



# Experimental Investigation On CNC Machining Of Freeform Curves Interpolated With NURBS Interpolator Using Macro Programming

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**Abstract** - The efficiency of the pump, blower, and turbine is highly dependent on the shape of the rotor's profile. Hence, it is required to machine die of such rotating parts specially used in aeronautical and power industries precise machines. Computer Numerical Control machines are used for the machining of such specific parts. By using freeform curves, the aesthetic aspect considering financial feasibility is added to enhance quality. CNC machining of the freeform curve is necessary to attain the quality needed in the current manufacturing world. The machining is quite difficult with the available limited numbers of interpolator codes. The recent practice of dividing curved geometry into linear segments causes toolpath deviation errors. The error distorts shape accuracy and surface quality. The number of linear segments to be increased is the only option. The aforesaid option is creating the problem of elevating the size of the part program. The problem ends in the dilemma of either reducing program size or enhancing surface quality. The solution is in the employment of MACRO programming. In the current research work, a Non-Uniform Rational B Spline (NURBS) interpolator is utilised to interpolate the Archimedean spiral as a freeform curve using MACRO programming. The machined curve is measured with a surface roughness tester. AutoCAD® is used to measure the deviation between the machined and calculated toolpath.

**Keywords** - NURBS Interpolation, MACRO programming, Archimedean Spiral, Surface quality, Shape accuracy

## I. INTRODUCTION

For a long time, CAD systems have used Non-Uniform Rational B-Splines (NURBS). That is why it appears so natural that CNCs should be able to use tool paths that are also defined in terms of NURBS. However, most CNCs now require contoured tool paths to be defined using straight lines or chords. This time-honoured method can result in inefficiencies familiar to almost any die or mould shop. Using chords to define complex geometries accurately results in large, data-dense programmed files—files that have historically been difficult to manage and slow to execute.

The development of NURBS-interpolating CNCs promised programs that could define the same complex geometries with fewer blocks of code, potentially alleviating data-flow bottlenecks. The interpolated path along with cutting tools or machine table movement, is known as a toolpath. Recent Computer-Aided Design (CAD)

System generates toolpath in the form of Cutter Location (CL) points automatically and provides to a Computer-Aided Manufacturing system [1]. The scope of the present research work is the precise and efficient generation of the toolpath with the help of the modern CAD/CAM system. The difference between the actual and machined freeform curved toolpath is known as error or noise. This is because the freeform toolpath is converted into linear segments to generate CL points [2]. Freeform curve geometry might be segmented into NURBS [3]. While machining the freeform curve, the cutting tool experienced a sudden direction change. It leads to the vibration that follows poor surface quality. It is required to reduce machining speed according to the curvature length of the freeform curve. Machining speed is required to decrease so that a parametric interpolator can be used. An analytical method is much needed to interpolate the freeform curve with the help of the NURBS by calculating the correct arc length of the curvature [4]. In general, it can be said that the cutting tool has to travel the freeform toolpath described by the manufacturer. The toolpath is produced by the CAM software in the form of G codes containing linear or circular arc segments. If the machining speed is not reduced according to curvature, it hampers the quality of the surface finish due to repetitive changes in acceleration. To reduce the changes in acceleration, a freeform curved toolpath is calibrated into several linear segments, which furthermore increases program size. NURBS can be introduced as a part of the solution [5]. The profile can be approximated with the NURBS in the case of CNC milling [6]. In total, the curve interpolator has to take care of surface quality while machining freeform geometry. A variable feed algorithm can be proposed to elevate surface quality in this case. NURBS interpolator could improve the surface finish for the desired feed rate at the given curvature [7]. The surface quality is improved by increasing the number of cutter location points and without increasing program length with the help of NURBS for the freeform toolpath [8]. From the literature reviewed, it can be narrated that an algorithm is to be developed to generate cutter location points for a freeform tool path by considering error tolerance. The NURBS interpolator is proposed to avoid curvature discontinuities and to attain smooth feed motion. Literature suggests that the NURBS can achieve higher surface quality compared to the Conventional interpolator. The algorithm is needed to develop to save the part programming time. In the present work, care has been taken that the developed algorithm shall be user-friendly and error-free while spooling the program.

