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Bianchi Type-VIII Magnetized Cosmological Model For Perfect Fluid Distribution In General Relativity

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ABSTRACT

Bianchi type-VIII magnetized cosmological model for perfect fluid distribution is investigated in general relativity. We assume that F_{23} is the only nonzero component of electromagnetic field tensor F_{ij}. For the complete assessment of the model, we assume that $\sigma_1^1 \propto \Theta$ which leads to $B = A^n$ where A and B are the metric potentials and n is constant; $\sigma_{11}$is the x-component of shear tensor σ_{ij} and Θ is the expansion in the model. The physical and geometrical significances of the model in the presence of magnetic field are discussed.

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Keywords- Bianchi type-VIII; Magnetic field; General Relativity.

1. INTRODUCTION

In the recent years, there has been a considerable interest in the study of large-scale structure of the universe because of the certainty that the origin of the fabrication in the universe is one of the biggest cosmological secrecy even today. The anisotropy plays a vital role in the early stage of evolution of the universe. The preference of anisotropic model in the Einstein's field equation allows us to find cosmological model more general than FRW models. Bianchi type VIII model is one of the important anisotropic cosmological models and hence it is broad studied in general relativity.

Special classes of Bianchi models are the magnetic universes, provided with a uniform fundamental magnetic field. This gives rise to a preferred spatial direction and so breaks isotropy. Magnetic fields are a crucial ingredient in the universe, whose presence is ubiquitous, but whose origin remains to be fully explained.

Magnetic fields are present in galaxies and clusters and play a crucial role in star and galaxy formation and evolution and they could even be significant on cosmological scales in the early universe. The theory of the magnetic universe has been developed by several authors. Lorentz [7] has investigated Exact Bianchi type VIII and type IX cosmological models with matter and electromagnetic fields. Anisotropic cosmological models including the so-called Bianchi cosmologies (Ellis et al. [5], Gron and Hervik[6]) are of great theoretical importance.

Bali and Jain [3] have studied Bianchi type –III non-static magnetized cosmological model for perfect fluid distribution in general relativity. Subramanian [10] has studied Magnetic fields in the early universe. Rao et.al. [9] have obtained Bianchi types II, VIII, and IX string cosmological models with bulk viscosity in a theory of gravitation.Tyagi and Chhajed [11] have investigated homogeneous anisotropic Bianchi type IX cosmological model for perfect fluid distribution with electro-magnetic field.

Adhavet. al. [1] have investigated Bianchi type VIII cosmological model with linear equation of state.Adhavet. al. [2] have also investigated Bianchi type-II, type-VIII and type-IX cosmological models within the frame work of $f(R,T)$ theory of gravity. Bali and Swati [4] have studied Bianchi type VIII inflationary universe in the presence of massless scalar field with flat potential. Tyagi et al. [12] investigated Magnetized Bianchi Type-VI_0 cosmological model for barotropic fluid distribution with variable magnetic permeability and dark energy.

Motivated by the above-mentioned studies, in this letter, we have investigated Bianchi type-VIII magnetized cosmological model for perfect fluid distribution in general relativity. The physical and geometrical attributes of the model in the presence of magnetic field are discussed.

2. THE METRIC AND FIELD EQUATIONS

The Bianchi type VIII line element is given by

$$ds^2 = dt^2 - B^2dx^2 - A^2dy^2 - [A^2\sinh^2 y + B^2\cosh^2 y]dz^2 - 2B^2 \cosh y\, dxdz \qquad \ldots (1)$$

where A and B are functions of time t only.

We assume the coordinates to be comoving so that

$$v^1=v^2=v^3=0 \text{ and } v^4=1 \qquad \ldots (2)$$

The energy momentum tensor T_i^j with magnetic field is given by

$$T_i^j = (p+\rho)v_iv^j - pg_i^j + \frac{1}{4\pi}\left[g^{rs}F_{ir}F_s^j - \frac{1}{4}F_{rs}F^{rs}g_i^j\right] \qquad \ldots (3)$$

with

$$v_4v^4 = 1 \qquad \ldots (4)$$

We assume that the current is flowing along x-axis so magnetic field is in the yz-plane. Thus F_{23} is the exclusive surviving component of F_{ij}.

The Maxwell's equation

$$\frac{\partial}{\partial x^j}\left(F^{ij}\sqrt{-g}\right) = 0 \qquad \ldots (5)$$

Equation (5) leads to

$$\frac{\partial}{\partial z}(F^{23}A^2B\sinh y) = 0$$

which again leads to

$$\frac{B}{A^2}\frac{\partial}{\partial z}\left(\frac{F_{23}}{\sinh y}\right) = 0$$

Then we get,

$$\frac{F_{23}}{\sinh y} = K$$

Therefore

$$F_{23} = K(\sinh y) \quad \dots (6)$$

where K is constant.

