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Expansion Of Bianchi Type-III Modified $f(R,T)$ Gravity Model In Lyra's Geometry

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Abstract: Bianchi type-III cosmological models within the frame work of $f(R,T)$ gravity have been investigated in the presence of Lyra's geometry. To obtain the solution of the field equation with $f(R,T)$ gravity based on Lyra's geometry, we consider second case of the $f(R,T)$ model i.e. $f(R,T) = f_1(R) + f_2(T)$, where R is the Ricci scalar and T is the trace of the source matter. Here we consider (EoS) state of the form $p = \gamma\rho$ where γ is a constant. The physical parameters have studied in this paper.

Keywords: Bianchi type-III, Lyra's Geometry, $f(R,T)$ gravity

1. Introduction:

The general theory of relativity formulated by Einstein in 1915, describe the relativity of all motions. General relativity is the theory of all motions, time and gravitation. The cosmology is perplexing fact that the universe in undergoing and accelerating expansion, but the main cause of this reason is still doubt in the cosmic time acceleration of the universe and the existence of dark matter. This has been confirmed various red-shift supernovae experiment [1-4]. Generally, several modification of Einstein theory is attracting furthermore to describe the dark energy and cosmic time also. Such modified theories are $f(R)$ gravity [5-7], $f(G)$ gravity [8], $f(T)$ gravity [9], $f(R,T)$ gravity theory [10]. In this paper, we are focusing in $f(R,T)$ theory of gravity. In $f(R,T)$ gravity, T is the trace of the energy momentum tensor and R is Ricci scalar.

A new modification of Riemannian geometry by introducing a gauge function which removes the non-integrability condition the length of a vector under parallel transport, which is investigates by Lyra [11]. A new scalar- tensor theory of gravitation and constructed an analog of geometry which is normal gauge function investigate by Sen and Dunn [12]. Sen writes the Einstein field equation with Lyra's geometry as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -8\pi G T_{ij} \quad (1)$$

$$\phi_i = (\beta(t), 0,0,0)$$

where $\emptyset_i$ is the displacement vector.

The constant vector displacement field $\emptyset_i$ in Lyra's geometry plays the role of cosmological constant $\wedge$ in the normal general relativistic treatment, investigated by Halford [13]. The scalar-tensor treatment based on Lyra's geometry predicts the same effects, within, which is again investigated by Halford [14]. The exact solution of Sen Equation in Lyra's geometry for the constant deceleration parameter, investigates by Singh and Desikan [15].

A homogeneous and anisotropic space-time described by Bianchi type-III metric with perfect fluid in Lyra geometry investigates by Mollah and Singh et al. [16]. Explore the cosmological solution of Bianchi type-III universe in the presence of massive field within the framework of Lyra's geometry, investigates by Sing and Rani [17]. The physical behavior of the Bianchi type-III model is studied within the framework of $f(R,T)$ gravity by Reddy, Santikumar and Naidu [18]. Tilted Bianchi type-III cosmological model filled with perfect fluid is investigated in $f(R,T)$ theory of gravity by Pawar and Shahare [19]. A dark energy model with EoS parameter is investigated in $f(R,T)$ theory of gravity in Bianchi type-III space-time in the presence of perfect fluid by Reddy et al. [20]. Homogeneous and anisotropic Bianchi type-III space- time in the presence of perfect fluid within the framework of $f(R,T)$ theory of gravity investigates by Chandel and Shriram [21]. Bianchi type-III space- time is considered in the presence of bulk viscous fluid containing one dimension cosmic strings in the frame work of $f(R,T)$ gravity investigates by Kiran and Reddy [22]. Explore the solution of Bianchi type-III cosmological model in Lyra's geometry in the background of anisotropic dark energy [23]

Bianchi type-III cosmological model for a cloud of string with bulk viscosity is studied in Lyra's geometry by Sahoo et al. [24]. Bianchi type-III bulk viscous dust filled cosmological models in Lyra's geometry are investigated by Bali and chandnani [25].