MACRO programming is an efficient tool to develop an algorithm to machine freeform curves [9]. Generation of the toolpath is very crucial to generate free-form shapes. Various algorithms have been developed for that, considering accuracy, surface quality, shape accuracy, etc. [10]. The state-of-the-art CAM/CAE software is helpful in planning tool paths for generating complex surfaces such as turbine blades [11]. Machining of dies, moulds, blades and various parts used in aerospace and automotive is a complex, time-consuming process as they are freeform surfaces [12]. The freeform shapes could be segmented into point sets for parameterisation [13]. A mathematical model can be developed to machine the blade considering the toolpath interval [14]. A method considering federate could be developed for the NURBS curve considering the toolpath. The algorithms could be developed to control chord errors [15].

An Archimedean spiral curved shape is utilised to design a horizontal-axis wind turbine. It leads to higher efficiency considering wind energy compared to conventional wind turbines [16]. During the present work, the Archimedean spiral is considered as it is asymmetric and has variable radii. The curve could be considered a freeform curve as it is fitted with NURBS in a later stage. Firstly, the curve has been drawn using AutoCAD® to determine CL points at regular intervals. G01 is the preparatory code that connects the CL points. Besides this approach, with the help of MACRO programming, the number of CL points is manipulated to influence surface quality. The X and Y coordinates of CL points are determined with the help of the curve parametric equation while using MACRO. Two successive CL points are interpolated linearly. MACRO programming is only dependent on the parametric equation of the curve. Hence, the number of CL points could be varied with the equation. So, it does not affect program length, and it can be said that surface quality is independent of program length while using MACRO. X and Y are the coordinates of the CL points of the curve having the radius (r) of the curve provided by Equation (1).

$$\left. \begin{aligned} r &= a\beta \\ X &= r \cdot \cos \beta \\ Y &= r \cdot \sin \beta \end{aligned} \right\} \quad (1)$$

The radius of the curvature (r) varies with the angle 'β' [17]. With the regular interval of 'β', the desired number of CL points can be achieved.

## II. MATHEMATICAL MODELLING

Considering the recent manufacturing scenario, precise machining of the freeform curve is much needed to satisfy the aesthetic design requirement at a reasonable cost. More CL points will increase surface quality, but it will generate more lines in the part program. Successive CL points will be connected with the linear segments. Each linear segment will add a single line in the part program. It results in a larger program size. In that case, the Conventional programming method is insufficient to produce a quality surface for the freeform curve. Fortunately, there is always a mathematical equation to represent a freeform curve. This equation can be part programmed using MACRO programming. The Achaemenian spiral is fitted with NURBS and represented as a freeform curve in the present research work. MACRO programming of the curve helps to add the desired number of CL points to achieve determined surface quality.

### 2.1 Representation of NURBS curve-

Non-symmetric curves like the Archimedean spiral can be represented by NURBS, as mentioned in equation (2) [18].

$$B(t) = \frac{\sum_{i=0}^n B_i W_i N_{i,k}(t)}{\sum_{i=0}^n W_i N_{i,k}(t)}, \quad 0 \leq t \leq n - k + 2, \quad (2)$$

Control points  $B_i = (n + 1)$ ,  
degree of curve =  $k - 1$ ,

each segment influenced by  $k$  will influence every  $(n - k + 2)$  segments

The basis function of NURBS and its boundary condition is stated by equation (3),

$$N_{i,k}(t) = \frac{(t-t_i)}{(t_{i+k-1}-t_i)} N_{i,k-1}(t) + \frac{(t_{i+k}-t)}{(t_{i+k}-t_{i+1})} N_{i+1,k-1}(t) \quad (3)$$

Where,  $t_i (0 \leq i \leq n + k)$  is knot value

$$t_i = 0, \quad \text{if } i < k$$

$$t_i = i - k + 1, \quad \text{if } k \leq i \leq n$$

$$t_i = n - k + 2, \quad \text{if } i > n$$

$$N_{i,1}(t) = 1, \quad \text{if } t_i \leq t \leq t_{i+1}, \quad \text{otherwise} = 0$$

For example, if the number of control points is six (06), in that case, 'n' will be five (05), and the control point segment will be three (03). Knot values can be calculated as per  $(0 - 0 - 0 - 1 - 2 - 3 - 4 - 4 - 4)$ . Coordinates of CL points of the curve midpoint will be calculated with equation (4).

$$P(t) = \left[ R_s - (R_s - R_e) \left( \frac{\beta_m}{360} \right) \right] \cos(\beta_m) \quad (4)$$

where,  $R_s$  = radius of curve at starting point = 30

$R_e$  = radius of curve at end point = 15

$\beta_m$  = angle traced by of midpoint of the curve with origin =  $45^\circ$

$$P(X) = 19.88, \quad P(Y) = 19.88$$

The coordinates of the control points  $P_1$  on the Archimedian spiral curve interpolated by the NURBS can be calculated using equation (5).

