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Wavelet transform in depth study and its application- Overview

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Abstract

Wavelet transform (WT) plays a significant role in representing the distribution theory also known as generalized functions. Several researcher [Pilipovi´c et al., 1985, 1987, 1989, 2011, 2016, 2020; Meyer, 1992b; Pathak, 2004a; Ashrrou & Butaev, 2011; Pandey, 2011, 2017; Pathak & Singh, 2006c, 2006d, 2016a, 2016b, 2016c] developed the theory of generalized function spaces using wavelet analysis. These studies are further applied in defining the standard characteristics (convergence, linearity, continuity, boundedness, inversion, uniqueness, etc.) of generalized function spaces [Zemanian, 1987]. There are various complex generalized function spaces also exist, that are still unpredictable in terms of their standard characteristics [Gelfand & Shilov, 1964, 1967, 1968]. The present paper aims to study the WT detail, types and applications in various fields.

Keywords Wavelet transform (WT) Fourier transform (FT) STFT

1.1 Introduction

In the transform theory, Fourier transform (FT) is known as a mathematical transform that decomposes a signal into its constituent frequencies using trigonometric functions (sine or cosine) [Debnath, 2002]. The approach of using the FT works very well when frequency spectrum is stationary [Mallat, 1999]. FT acts as a high resolution in the frequency domain and zero resolution in the time domain at a particular time (Fig. 1.1 a & b) [Pathak, 2009a]. It is an auxiliary tool to find out the frequency components of the signals [Mateo & Talavera, 2018]. FT is generally considered ideal for analysing the periodic signals, as it uses periodic sine and cosine functions (Fig. 1.3 a) [Bochner & Chandrasekharan, 1950]. Generally, the frequencies present in the signal are not time-dependent and spread around anywhere in the signal domain [Mallat, 1989b]. If the signal is more nonstationary/dynamic, the results of analysis will be worse as it uses identical techniques for both stationary and non-stationary type of signals [Champney, 1987]. However, in real life, most of the signals dealing with dynamic systems (related to ECG, stock market, equipment or sensor data, etc.) are non-stationary in nature [Karthikeyan & Kumar, 2013]. The main drawback of FT in signal analysis is that it provides only one feature of signal either time or frequency, in a single window at a particular time (Fig. 1.1 a & b) [Pathak, 1997]. To overcome this problem, much better approaches are available for analysing the dynamic signals instead of using the FT, which suggests that use of other transforms can provide more significant results [Daubechies, 1990]. Several researchers suggested various techniques as a solution to deal with such type of problems [Kaiser, 1994; Pathak, 2009a]. To find both the informations (time and frequency) of a signal in a single window at a particular time with better resolution, two other transforms are widely used

(i) Short-time Fourier transform (STFT)

(ii) Wavelet transform (WT).