The above discussion and the investigation have inspired us to take up the investigation of Lyra's geometry in Bianchi type-III space-time in $f(R,T)$ theory of gravity. The outline of the work is as follows: In section 2, modified $f(R,T)$ gravity. In section 3, we explain metric and field equation. In section 4,

2. Modified $f(R,T)$ Gravity:

The field equations of $f(R,T)$ gravity are derived from the Hilbert-Einstein variational principle by assuming the metric- dependent Lagrangian density L_m. The action for $f(R,T)$ gravity is given as

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x$$

$$S = \int \left[\sqrt{-g}\left(\frac{1}{16\Pi G}f(R,T) + L_m\right)\right] d^4x \tag{2}$$

where $f(R,T)$ an arbitrary function of Ricci scalar R is and trace of stress- energy tensor T. g is the determinant of the metric tensorg_{ij}, and L_m is the matter lagrangian density. For the matter source, the stress energy tensor T_{ij} is given by,

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta\sqrt{-gL_m}}{\delta g^{ij}} \tag{3}$$

And its trace is

$$T = g^{ij}T_{ij} \tag{4}$$

Here, we have assumed that the matter Lagrangian L_m is depends only on the metric tensor component g_{ij} rather than its derivatives. Hence, we obtain

$$T_{ij} = g_{ij}L_m - \frac{\partial L_m}{\partial g^{ij}} \tag{5}$$

By varying the action S in equation (2) with respect to g_{ij}, the $f(R,T)$ gravity field equations are obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right)f_R(R,T) = -\frac{8\pi G}{c^2}T_{ij} - f_T(R,T)\theta_{ij} - f_T(R,T)T_{ij} \tag{6}$$

where

$$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\delta g^{lm}} \tag{7}$$

Here

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R}, \qquad f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$$

and ∇_i denotes the covariant derivative, $\Box = \nabla^u\nabla_u$ the standard stress energy tensor for matter Lagrangian is given by

$$T_{ij} = (p+\rho)u_iu_j + pg_{ij} \tag{8}$$

Where ρ is the energy density and p is the pressure of the fluid. Here $u^i = (0,0,0,1)$ is the four-velocity vector in the co-moving coordinate system satisfying $u_iu^i = -1$ and $u^i\nabla_j u_i = 0$.

Moreover, the matter Lagrangian is not uniquely considered. So, the source term is described as a function of Lagrangian matter choose a perfect fluid matter as $L_m = -p$ which yields

$$\theta_{ij} = -2T_{ij} - pg_{ij} \tag{9}$$

It is worth to mention that the physical nature of the matter field through θ_{ij} is used to from the field equations of $f(R,T)$ gravity are possible depending on the nature of matter source.

Harko et al. [10] constructed three types of frames of $f(R,T)$ gravity as follows

$$f(R,T) = \begin{cases} R + 2f_1(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \tag{10}$$

In this paper, we have consider second case as

$$f(R,T) = f_1(R) + f_1(T) \tag{11}$$

$$f_1'(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right)f_1' = -\frac{8\pi G}{c^2}T_{ij} + f_2'(T)T_{ij} + [f_2'(T)p + \frac{1}{2}f_2(T)]g_{ij} \tag{12}$$

Let us assume that $f_1(R) = uR$ and $f_1(T) = uT$, where u is arbitrary constant.

For a perfect fluid matter source the field equation of $f(R,T)$ gravity, the equation in (12) becomes

$$R_{ij}-\frac{1}{2}Rg_{ij}=-\frac{8\pi G-\mu c^2}{\mu c^2}T_{ij}+\left[p+\frac{1}{2}T\right]g_{ij} \quad (13)$$

From equation (1) and (12), we get

$$R_{ij}-\frac{1}{2}Rg_{ij}+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}g_{ij}\phi_i\phi^j=-\alpha T_{ij}+\left[p+\frac{1}{2}T\right]g_{ij} \quad (14)$$

Here the displacement vector field is

$\phi^i=(0,0,0,\beta)$ and scalar factor $\alpha=\frac{-8\pi G-\mu c^2}{\mu c^2}$

3. Metric and Field equation:

We consider the spatially homogeneous and anisotropic Bianchi type-III metric in the form

$$ds^2=dt^2-A^2dx^2-B^2e^{-2hx}dy^2-C^2dz^2 \quad (15)$$

where,

A, B and C are the function of cosmic time t alone and h is a constant.

For the metric (15), the Einstein field equation (14) reduce to the form as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{3}{4}\beta^2=\alpha p-\frac{7p+\rho}{2} \quad (16)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{3}{4}\beta^2=\alpha p-\frac{7p+\rho}{2} \quad (17)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{h^2}{A^2}-\frac{3}{4}\beta^2=\alpha p-\frac{7p+\rho}{2} \quad (18)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{h^2}{A^2}+\frac{3}{4}\beta^2=-\alpha(2p+\rho)+\frac{7p+\rho}{2} \quad (19)$$

$$h\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0 \quad (20)$$

Where, the overhead dot denotes derivatives with respect to the cosmic time t.