$$P(t) = \frac{(1-t)^2 \cdot w_0 \cdot P_0 + 2t(1-t) \cdot w_1 \cdot P_1 + t^2 \cdot w_2 \cdot P_2}{(1-t)^2 \cdot w_0 + 2t(1-t) \cdot w_1 + t^2 \cdot w_2} \quad (5)$$

Table 1 depicts the X and Y coordinates of the control point of the curve obtained from equation (5)

Table 1 Coordinates of the curve control points

Control points	X	Y
P <sub>0</sub>	30	0
P <sub>1</sub>	26.799	29.45
P <sub>2</sub>	0	26.25
P <sub>3</sub>	-25.7	23.049
P <sub>4</sub>	-22.5	0
P <sub>5</sub>	-19.299	-21.95
P <sub>6</sub>	0	-18.75
P <sub>7</sub>	18.2	-15.549
P <sub>8</sub>	15	0

Similar calculations are shown in Table 2 for the value of 't' varies from 0 to 1 with the interval of 0.01 to find the X and Y coordinates of the CL points of the curve in all four quadrants calculated from equation (5).

After calculating CL points, as shown in Table 2, the curve is plotted using CAD software (i.e., AutoCAD®) to generate part programs using the conventional programming method mentioned in Figure 1.

Table 2 X and Y coordinates of the curve CL points for a regular interval of 't'

Quadrant	T	X	Y	Quadrant	t	X	Y	
1	0	30	0	3	2	-22.5	0	
	0.25	26.94	10.61		2.25	-	19.96	-7.85
	0.5	19.88	19.88		2.5	-	14.58	-14.58
	0.75	10.09	25.36		2.75	-7.32	-18.38	
	1	0	26.25		3	0	-18.75	
2	1	0	26.25	4	3	0	-18.75	
	1.25	-9.23	23.45		3.25	6.47	-16.48	
	1.5	-	17.23		3.5	11.93	-11.93	
	1.75	-	8.70		3.75	14.90	-5.94	
	2	-22.5	0		4	15	0	

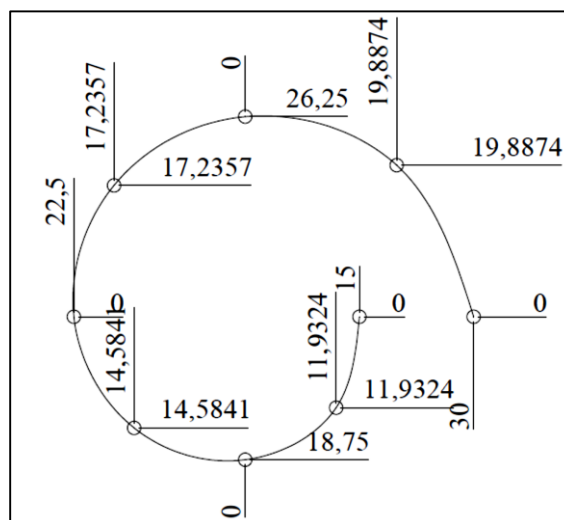


Figure 1. The curve was plotted using CAD software for the conventional programming method

G code is generated for the curve (i.e., Archimedean spiral) drawn in AutoCAD® software using CAM software CIMCO® [19]. Figure 2(a) states conventional part programming for the CL points obtained from the curve drawing using CAD software AutoCAD®. Figure 2(B) depicted MACRO programming for the same curve but approximated with the NURBS interpolator.