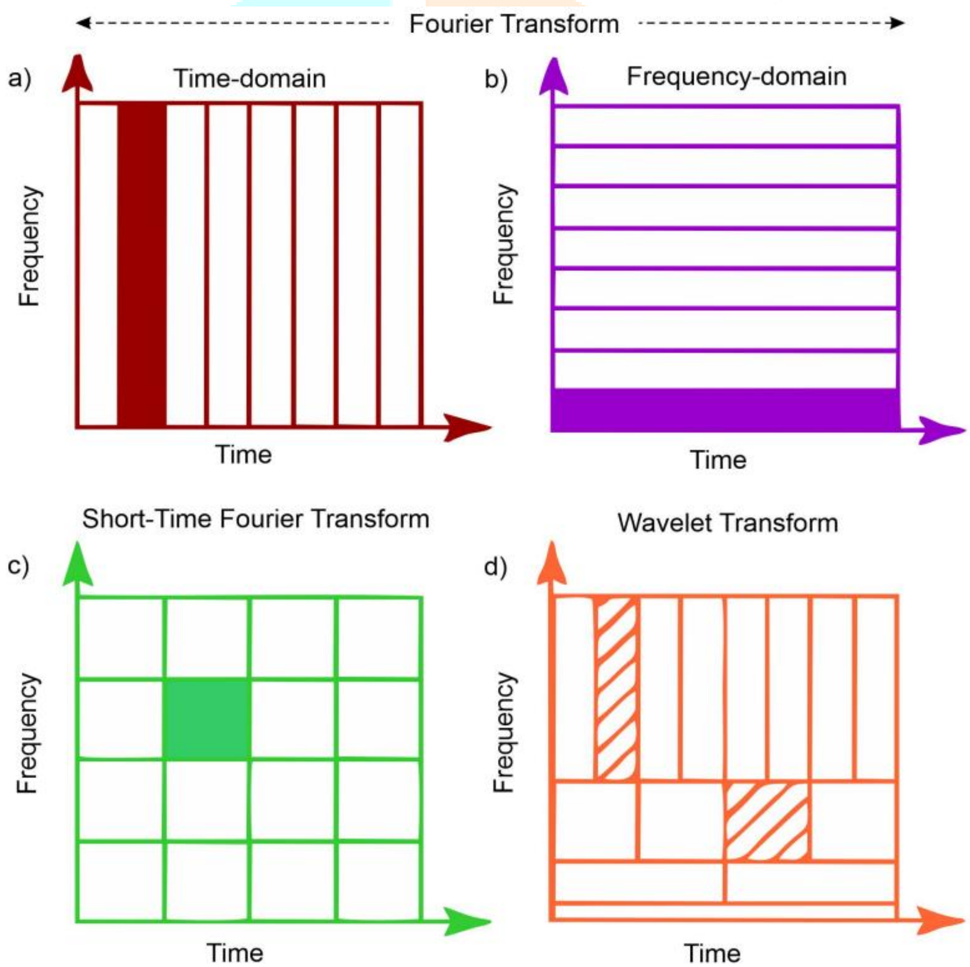

Fig. 1.1: Signal analysis using Fourier Transform (FT), Short-time Fourier Transform (STFT) and Wavelet Transform (WT) representing time and frequency graph in a single window with different resolution, (a) Signal analysis using FT in time domain and filled slit shows minimum resolution (constant) set by FT for different values of signal/data, (b) Signal analysis using FT in frequency domain and filled slit shows minimum resolution (constant) set by FT for different values of signal/data, (c) Signal analysis using STFT and filled square box shows

the medium (constant) resolution of various values of signal/data in time and frequency domain in a single window, (d) Signal analysis using WT on various frequency level and different section shows, how resolution varies for different values of signal/data during analysis in a single window at a particular time [Source: Debnath, 2002]

1.1.1 Short-time Fourier Transform (STFT)

The technique of Short-time Fourier transform (STFT) acts finer than the conventional Fourier transform [Bochner & Chandrasekharan, 1950]. It has medium sized resolution in both the frequency and time domain at a particular time (Fig. 1.1 c). In signal extraction, the STFT divides a large sample of dataset into a smaller window containing various smaller intervals and analyse each smaller interval, separately [Chamneney, 1987; Kaiser, 1994; Debnath & Shah, 2015]. STFT uses a separate window in scrutinize way to analyse the signal and as a resultant spectrogram of the signal appeared [Daubechies, 1992] (Fig. 1.2). It gives the information of both time and frequency components in a single widow at a particular time [Meyer, 1993]. However, the length of window limits the resolution of the frequency (Fig. 1.1 c).

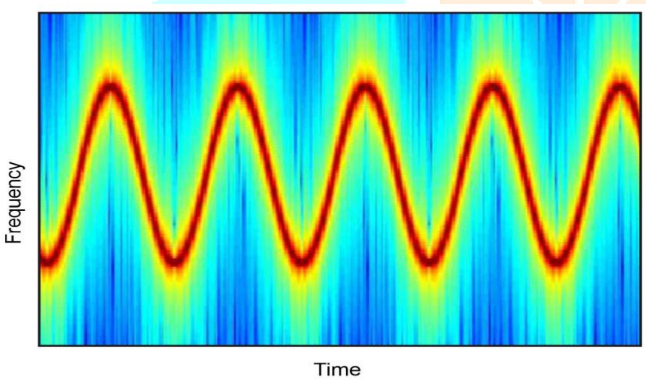

Fig. 1.2: Spectrogram of the signal analysis using Short-time Fourier transform (STFT) [Source: Debnath, 2002].