The covariant derivative of the field equation (14) of right hand side gives the energy conservation law as,

$$\alpha[\dot{\rho}+3H(\rho+p)]-\frac{1}{2}(\dot{\rho}-7\dot{p})=0 \quad (21)$$

And the covariant derivative of the field equation (14) of left hand side gives the energy conservation law as

$$\frac{3}{2}\beta\dot{\beta}+\frac{3}{2}\dot{\beta}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \quad (22)$$

The average scale factor $a(t)$ of the Bianchi type-III space-time is defined as

$$a(t)=(ABC)^{\frac{1}{3}} \quad (23)$$

The spatial volume (v) is defined as

$$v=(a)^3=ABC \quad (24)$$

The generalized Hubble parameter (H) and the scalar expansion (θ) are defined as

$$H=\frac{\dot{a}}{a}=\left(H_x+H_y+H_z\right) \quad (25)$$

$$\theta = 3H = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \quad (26)$$

The directional Hubble parameters are

$$H_x = \frac{\dot{A}}{A}, H_y = \frac{\dot{B}}{B}, \text{ and } H_z = \frac{\dot{C}}{C} \quad (27)$$

The average Hubble parameters is

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\frac{\dot{v}}{v} = \frac{1}{3}\left[\frac{\frac{\partial}{\partial t}(ABC)}{ABC}\right]$$

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \quad (28)$$

The dynamical scalar expansion θ is,

$$\theta = 3H$$

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (29)$$

The shear expansion (σ^2) is defined as

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right] \quad (30)$$

The anisotropy parameter (Δ) is defined as,

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \quad (31)$$

4. A Solution of field equation:

To find the solution of the system we require one condition and this condition we get from equation (20) and on integrating equation (20), we get

$$A = kB \quad (32)$$

For simplicity we choose $k = 1$, and k is integrating constant

$$\Rightarrow \quad A = B \quad (33)$$

Therefore, the equation (16)-(19) reduce to

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{3}{4}\beta^2 = \alpha p - \frac{7p+\rho}{2} \quad (34)$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{h^2}{A^2} - \frac{3}{4}\beta^2 = \alpha p - \frac{7p+\rho}{2} \quad (35)$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{C}}{AC} - \frac{h^2}{A^2} + \frac{3}{4}\beta^2 = -\alpha(2p+\rho) + \frac{7p+\rho}{2} \quad (36)$$

The above equations (34)-(36) involve five unknown parameter i.e. A, C, $\boldsymbol{\beta}$, ρ and p. To solve the above equation, it is required two more physical conditions involving these parameter.

First condition as follows

We considered the equation of state in the form

$$p = \gamma\rho \tag{37}$$

Where γ is a constant, and the second condition as follows

We assume that the expansion scalar θ is proportional to the scalar tensorσ', so that we get

$$BC = A^n \tag{38}$$

Where n is a constant.

$$\Rightarrow\ C = A^{n-1}$$

From the equation (34)-(36) and (38), we get the following quadratic equation

$$n\frac{\ddot{A}}{A} + (n-1)^2\left(\frac{\dot{A}}{A}\right)^2 - \frac{3}{4}\beta^2 = \alpha p - \frac{7p+\rho}{2} \tag{39}$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{h^2}{A^2} - \frac{3}{4}\beta^2 = \alpha p - \frac{7p+\rho}{2} \tag{40}$$

$$(2n-1)\left(\frac{\dot{A}}{A}\right)^2 - \frac{h^2}{A^2} + \frac{3}{4}\beta^2 = -\alpha(2p+\rho) + \frac{7p+\rho}{2} \tag{41}$$

From equation (39) and (40), we get

$$(n-2)\frac{\ddot{A}}{A} + (n^2-2n)\left(\frac{\dot{A}}{A}\right)^2 + \frac{h^2}{A^2} = 0 \tag{42}$$

From equation (40) and (41), we get

$$\frac{\ddot{A}}{A} + (n)\left(\frac{\dot{A}}{A}\right)^2 - \frac{h^2}{A^2} = -\alpha(\rho+p) \tag{43}$$

By solving above two quadratic equations (42) and (43), we get

$$\frac{\ddot{A}}{A} + (n)\left(\frac{\dot{A}}{A}\right)^2 = -\frac{\alpha(\rho+p)}{(n-1)} \tag{44}$$

The equation (24) and equation (44) will give us

$$\dot{V} = \sqrt{-\frac{\alpha(\rho+p)}{(n-1)}\rho V^2 + k_1} \tag{45}$$

Where k_1 is a integrating constant.

Integrating equation (45), we get

$$\int \frac{dv}{\sqrt{-\frac{\alpha(\rho+p)}{(n-1)}\rho V^2 + k_1}} = t + k_2 \tag{46}$$

Where k_2 is an integrating constant that represent a shift of cosmic time t. therefore, it can be chosen as zero.