<pre> 1 G90 G40 G54 2 M06 T09 D01 ;8 MM END MILL CUTTER 3 G00 X0.0 Y0.0 Z15.0 4 M03 S3000 ;SPINDLE SPEED 5 R41=200 ;FEED RATE IN MM PER MIN 6 R42=-0.5 ;DEPTH 7 N10 G00 X30 Y0 Z5 8 N25 G01 Z=R42 F50 9 N30 X30.000 Y0.000 F=R41 10 N35 X29.555 Y1.401 11 N40 X29.331 Y2.100 12 N45 X29.105 Y2.798 13 N50 X28.877 Y3.494 14 N55 X28.646 Y4.188 15 N60 X28.412 Y4.879 16 N65 X28.175 Y5.566 17 N70 X27.933 Y6.250 </pre>	<pre> 1 G90 G40 G54 2 M06 T09 D01 ;8 MM END MILL CUTTER 3 G00 X0.0 Y0.0 Z15.0 4 M03 S3000 ;SPINDLE SPEED 5 R40=800 ;NO. OF CONTROL POINTS 6 R41=200 ;FEED RATE IN MM PER MIN 7 R42=-0.5 ;DEPTH 8 R52=30.0 ;INITIAL RADIUS OF A SPIRAL 9 R53=15.0 ;FINAL VALUE OF A SPIRAL 10 R4=1.0 ;NO. OF CONVOLUTION 11 R5=1.0 ;CONVOLUTION COUNTER 12 G00 X=R52 Y0.0 Z5.0 13 G01 Z=R42 F50 14 R2=R52 15 R3=(R52-((R52-R53)*R5/R4)) 16 EEE: 17 R1=0.0 18 R12=(R2-((R2-R3)*(0/360)))*COS(0) 19 R13=(R2-((R2-R3)*(45/360)))*COS(45) </pre>
(a)	(b)

Figure 2 (a) Conventional programming of the curve

(b) MACRO programming of the curve

After programming with conventional and MACRO programming methods for the CL points obtained from the AutoCAD® and CL points obtained from the equation of the NURBS, actual CNC machining proceeded. DOE and MACRO programs are prepared to machine non-symmetric profiles on M.S. material to determine the effect of programming methods and interpolating curves on the Machining Time, Program size, Memory requirement, and shape accuracy.

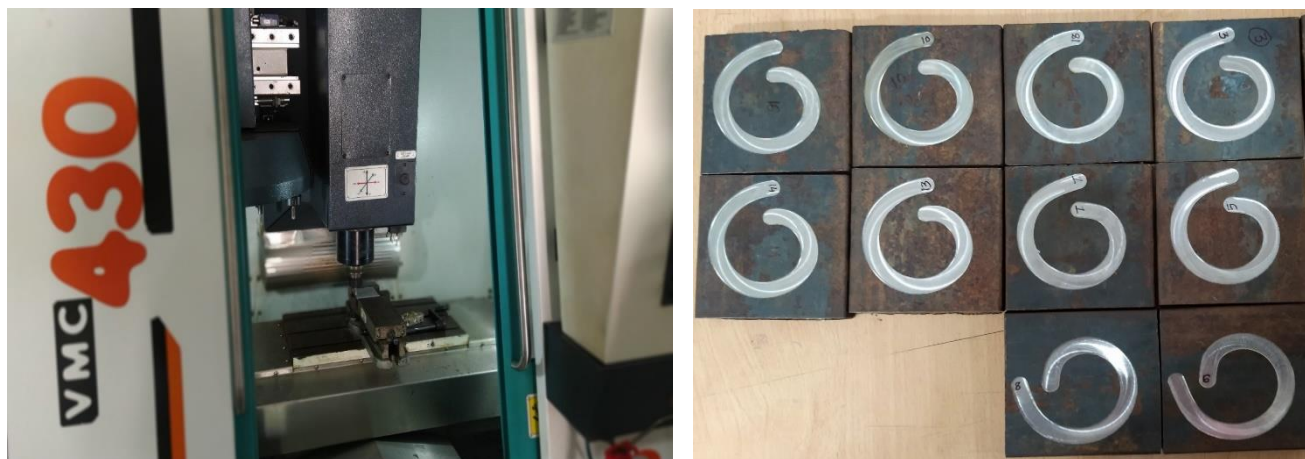
### III. EXPERIMENTAL SETUP:

Cutting parameters are needed to optimise the selection of the suitable combination for the desired output. The RSM and ANOVA are employed to design the experiment. A pilot experiment is done, and the following parameters are selected based on the pilot experiment. The parameters are selected considering the tool maker handbook and machine tool capacity. Feed and Speed have been selected as cutting parameters, while conventional and NURBS interpolation using the MACRO programming method have been selected as interpolation parameters, as shown in Table 3.

Table 3 Parameters for response surface method

Factor	Level
Feed	100, 150, 200 mm/min
Speed	2000,2500,3000 rpm
Method	Conventional, NURBS (MACRO)

Mild Steel (M.S.) is taken as workpiece material, while Solid Carbide is cutting tool material for the end mill cutter. After the determining design of the experiments, actual machining is done using vertical machining center-Siemens controlled-VMC 430 at ITI, Surat, as shown in figure 3(a) and (b).



