3 & 3 & 0.5 & 16 \end{bmatrix}$$

We solve it by Hungarian methods to get the following optimal solution.

Step 1:

Row reduction. Subtract the minimum element of each row from all elements of that row

$$\begin{bmatrix} 2.75 & 3.5 & 0 & 3 & 5.5 \\ 1.53 & 0.52 & 0 & 2.25 & 2.25 \\ 0 & 4 & 0.25 & 2.25 & 1.5 \\ 5.25 & 5 & 0.75 & 0 & 3.75 \\ 9.75 & 2.5 & 2.5 & 0 & 15.5 \end{bmatrix}$$

Step 2: Column reduction. Subtract the minimum element of each column from all elements of that column

$$\begin{bmatrix} 2.75 & 1.5 & 0 & 3 & 4 \\ 3 & 0 & 0 & 2.25 & 0.75 \\ 0 & 2 & 0.25 & 2.25 & 0 \\ 5.25 & 3 & 0.75 & 0 & 2.25 \\ 9.75 & 0.5 & 2.5 & 0 & 14 \end{bmatrix}$$

Step 3: For making assignments, draw minimum possible horizontal and vertical lines covering all zeros.

$$\begin{bmatrix} 2.75 & 1.5 & 0 & 3 & 4 \\ 3 & 0 & 0 & 2.25 & 0.75 \\ 0 & 2 & 0.25 & 2.25 & 0 \\ 5.25 & 3 & 0.75 & 0 & 2.25 \\ 9.75 & 0.5 & 2.5 & 0 & 14 \end{bmatrix}$$

Step 4: Modify the above table by subtracting the smallest uncovered number from all the elements not covered by lines and adding same at the intersection of the two lines.

$$\begin{bmatrix} 2 & 2.25 & 0 & 3 & 4 \\ 0.75 & 0 & 0 & 2.25 & 0 \\ 0 & 2.75 & 1 & 3 & 0 \\ 4.5 & 3 & 0.75 & 0 & 1.5 \\ 9 & 0.5 & 2.5 & 0 & 13.25 \end{bmatrix}$$

Step 5: For making assignments, draw minimum possible horizontal and vertical lines covering all zeros.

$$\begin{bmatrix} 2 & 2.25 & 0 & 3 & 4 \\ 0.75 & 0 & 0 & 2.25 & 0 \\ 0 & 2.75 & 1 & 3 & 0 \\ 4.5 & 3 & 0.75 & 0 & 1.5 \\ 9 & 0.5 & 2.5 & 0 & 13.25 \end{bmatrix}$$

Step 6: Modify the above table by subtracting the smallest uncovered number from all the elements not covered by lines and adding same at the intersection of the two lines.

$$\begin{bmatrix} 2 & 1.5 & 0 & 3 & 4 \\ 0.75 & 0 & 0 & 2.25 & 0 \\ 0 & 2.75 & 1 & 3 & 0 \\ 4.5 & 3 & 0.75 & 0 & 1.5 \\ 9 & 0.25 & 2.5 & 0 & 13.25 \end{bmatrix}$$

Step 7: For making assignments, draw minimum possible horizontal and vertical lines covering all zeros.

$$\begin{bmatrix} 2 & 1.5 & 0 & 3 & 4 \\ 0.75 & 0 & 0 & 2.25 & 0 \\ 0 & 2.75 & 1 & 3 & 0 \\ 4.5 & 3 & 0.75 & 0 & 1.5 \\ 9 & 0.25 & 2.5 & 0 & 13.25 \end{bmatrix}$$

Step 8: Modify the above table by subtracting the smallest uncovered number from all the elements not covered by lines and adding same at the intersection of the two lines.

$$\begin{bmatrix} 1.75 & 1.25 & 0 & 3 & 3.75 \\ 0.75 & 0 & 0.25 & 2 & 0 \\ 0 & 2.75 & 1.25 & 2.75 & 0 \\ 4.25 & 2.75 & 0.75 & 0 & 1.25 \\ 8.75 & 0 & 2.5 & 0 & 13 \end{bmatrix}$$

Step 9: Making assignments

$$\begin{bmatrix} 1.75 & 1.25 & 0 & 3 & 3.75 \\ 0.75 & 0 & 0.25 & 2 & 0 \\ 0 & 2.75 & 1.25 & 2.75 & 0 \\ 4.25 & 2.75 & 0.75 & 0 & 1.25 \\ 8.75 & 0 & 2.5 & 0 & 13 \end{bmatrix}$$