From the equation (21) and equation (37), we get

$$\rho = (V)^{\frac{2\alpha(1+\gamma)}{1-(7\gamma+2\alpha)}} \tag{47}$$

For solving integration in equation (46) we take $k_1 = k_2 = 0$, we get the volume V as,

$$V = \left[\frac{\alpha^{\frac{3}{2}}(\gamma+1)kt}{2\alpha+7\gamma-1}\right]^{\frac{2\alpha+7\gamma-1}{\alpha(\gamma+1)}} \tag{48}$$

Where $k = \left[-\frac{(n+1)}{(n-1)}\right]^{\frac{1}{2}}$ is a constant term.

So, the scalar factors A and B are obtained from equation (24) and equation (48)

$$A = B = \left[\frac{\alpha^{\frac{3}{2}}(\gamma+1)}{2\alpha+7\gamma-1}kt\right]^{\frac{2\alpha+7\gamma-1}{\alpha(n+1)(\gamma+1)}} \tag{49}$$

The scalar factor C obtained from equations (38) and (49),

$$C = \left[\frac{\alpha^{\frac{3}{2}}(\gamma+1)}{2\alpha+7\gamma-1}kt\right]^{\frac{(n-1)2\alpha+7\gamma-1}{\alpha(n+1)(\gamma+1)}} \tag{50}$$

Using equations (49) and (50) in equation (15), we get the Bianchi type-III metric in the form

$$ds^2 = dt^2 - \left[\frac{\alpha^{\frac{3}{2}}(\gamma+1)}{2\alpha+7\gamma-1}kt\right]^{\frac{2\alpha+7\gamma-1}{\alpha(n+1)(\gamma+1)}}[dx^2 + e^{-2hx}dy^2] - \left[\frac{\alpha^{\frac{3}{2}}(\gamma+1)}{2\alpha+7\gamma-1}kt\right]^{\frac{(n-1)2\alpha+7\gamma-1}{\alpha(n+1)(\gamma+1)}} dz^2 \tag{51}$$

The energy density ρ is obtained by using the equations (47) and (48)

$$\rho = \left[\frac{\alpha^{\frac{3}{2}}(\gamma+1)kt}{2\alpha+7\gamma-1}\right]^2 \tag{52}$$

Therefore, from equation (37), the pressure p can be obtained as

$$p = \gamma\left[\frac{\alpha^{\frac{3}{2}}(\gamma+1)kt}{2\alpha+7\gamma-1}\right]^2 \tag{53}$$

From equation (39),the value of β can be obtained as

$$\frac{3}{4}\beta^2 = \frac{1}{2}\left[\frac{\alpha^{\frac{3}{2}}(\gamma+1)}{2\alpha+7\gamma-1}kt\right]^2 (2\alpha-7)(\gamma-1) - \frac{2\alpha+7\gamma-1}{\alpha(n+1)(\gamma+1)}t^2\left[\frac{2\alpha+7\gamma-1}{\alpha(n+1)(\gamma+1)}n(1-n)+1\right] - n \tag{54}$$

The scalar expansion θ is obtained from the equation (29) as,

$$\theta = \left[\frac{2\alpha+7\gamma-1}{\alpha(\gamma+1)}t^{-1}\right] \tag{55}$$

The Hubble parameter H obtained from equation (28) and shear scalar σ obtained from equation (30) as follows

$$H = \frac{1}{3}t^{-1}\left[\frac{2\alpha+7\gamma-1}{\alpha(\gamma+1)}\right] \tag{56}$$

$$\sigma^2 = \frac{(n-2)^2}{3}\left[\frac{2\alpha+7\gamma-1}{\alpha(n-1)(\gamma+1)}t^{-1}\right]^2 \tag{57}$$

The anisotropy parameter (Δ) is

$$\Delta = \frac{2(n^2-2n+4)}{(n-1)^2} \tag{58}$$

The deceleration parameter is s

$$q = \frac{3\alpha(\gamma+1)}{2\alpha+7\gamma-1} - 1 \tag{59}$$

5. Conclusion:

In this paper we will study Bianchi type-III cosmological model within the framework of $f(R,T)$ gravity in presence of perfect fluid and Lyra geometry. In a Bianchi type-III cosmological model volume ≠0 and density ≠∞, it means that this cosmological model has no singularity and our model represent expanding universe. This model under the assumption of EoS it has been obtained the exact solution of the Einstein field equation. The vector $\boldsymbol{\beta}$ and shear scalar ρ becomes zero as t⟶∞, so our model represents a shear free dark energy cosmological model universe.

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