(a)

(b)

Figure 3(a) and (b) Actual machining of the curve Archimedean Spiral

After machining, surface roughness is measured at the regular angular interval with the MITUTOYO ® surf test SJ-210. The number of runs, selected factors and responses are shown in Table 4, and it is analysed with RSM and ANOVA as shown in Tables 5 and 6.

Table 4 Number of runs, factors and response for RSM

Run	Factor			Roughness (Ra) in $\mu\text{m}$					Average Ra in $\mu\text{m}$
	A: SPEED (RPM)	B: FEED (mm/min)	C: METHOD	1	2	3	4	5	
1	3000	100	NURBS	0.175	0.198	0.235	0.172	0.254	0.2068
2	2500	100	NURBS	0.317	0.245	0.311	0.27	0.229	0.2744
3	2000	150	CONVENTIONAL	0.412	0.32	0.421	0.37	0.408	0.3862
4	2000	100	CONVENTIONAL	0.261	0.302	0.357	0.293	0.257	0.294
5	2000	200	CONVENTIONAL	0.45	0.514	0.541	0.478	0.444	0.4854
6	2500	150	CONVENTIONAL	0.439	0.513	0.518	0.341	0.406	0.4434
7	3000	100	CONVENTIONAL	0.469	0.4	0.464	0.405	0.305	0.4086
8	2000	100	NURBS	0.296	0.174	0.19	0.153	0.116	0.1858
9	3000	150	NURBS	0.259	0.388	0.344	0.37	0.335	0.3392
10	2500	150	NURBS	0.332	0.302	0.315	0.308	0.244	0.3002
11	3000	150	CONVENTIONAL	0.561	0.455	0.461	0.489	0.312	0.4556
12	2000	200	NURBS	0.357	0.373	0.331	0.368	0.327	0.3512
13	2500	100	CONVENTIONAL	0.285	0.417	0.401	0.31	0.354	0.3534
14	2000	150	NURBS	0.345	0.35	0.251	0.321	0.296	0.3126
15	3000	200	CONVENTIONAL	0.371	0.369	0.321	0.43	0.367	0.3716
16	2500	200	CONVENTIONAL	0.572	0.365	0.458	0.438	0.492	0.465
17	2500	200	NURBS	0.279	0.289	0.46	0.278	0.381	0.3374
18	3000	200	NURBS	0.267	0.277	0.403	0.418	0.327	0.3384

Table 5 ANOVA for Linear model

Source	Sum of Squares	df	Mean Square	F-value	p-value	
<b>Model</b>	0.0911	3	0.0304	16.22	< 0.0001	significant
A-SPEED	0.0009	1	0.0009	0.4909	0.4950	
B-FEED	0.0327	1	0.0327	17.45	0.0009	
C-METHOD	0.0575	1	0.0575	30.71	< 0.0001	
<b>Residual</b>	0.0262	14	0.0019			
<b>Cor Total</b>	0.1173	17				

Table 6 Fit Statistics

<b>Std. Dev.</b>	0.0433	<b>R<sup>2</sup></b>	0.7765
<b>Mean</b>	0.3505	<b>Adjusted R<sup>2</sup></b>	0.7287
<b>C.V. %</b>	12.34	<b>Predicted R<sup>2</sup></b>	0.5952
		<b>Adeq Precision</b>	11.5160

Similarly, shape accuracy is also compared for the curved approximated by the NURBS interpolator and using an explicit curve equation plotted by AutoCAD® software, as shown in Figures 4(a) and 4(b).