The energy momentum tensor for the line element (1) using (6) is obtained as

$$T_1^1 = -p - \frac{K^2}{8\pi A^4} \quad \dots (7)$$

$$T_2^2 = -p + \frac{K^2}{8\pi A^4} \quad \dots (8)$$

$$T_3^3 = -p + \frac{K^2}{8\pi A^4} \quad \dots (9)$$

$$T_4^4 = \rho - \frac{K^2}{8\pi A^4} \quad \dots (10)$$

The Einstein's field equation is given by

$$R_i^j - \frac{R}{2} g_i^j = -8\pi T_i^j \quad \dots (11)$$

The Einstein's field equation for the line element (1) lead to the following system of equations:

$$2\frac{A_{44}}{A} + \frac{{A_4}^2}{A^2} - \frac{1}{A^2} - \frac{3B^2}{4A^4} = -8\pi p - \frac{K^2}{A^4} \quad \dots (12)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B^2}{4A^4} = -8\pi p + \frac{K^2}{A^4} \quad \dots (13)$$

$$2\frac{A_4 B_4}{AB} + \frac{{A_4}^2}{A^2} - \frac{1}{A^2} - \frac{B^2}{4A^4} = 8\pi\rho - \frac{K^2}{A^4} \quad \dots (14)$$

where suffix 4 stand for derivative with respect to t.

The scalar expansion (θ)is given by

$$\theta = 2\frac{A_4}{A} + \frac{B_4}{B} \quad \dots (15)$$

The components of shear tensor (σ_i^j) and scalar shear (σ) are given by

$$\sigma_1^1 = \frac{2}{3}\left(\frac{B_4}{B} - \frac{A_4}{A}\right) \quad \dots (16)$$

$$\sigma_2^2 = \sigma_3^3 = \frac{1}{3}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) \quad \dots (17)$$

$$\sigma_4^4 = 0 \quad \dots (18)$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) \quad \dots (19)$$

The directional Hubble parameters in the direction of x, y and z respectively for the Bianchi type VIII line element are

$$H_x = H_y = \frac{A_4}{A} \text{ and } H_z = \frac{B_4}{B} \quad \dots (20)$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{3}(H_x + H_y + H_z) \quad \dots (21)$$

A physical sum deceleration parameter q defined by

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \qquad \dots (22)$$

3. SOLUTION OF THE FIELD EQUATIONS

The field equations are (12) - (14) in four unknowns A, B, p and ρ. In order to obtain compatible solutions, we need one excess condition.

For the complete assessment of the model, we assume that $\sigma_1^1 \propto \Theta$ which leads to

$$B = A^n \qquad \dots (23)$$

From equation (12) and equation (13), we obtain

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{{A_4}^2}{A^2} - \frac{1}{A^2} - \frac{A_4B_4}{AB} - \frac{B^2}{A^4} + 2\frac{K^2}{A^4} = 0 \qquad \dots (24)$$

Substituting equation (23) into equation (24), yields

$$2A_{44} + 2\frac{(1-n^2)}{(1-n)}\frac{{A_4}^2}{A} - \frac{2}{(1-n)A} - \frac{2}{(1-n)A^{3-2n}} + \frac{4K^2}{(1-n)A^3} = 0 \qquad \dots (25)$$

Let $A_4 = f(A)$, $A_{44} = f\frac{df}{dA}$, in equation (25), we obtain

$$2ff^1 + 2(n+1)\frac{f^2}{A} = \frac{2}{(1-n)A} + \frac{2}{(1-n)A^{3-2n}} + \frac{4K^2}{(n-1)A^3} \qquad \dots (26)$$

which leads to

$$\frac{d}{dA}(f^2) + 2(n+1)\frac{f^2}{A} = \frac{2}{(1-n)A} + \frac{2}{(1-n)A^{3-2n}} + \frac{4K^2}{(n-1)A^3} \qquad \dots (27)$$

Equation (27), after integration, reduces to

$$f^2 = {A_4}^2 = \frac{1}{1-n^2} + \frac{A^{2n-2}}{2n(1-n)} + \frac{2K^2}{n(n-1)A^2} + CA^{-2(n+1)} (\text{where } n \neq -1,0,1) \qquad \dots (28)$$

where C is the integrating constant.

Equation (28) leads to

$$\int \frac{dA}{\sqrt{\frac{1}{1-n^2} + \frac{A^{2n-2}}{2n(1-n)} + \frac{2K^2}{n(n-1)A^2} + CA^{-2(n+1)}}} = \int dt + M = t + M \qquad \dots (29)$$

where M is the integrating constant.