1.1.2 Wavelet Transform (WT)

Wavelets deals with analysing of a signal in both frequency and time domain in a single window at a particular time (Fig. 1.3 b) [Daubechies, 1990]. The method of signal extraction using wavelets are known as "Wavelet Transform" (WT) [Pathak, 2004a]. WT has high resolution in the frequency domain for small frequency values and low resolution in the time-domain (Fig. 1.1 d) [Debnath, 2002]. However, it has low resolution in the frequency domain for large frequency values and high resolution in the time domain [Daubechies et al., 1986]. In other words, WT makes a trade-off; if time-dependent features are interesting then it highlights as a high resolution at a scale in the time-domain and conversely, if frequency-dependent features are interesting, then it highlights as a high resolution at a scale in the frequency domain [Meyer, 1992b].

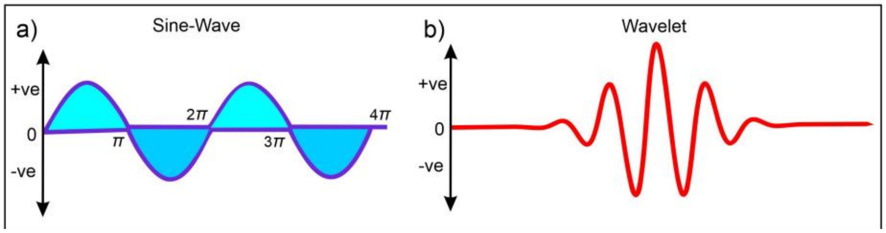

Fig. 1.3: Schematic diagram showing the general structure of sine-wave and wavelet, (a) Sine-wave, and (b) Wavelet.

2. Objectives

The main aim of the present research work is to extend the wavelet theory study and finding the applications of wavelet theory in the field of real-world problems.

3. Overview

WT is considered as a suitable alternative to overcome the signal resolution and time localization difficulties [Daubechies, 1992]. WT acts using supported small wavelets with limited duration (Fig. 1.3 b) [Meyer, 1992a]. It is known as a advanced mathematical tool for decomposing a signal into graph function, that shows the signal details and trends as a function of time [Mallat, 1989b]. This method, firstly breaks the received signal into various frequency components in the relatively small window and then studies each components separately with a resolution matched to its scale [Koornwinder, 1993]. In WT technique, a good approximation of the given function using only a few coefficients is obtained [Meyer, 1992b]. Most of the wavelet coefficients vanish for large number of datasets in the wavelet transform. However, FT fails to deal with large number of data sets due to temporal loss [Meyer, 1993]. This property is considered as the most significant feature of wavelet analysis dealing with large data sets (e.g. Climatology, Engineering, Astrophysics, Medical Sciences, etc.) [Mallat, 1989b].

3.1 Fourier Transform (FT) and its advancement

Fourier transform (FT) was suggested in the early years of 1800 [Pathak, 1997]. In this method, a signal is represented through a linear combination of sine-waves (Fig. 1.3 a) [Holscneider, 1995]. FT acts by multiplying a signal with a series of sine-waves with different frequencies so that, information of frequencies present in a signal can be identified [Butzer & Nessel, 1971]. FT is an extension of the Fourier series that represent the information of resultant function of long duration and allowed the function to approach up to infinity [Debnath, 2002]. FT changes a signal from time-domain to frequency domain [Titchmarsh, 1967]. The peaks in the frequency spectrum indicate the most occurring frequencies in the signal (Fig. 3.1) [Champeney, 1987].

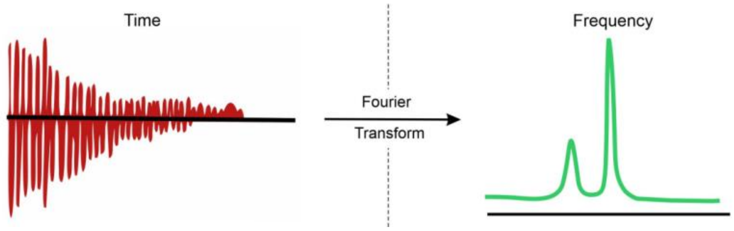

Fig. 3.1: Conversion of a signal from time domain to frequency domain using Fourier transform [Source: Dinc & Yazan, 2018].