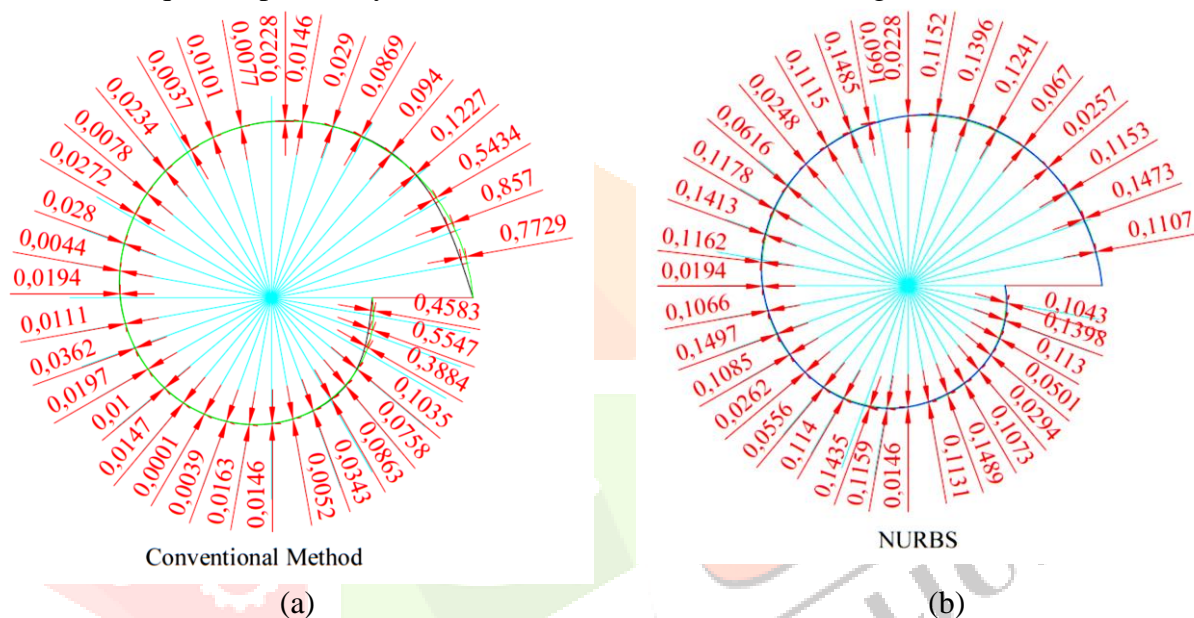


Figure 4(a) and (b) Shape accuracy for conventional and NURBS

After performing the experiments, it is determined by the design of the experiments. Surface roughness and shape accuracy obtained are analysed.

#### IV. RESULT AND DISCUSSION

From ANOVA, the F-value of 16.22 implies that the model is significant. There is a 0.01% chance that the F-value could be more significant than attained in the present work due to noise. The P-value is less than 0.05, which indicates that model terms are significant. In this case, it can be stated that feed and programming methods are significant model terms, as shown in Table 5. The predicted R<sup>2</sup> value of 0.5952 is in reasonable agreement with the adjusted R<sup>2</sup> of 0.7287; i.e. the difference between both values is less than 0.2. Adequate precision measures the signal-to-noise ratio. The ratio should be greater than 4. In the present work, the ratio has a value of 11.516, which indicates an adequate signal. Machining time, Program length, file sizes and radial error are compared, as mentioned in Table 7.

Table 7: Comparison of conventional and MACRO programming methods

Feed (mm/min)	Programming Method	Machining Time (min)	Program Length (No. of blocks)	File size (Bytes)	RMS Radial error (mm)
100	Conventional	1.75	489	10722	0.419
150		1.26			
200		1.03			
100	MACRO (NURBS)	1.75	80	3247	0.075
150		1.26			
200		1.03			

From Table 7, it can be said that the program length (489 block lines), file size (10722 bytes) and radial error (0.419 mm) are more remarkable for the conventional programming method compared to the program length (80 block lines), file size (3247 bytes) and Radial error (0.07 mm) for the MACRO programming method.

## V. CONCLUSIONS

Archimedean spiral is approximated as an asymmetric curve having a variable radius. The curve is fitted by the NURBS interpolator and programmed by the MACRO programming method. The CL points are obtained and utilised in the conventional programming method. Both programming methods are compared regarding file size, program length, surface quality and shape accuracy. From the experiments, the following conclusions can be narrated.

1. ANOVA is adopted to analyse results obtained from the experiments performed according to the design of the experiment. It is found that the method of programming is the most significant factor, while the feed is the second most significant factor in attaining surface quality.
2. In the case of the MACRO programming method, root means square (RMS) values of the chordal and radial error are the least when the curve is interpolated with the NURBS compared to the conventional programming method. NURBS can be determined by control points and weights, which allows manipulation of control points to attain smooth curves. It helps to find shape accuracy better.
3. While machining with MACRO programming, the surface quality achieved is better than the conventional method. The MACRO program utilises equations. So, the processor could calculate the immediate CL point while the cutter is located at the previous CL point. It results in the least vibration of the cutting tool.
4. It is observed that the program length and file size are more for the conventional programming method compared to the program length and file size for the MACRO programming method. That is because MACRO programming is equation-driven, and the program calculates the X and Y coordinates of CL points using the curve equation.

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