Appropriate transformation of coordinates

A=T, x=X, y=Y, z=Z

The line element (1) reduces to,

$$ds^2 = \frac{dT^2}{\frac{1}{1-n^2} + \frac{T^{2n-2}}{2n(1-n)} + \frac{2K^2}{n(n-1)T^2} + C\,T^{-2(n+1)}} - T^{2n}dX^2 - T^2dY^2 - [T^2\sinh^2 Y + T^{2n}\cosh^2 Y]dZ^2 - 2T^{2n}\cosh Y\, dX\, dZ \qquad \dots (30)$$

4. THE GEOMETRICAL AND PHYSICAL ATTRIBUTES OF MODEL IN THE PRESENCE OF MAGNETIC FIELD

The energy density (ρ), isotropic pressure (p), the expansion (θ),shear (σ), Proper volume (V^3), Hubble directional parameters (H_x, H_y and H_z),Hubble parameter (H) and deceleration parameter (q) for the model (30) are given by

$$8\pi\rho = \frac{n(n+2)}{(1-n^2)T^2} + \frac{(n+1)(n+2)K^2}{n(n-1)\,T^4} + \frac{(n+1)(n+2)}{4n(1-n)}\frac{1}{T^{4-2n}} + (2n+1)\frac{C}{T^{4+2n}} \quad \dots (31)$$

$$8\pi p = \frac{n^2}{(n^2-1)T^2} + \frac{(n+1)(n-2)K^2}{n(1-n)\,T^4} + \frac{(n+1)(3n-2)}{4n(n-1)}\frac{1}{T^{4-2n}} + (2n+1)\frac{C}{T^{4+2n}} \quad \dots (32)$$

$$\theta = (n+2)\sqrt{\frac{1}{(1-n^2)T^2} + \frac{2K^2}{n(n-1)\,T^4} + \frac{1}{2n(1-n)T^{4-2n}} + \frac{C}{T^{4+2n}}} \quad \dots (33)$$

$$\sigma_1^1 = \frac{2(n-1)}{3}\sqrt{\frac{1}{(1-n^2)T^2} + \frac{2K^2}{n(n-1)\,T^4} + \frac{1}{2n(1-n)T^{4-2n}} + \frac{C}{T^{4+2n}}} \quad \dots (34)$$

$$\sigma_2^2 = \sigma_3^3 = \frac{(n-1)}{3}\sqrt{\frac{1}{(1-n^2)T^2} + \frac{2K^2}{n(n-1)\,T^4} + \frac{1}{2n(1-n)T^{4-2n}} + \frac{C}{T^{4+2n}}} \quad \dots (35)$$

$$\sigma_4^4 = 0 \quad \dots (36)$$

$$\sigma = \frac{(1-n)}{\sqrt{3}}\sqrt{\frac{1}{(1-n^2)T^2} + \frac{2K^2}{n(n-1)\,T^4} + \frac{1}{2n(1-n)T^{4-2n}} + \frac{C}{T^{4+2n}}} \quad \dots (37)$$

From equation (33) and equation (37), we obtain

$$\frac{\sigma}{\theta} = \frac{(1-n)}{\sqrt{3}(n+2)} = \text{Constant}, (n \neq -2) \quad \dots (38)$$

$$V^3 = A^2B = T^{n+2} \quad \dots (39)$$

$$H_x = H_y = \sqrt{\frac{1}{(1-n^2)T^2} + \frac{2K^2}{n(n-1)\,T^4} + \frac{1}{2n(1-n)T^{4-2n}} + \frac{C}{T^{4+2n}}} \quad \dots (40)$$

$$H_z = n\left(\sqrt{\frac{1}{(1-n^2)T^2} + \frac{2K^2}{n(n-1)\,T^4} + \frac{1}{2n(1-n)T^{4-2n}} + \frac{C}{T^{4+2n}}}\right) \quad \dots (41)$$

$$H = \frac{n+2}{3}\left(\sqrt{\frac{1}{(1-n^2)T^2} + \frac{2K^2}{n(n-1)\,T^4} + \frac{1}{2n(1-n)T^{4-2n}} + \frac{C}{T^{4+2n}}}\right) \quad \dots (42)$$

$$q = -1 + \frac{3}{n+2}\left\{\frac{\frac{1}{(1-n^2)T^2} + \frac{4K^2}{n(n-1)\,T^4} + \frac{(n-2)}{2n(n-1)T^{4-2n}} + \frac{(n+2)C}{T^{4+2n}}}{\frac{1}{(1-n^2)T^2} + \frac{2K^2}{n(n-1)\,T^4} + \frac{1}{2n(1-n)T^{4-2n}} + \frac{C}{T^{4+2n}}}\right\} \quad \dots (43)$$