Fourier Transform Fourier transform (FT) of a function f is denoted by ˆf or F(f) and represented as [Debnath, 2002]-

$$\mathscr{F}(f(x)) = \hat{f}(\xi) = \int_{-\infty}^{+\infty} e^{-2i\pi x\xi} \; f(x)dx, \qquad (2.1)$$

and the inverse Fourier transform of a function f is define by [Debnath, 2002]-

$$f(x) = \int_{-\infty}^{+\infty} \hat{f}(\xi)\, e^{2i\pi x\xi}\, d\xi. \qquad (2.2)$$

FT further divided into two types- (i) Continuous Fourier transform (CFT), and (ii) Discrete Fourier transform (DFT) [Debnath, 2002]. The basic difference between the methodologies used by the CFT and DFT is that CFT uses the Fourier integral theorem and DFT works using the Fourier series [Titchmarsh, 1967].

3.2 Short-time Fourier Transform

Short-time Fourier transform (STFT), is a Fourier transform based method. STFT is useful to identify the local information of signal as this varies over time (sinusoidal frequency and phase content) [Champeney, 1987]. STFT is represented as follows [Kaiser, 1994]-

$$STFT\,f(t)(a,b) = \mathscr{F}(a,b) = \int_{-\infty}^{+\infty} f(t)\, g(t-a)\, e^{-ibt}\, dt, \qquad (2.3)$$

where g (t) denotes the window function and F(a,b) is representing the convolution of functions f (t) and g (t). In this method, original signal is splitted into several parts of equal length using a sliding window (which may or may not have an overlap) [Bochner & Chandrasekharan, 1950].

Introduction to Wavelets The concept of “wavelets” or “ondelettes” started to appear in the literature in the early 1980’s [Debnath, 2002]. Wavelet is a short “wave-like” oscillation or pulses, which never goes to zero and stays there, they carry on forever like sine or cosine [Meyer, 1992a] (Fig. 1.3 a & b). The precisely localized wavelike function is a more accurate description of a little wave called wavelet [Meyer, 1989b].

There are two properties exist with respect to wavelet that are necessarily satisfied by any of wavelet. They are knows as normalization and orthogonalization of wavelets [Mallat, 1999]. These properties are useful to generate a new type of wavelet from existing ones [Mallat, 1993]. According to these properties a wavelet must have- (i) zero average, and (ii) finite energy. Wavelet is a function $\psi \in L^2(R)$ (square integrable), that possess the following condition as

$$\int_{-\infty}^{+\infty} \hat{\psi}(\omega)d\omega = 0. \qquad (2.4)$$

referred as zero-average condition and this condition implies that ψ changes sign in (−∞,+∞) and also vanishes at (−∞,+∞) [Pathak, 2009a]. Zero-average ensure that wavelet is integrable and the inverse of the WT can be computed [Daubechies, 1992]. The Finite energy represents that wavelet is localized in time and frequency which shows the existence of integrability and the inner product between the wavelet and the signal,

$$\int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty. \qquad (2.5)$$

Where, ψˆ (ω) is the Fourier transform of ψ(x), and defined by as follows [Pathak, 2009a]-

$$\hat{\psi}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-i\omega x} \psi(x)\, dx. \qquad (2.6)$$

Eq. (2.5) is known as admissibility condition, which is useful to obtain inversion of wavelet transform. The admissibility condition implies a wavelet has zero average in the timedomain (a zero frequency in the time-domain).

3.3 Mother Wavelet Wavelets

(ψ(t)) are generated from one single function called mother wavelet with translation and dilation [Pathak, 2009a]. Morlet [1982a] introduced the family of function constructed using dilation a and translation b of ψ(t), represent as ψb,a(t), which is define as follows [Morlet, 1982a, 1982b]-

$$\psi_{b,a}(t) = a^{-n/2}\, \psi\left(\frac{t-b}{a}\right), \quad t \in \mathbb{R}^n \qquad (2.7)$$

where, a is called a scaling parameter, which measures the degree of compression or scale and b is known as the translation parameter.