The fuzzy optimal total cost

$$\bar{a}_{13} + \bar{a}_{25} + \bar{a}_{31} + \bar{a}_{44} + \bar{a}_{52} = (1,2,3) + (4,5,9) + (0,2,4) + (1,2,3) + (1,3,4) = (7,14,23)$$

$$Mag(7,14,23) = \frac{1}{2} \int_0^1 (a^* + 3a_0 - a_*) r dr = 14.5$$

Hence the fuzzy optimal cost is 14.5.

Example: 4.2

Consider a fuzzy assignment problem with rows representing eight persons and columns representing the eight jobs with Assignment cost. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum. The cost matrix $((\tilde{c}_{ij})_{n \times n})$ is given whose elements are triangular fuzzy numbers as follows in the below table:

JOBS/	Job 1	Job 2	Job 3	Job 4	Job 5	Job 6	Job 7	Job 8
PERSONS								
Person 1	5,6,7	2,4,8	6,9,12	9,11,13	7,9,11	3,8,13	3,8,17	2,9,14
Person 2	6,9,12	6,8,10	11,12,13	-16,4,9	6,7,9	6,8,10	-6,4,10	12,14,16
Person 3	4,6,12	14,15,16	12,15,18	4,5,7	7,9,11	8,10,16	15,19,20	14,16,19
Person 4	3,9,15	4,6,12	8,15,18	9,10,11	1,2,3	4,5,6	6,7,8	9,10,11
Person 5	7,8,9	4,5,7	1,8,13	10,11,12	13,14,15	16,17,18	11,12,13	8,9,14
Person 6	9,10,11	3,6,9	4,8,12	2,4,6	5,10,15	1,3,5	7,9,11	7,9,12
Person 7	15,16,17	6,12,18	7,14,21	3,6,9	12,15,18	2,4,6	8,9,12	3,13,16
Person 8	10,14,16	8,12,14	4,8,10	6,7,8	2,8,10	5,6,7	1,4,6	13,17,21

Solution:

The same procedure as we applied in example 4.1, we obtain the fuzzy optimal cost is

$$Z = (2, 42, 69).$$

5. Conclusion

In this paper, the assignment cost has been considered as imprecise numbers that can be described by Triangular Fuzzy Numbers which are more realistic and general in nature. Here, the fuzzy assignment problem has been converted into crisp assignment problem using Method of Magnitude and Ranking Function for Robust Ranking Technique for fuzzy costs matrix and solving it by Hungarian Method has been applied to find an optimal solution.

The solution to the fuzzy assignment problem is more relevant and gives the effective solution to place a suitable job for the suitable person. Moreover, one can conclude that the solution to fuzzy problems can be obtained by Robust's Ranking methods effectively. This technique can also be used in solving other types of problems like project schedules, transportation problems and network flow problems.