The magnitude of rotation

$$\omega = 0 \quad \dots (44)$$

Weak energy condition

$\rho + p > 0$ which leads to

$$\frac{2n}{(1-n^2)T^2} + \frac{4(n+1)K^2}{n(n-1)\,T^4} + \frac{(n+1)(n-2)}{2n(n-1)}\frac{1}{T^{4-2n}} + \frac{2C(2n+1)}{T^{4+2n}} > 0 \text{ (where } n \neq -1, 0, 1)$$

Strong energy condition

$\rho + 3p > 0$ which leads to

$$\frac{2n}{(n+1)T^2} + \frac{2(n+1)(n-4)K^2}{n(1-n)\,T^4} + \frac{2(n+1)}{n}\frac{1}{T^{4-2n}} + \frac{4C(2n+1)}{T^{4+2n}} > 0 \text{ (where } n \neq -1, 0, 1)$$

5. DISCUSSION

In the present paper, we have constructed Bianchi type-VIII magnetized cosmological model for perfect fluid distribution in general relativity, we get a new explicit solution of Einstein's field equations.

The model (30) starts expanding with big bang at T = 0and the expansion of the model decreases as time increases, for -2 < n < 2. The expansion tends to zero as T→∞.As T → 0, the spatial volume (V^3)→ 0 and V is increasing function of T for n > -2.When n=2, T tends to infinity, deceleration parameter (q) tends to -1 therefore the model represent accelerating phase of universe.

The comparative proportion of the shear scalar σ and expansion θ tends to finite value $i.e.\ \frac{\sigma}{\theta} = \frac{(1-n)}{\sqrt{3}(n+2)}$, (n ≠ -2). Since T→∞, $\frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy. However, if n = 1then $\sigma = 0$. Thus the model isotropizes for n =1. Weak energy condition $\rho + p > 0$ and Strong energy condition $\rho + 3p > 0$ satisfied except n = -1, 0, 1.

Wetake notice that the energy density, pressure and Hubble parameter are decreasing function of time and tend to zero as T→∞. The energy condition ρ ≥ 0 is satisfied for all values of T. All physical parameters decrease more expeditiously in the presence of magnetic field.

The model (30) has Point Type singularity at T=0 for n > 0 (MacCallum [8]). In general, the present model represents shearing, expanding and non-rotating universe.

REFERENCES

1. Adhav, K.S. and Pawade I. D., (2013). "Bianchi type VIII cosmological model with linear equation of state".Int. J. Sci. Adv. Tech., **3**, 30-33.
2. Adhav, K. S., Dawande, M. V. and Deshmukh, R. G. (2013). "Bianchi type II, VIII and IX universe fiilled with wet dark fluid in *f(R,T)* Theory of gravitation". Int. J. Theor. Phys., **3**, 139-146.
3. Bali, R. and Jain, S. (2007). "Bianchi type–III non-static magnetized cosmological model for perfect fluid distribution in general relativity". Astrophys. Space Sci., **311**, 401-406.
4. Bali, R. and Swati (2015). "Bianchi Type VIII Inflationary Universe with Massless Scalar Field in General Relativity".Prespacetime journal.**6**, 679- 683.
5. Ellis, G. F. R., Maartens, R. and MacCallum, M. A. H. (2012). "Text Book on Relativistic Cosmology". Cambridge University Press.
6. Gron, O. and Hervik, S. (2007). "Text Book on Einstein's General Theory of Relativity with Modern Applications in Cosmology". Springer.
7. Lorentz D., (1980). "Exact Bianchi type VIII and IX cosmological models with matter and electromagnetic fields". Phys. Rev. D **22**, 1848.
8. MacCallum M A H (1971). Comm. Math. Phys. **20,** 57.
9. Rao, V. U. M., Sireesha, K. V. S. andSanthi M.V. (2011). "Bianchi types II, VIII, and IX string cosmological models with bulk viscosity in a theory of gravitation." ISRN Mathematical Physics. **2012**.
10. Subramanian, K., (2010). "Magnetic fields in the early universe". AN. 1-11.
11. Tyagi, A. and Chhajed, D. (2012). "Homogeneous Anisotropic Bianchi type IX cosmological model for perfect fluid distribution with electro-magnetic field". American Journal of Mathematics and Statistics.**2,** 19-21.
12. Tyagi, A., Jorwal, P. and Chhajed, D. (2020). "Magnetized Bianchi Type VI_0 cosmological model for barotropic fluid distribution withvariable magnetic permeability and dark energy". J. Rajasthan Acad. Phys. Sci., **19**, 207-216.