3.3.1 Types of Wavelets

Wavelets are having various type, but some of them are as follows-(i) Haar-wavelet, (ii) Daubechies-wavelet 4 & 20, (iii) Shannon-wavelet, (iv) Coiflet-wavelet, (v) Mexican hat wavelet, (vi) Bi-orthogonal, and (vii) Gaussian/Spline wavelet (Fig. 3.3). These wavelets have various characteristics (viz. shape, smoothness, compactness, etc.) that is useful to determine the different properties of signal [Meyer, 1992b]. The main advantage of choosing wavelets in the signal extraction is due to zero-average condition (Eq. 2.4), which reveals local information along with global informations related to the signal [Pathak, 2009a]. However, some of wavelets do not satisfy the zero-average condition (viz. Mexican hat and Morlet wavelets) and their average are approximately tends to zero [Chui, 1992b]. Furthermore, some of different characteristics of wavelet are also exist, which are listed as follows [Debnath, 2002; Pathak, 2009a]- (a) Wavelet can be orthogonal or non-orthogonal (a) Wavelet can be orthogonal or non-orthogonal. (b) A wavelet can be bi-orthogonal or not. (c) A wavelet can be symmetric or not. (d) A wavelet can be complex or real (real part representing the amplitude of the wavelet and an imaginary part representing the phase of wavelet). (e) A wavelets is normalized to have unit energy. Wavelet family (having various types of wavelets based on different characteristics and their range of analysis) are further categorised as real-valued wavelets (Beta, Hermtian, Hermitian hat, Mexican hat, Poisson, Shannon, Stomberg, Morlet, etc) and complex valued wavelet (complex Mexican hat, Shannon, frequency B-spline, Morlet & Modified Morlet, etc). Each wavelet family consist various wavelet based on divided subcategories within the family (such as Daubechies wavelet family has different wavelet Daubechies 2, 4, 6,10, 20, etc) [Meyer, 1993].

The different subcategories of wavelets are based on the number of coefficients (the number of vanishing moments) and level of the decomposition [Karthikeyan & Kumar, 2013]. In Daubechies orthogonal wavelets family the wavelets

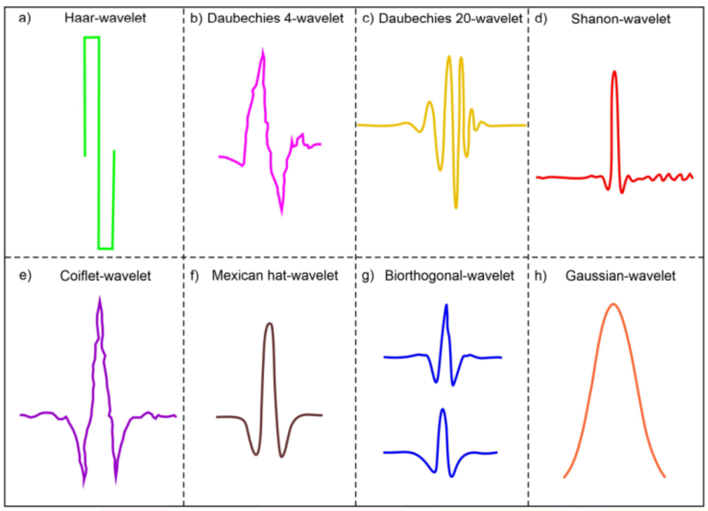

Fig. 3.3: Graphical representation of different wavelets as a) Haar (1909), b) Daubechies 4, c) Daubechies 20 (1988), d) Shannon (1999), e) Coiflet (1991), f) Mexican hat (1980), g) Bi-orthogonal (1990) and h) Gaussian/ Spline (1992) [modified after Nasir et al., 2016].

db1–db10 are commonly most used [Daubechies, 1992]. The index number of wavelets represent the number (N) of coefficients. Each wavelet has a numbers of zero moments or vanishing moments, equals to half the number of coefficients viz. db 2 has one vanishing moment, db 4 has two and similarly db 6 has 3 and so on [Daubechies, 1990]. Basically, a vanishing moment controls the ability of wavelets to represent polynomial behaviour or information of signal [Daubechies, 1992] and wavelet decomposition is useful in several applications such as noise removal, dimension deduction, trend analysis, etc.