6: References

1. Ammar, E. and Muamer, M. " Algorithm for solving multi objective linear fractional programming problem with fuzzy rough coefficients" *Open Science Journal of Mathematics and Application*, 4(1), (2016), 1- 8.
2. Chen, M.s.(1985), On a fuzzy assignment problem, *Tamkang J.*22,407- 411.
3. Dubois, D. and Fortemps, p. (1999), Computing improved optimal solutions to max – min flexible constraint satisfaction problem, *European J. Op. R.* 118,95 – 126.
4. Dubois, D. and Prade, H. " Fuzzy Sets and system: Theory and Applications" Academic Press, New York, (1980).
5. Gani, A.N. and Assarudeen, S.N.M. (2012), A new operation on triangular fuzzy number for solving fuzzy linear programming problem, *Appl.Math.Sci.*6, 525-532.
6. Hajjari T, Abbasbandy S. A promoter operator for defuzzification methods. *Australian Journal of Basic and Applied Sciences.*2011; 5(10): 1096-1105.
7. Huang, L. and Xu, G. (2005), solution of assignment for restriction of qualification, *Operation Research and Management Science* 14, 28 – 31.
8. Isabel, K.R. and Uthra, D.G. (2012); An application of linguistic variables in assignment problem with fuzzy costs, *International Journal of Computational Engineering Research*, 2(4), 1065- 1069.
9. Jatinder Pal Singh and Neha Ishesh Thakur (2015), A Novel Method to solve Assignment Problem in Fuzzy Environment, *Industrial Engineering Letters* 1, ISSN 2224-6096 (paper) ISSN 225-0581 (online).
10. Kaufman, A. and Gupta, M. M. "Fuzzy Mathematical Models in Engineering and Management Science" Els. Scie. Publisher, B.v., New York (1988).
11. Kalairasi, k., Sindhu, s., and Arunadevi, M. (2014), Optimization of fuzzy assignment model with triangular fuzzy numbers using Robust Ranking Technique, *International Journal of Innovative Science, Engg. & Technology* 1, 10 – 15.
12. Krishna Prabha S. and Vimala S. (2016), Implementation of BCM for solving Fuzzy Assignment Problem with various Ranking Technique, *Assian Research Journal of Mathematics* 1 (2): 1-11.
13. Kuhn, H.W. (1955), The Hungarian method for the assignment problem, *Naval Re. Logistics Quarterly* 2, 83 – 97.
14. Kumar, A. and Gupta, A. (2011), Methods for solving fuzzy assignment problems and fuzzy traveling salesman problems with different membership functions. *Fuzzy Information and Engineering* 3, 332 – 335.
15. Kumar, A. and Gupta, A. (2012), Assignment and Traveling salesman problems with Coefficients as LR Fuzzy parameters, *Int. J. of App. Sci. and Engg.*10, 155 -170.
16. Kumar, A. and Gupta, A. and Kaur, A.(2009), Methods for solving fully fuzzy assignment problems using triangular fuzzy numbers, *World Academy of Science, Engineering and Technology* 3, 968- 971.
17. Kumar, A. and Bhatia, N. (2011), A new method for solving fuzzy sensitivity analysis for fuzzy linear programming problems, *Int. J. of App. Science and Engineering and Technology* 3, 169 – 176.
18. Li, F. et.al. (2012); Study on solution models and methods for the fuzzy assignment problems, *Expert Systems with Applications*, 39. 11276 – 11283.
19. Lin J.C. and Wen, P.U. (2004); The labelling algorithm for the assignment and transportation problem, *Naval Research Logistics Quarterly*, 2, 83 -97.
20. Lu, H., Huang, G., and He, L.; (2011), An inexact rough – interval fuzzy linear programming method for generating conjunctive water. *Applied mathematics modelling*, 35, 4330 – 4340.

21. Lin, C.J. and Wen, U. P. (2004). The labeling algorithm for the fuzzy assignment problem, Fuzzy sets and Systems 142, 373 - 391.
22. Maleki, H. R. at (2000); Linear programming with fuzzy variables, fuzzy sets and systems 109.21 - 33.
23. Mukherjee, S. and Basu, K. (2010), Application of fuzzy ranking method for solving assignment problems with fuzzy costs, Int.J.I of Computational and App. Math.5, 359 – 368.
24. Mohanaselvi S, Ganesan K. Fuzzy optimal solution to fuzzy transportation problem: A new approach. International Journal on Computer Science and Engineering. 2012;4(3).
25. Nagarajan, R. and Solairaju, A. (2010); Applied Robust ranking technique for solving assignment problems with fuzzy costs, International Journal of Computer Applications,(0975 – 8887). Volume 6. No.4
26. Pal, S. K.; (2004), Soft data mining computational theory of perceptions and rough fuzzy approach, Information Sciences, 163, (1-3), 5-12.
27. Salehi, A.; (2014), An approach for solving multi- objective assignment problem with interval parameters. Management Sciences,4(2014),2155-2160.
28. Sakawa, M. "Fuzzy sets and interactive multi objective optimization" Plenum press, New York, (1993).
29. Shanmugasundari, M. and Ganesan, K. (2013), A novel approach for the fuzzy optimal solution of fuzzy transportation problem, Int.J. of Engg. Research and Applications 3, 1416 – 1424.
30. Wang, X. (1987), Fuzzy optimal assignment problem, Fuzzy Math 3, 101 -108.
31. Yager RR. Ranking fuzzy subsets over the unit interval, in Proceedings of the IEEE Conference on Decision and Control (CDC'78), Albuquerque, NM, USA. 1978; 1435- 1437.
32. Zadeh, L.A. (1965), Fuzzy Sets, Inf. And Control 8, 338-353.