3.3.2 Wavelet Transform and its Characteristics

The WT is known as a modified mathematical tool for decomposing a signal into graph function, which shows the signal details and trends as a function of time [Mallat, 1999]. WT was firstly suggested by Jean Morlet (1982a) in the context of seismic signal analysis [Pathak, 2009a]. The developed new wavelet theory is somewhat similar to coherent states technique in quantum mechanics already developed in year 1939 [Cohen & Ryan, 1995]. Further, Alex Grossman [1984] recognized the importance of the WT and provide a detailed theory of wavelet transform along with its various applications [Morlet et al., 1982b]. The joint collaboration of Grossman and

Morlet suggested an exact inversion formula of WT [Grossman & Morlet, 1984]. Now, it has become clear that from analogues to the Fourier expansions, the wavelet theory had provided a new improve method for decomposing a signal/data into bunch of informations [Meyer, 1992c]. This new concept can be viewed as a synthesis of several ideas which originated from different discipline of Sciences including Mathematics, Physics and Engineering [Meyer, 1992b]. The main advantages of using wavelet methods over traditional FT method are the practice of localized basis function (provides details of discontinuities and sharp spikes) and faster computation speed [Grossman & Morlet, 1984].

3.3.3 Types of Wavelet Transform

Wavelet transform (WT) is divided into two types- (i) continuous wavelet transform (CWT), and (ii) discrete wavelet transform (DWT). Generally, WT is considered as CWT, whereas DWT is understood as discrete form of WT [Pathak, 2009a]. For the CWT, several kinds of wavelet basis functions are developed, which all have specific different characteristics [Pathak, 2009a]. The CWT is computed by the convolution of the signal and a wavelet function (a small oscillatory wave which contains convolution analysis and window function) [Debnath, 2002]. However, DWT is computed using filter banks (contain wavelet filters and summarize the frequency content of the signal in diverse sub-bands) [Butzer & Nessel, 1971].

3.3.4 Applications of Wavelet Transform

Wavelet transform has several scientific and industrial applications. Significance of the research on WT with respect to the state-of-the-art in the field of various sciences are given below as (Fig. 2.4)–

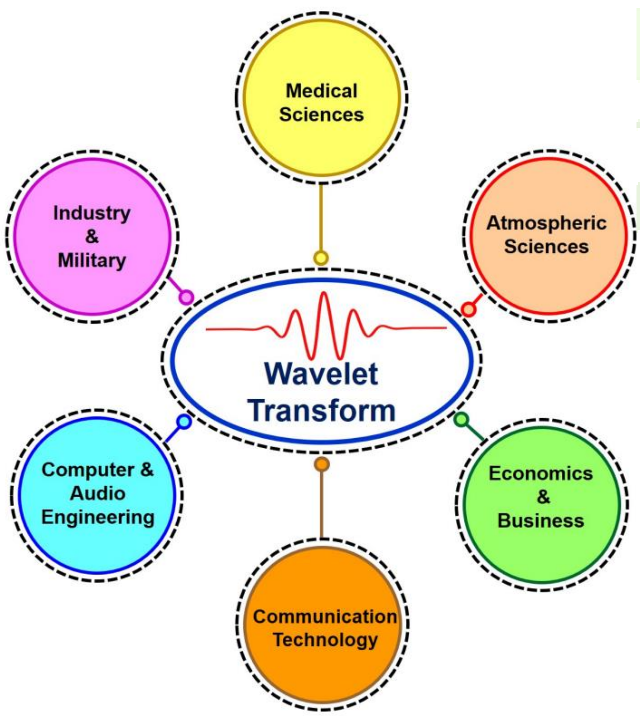

Fig. 3.4: Pictorial representation depicting the several applications of wavelet transform in various sciences.

Summary and Conclusion

In this thesis, an attempt has been made to explore the wavelet transforms of various generalized function spaces and finding the applications of wavelet theory in various fields. In the paresent paper we presented briefly introduction about the background of wavelet transform (WT) and distribution theory, are discussed including the importance of WT technique over existing transforms techniques viz. Fourier transform, Short-time Fourier transform (STFT) or Window Fourier transform (WFT). The preferential of using WT technique is discussed in terms of resolution, frequency, time localization, etc. The work will also be beneficial for understanding the basic properties related WT. The key advantage of this work attempt will be defining of how a purely mathematical theory is useful in real-world applications.

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