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HYDRODYNAMIC FLOWO OF A MICRO-POLAR FLUID BETWEEN TWO SYMMETRIC ROLLERS

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Abstract

Presented herein are in the analytical studies of hydrodynamic lubrication of micro-polar fluid in symmetric rollers. The load capacity have been examined for different values of parameter λ^x, $\frac{\mu}{\gamma}$, and $\frac{k_v}{\gamma_v}$. The load capacity depends different values of parameters. It has been observed that the load capacity increase with the increase of axial distance.

Keywords; Micro-polar fluid, coupling constant, load capacity, axial distance.

Introduction

If a fluid is forced between two symmetric rolls, which are subjected to rotate with equal speed, a thin film, in the form of material sheet of uniform thickness comets out. This process has been named as calendaring .

The process of calendaring can process then sheeting from polymeric materials, when the rolls are driven at very high speeds. According to marshall[2] many thermoplastic materials can be calendared at speeds up to 300 ft/minutes, less with the variable thickness up to 1 millimeter.

The analysis enables us to find the dimensions of the rolls, their speed, and gap distance between them, to calculate the rate at which the material is being driven though the rolls, the thickness of the film produced and the force required to drive the rolls for calendaring. The classical theory of micro-polar fluid has been described by Erigen, and in short.

The importance of micro-polar fluid in gyring rapidly, due to the typical behave our imparted by such type of fluids and scientists are studying the behavior of these fluids from different angles . Yadav and kumar of discussed the behavior of micro-polar fluids of slider bearing which are important from industrial point if view. Willson investigated the stability of micro-polar fluid down as inclined plane.

In this paper we have made in an attempt to analyses the load capacity in a micro-polar fluid between two symmetric rollers.

Formulation of the problem

The basic field equation for micro-polar fluids in the vector are given by the following set of equation ;

$$\nabla \bar{v} = 0 \qquad (1)$$

$$(\lambda_v + 2\mu_v + K_v)\nabla\nabla.\bar{v} - (\mu_v + K_v)\nabla\times\nabla\times\bar{v} + K_v\nabla\times\bar{v} - \nabla p + \rho f = \rho\frac{d\,\bar{v}}{dt} \qquad (2)$$

$$(\alpha_v + \beta_v + \Upsilon v)\nabla\nabla.\bar{v} - \Upsilon_v\nabla\times\nabla\times v + K_v\nabla\times\bar{v} - 2k_v\bar{v} + \rho l = \rho j v \qquad (3)$$

Where α_v, β_v, Υ_v, λ_v, μ_v and K_v are sis material constants, represented body force, l respected the body couple and $\bar{v}$ and j respected the micro rotation and gyration constant respectively.

The present analysis is devoted to the study of calendaring process, forced between two symmetrical rollers of radii R and driven at a uniform speed U_0 simultaneously. The separation at the nip is $2h_0$. for the sake of mathematical simplifications, we assume that flow of the fluid is relatively slow then the speed of rollers, because the micro-polar force are large. These assumptions are restricted to the central region only where the cylindrical surfaces can be treated as parallel plates.

The equation determining the velocity field with the fluid are in general inter-related. Neglecting the velocity derivatives with respect to axial distance and leaving the inertia terms. We use the usual assumptions of lubrication theory which are valid for the present problem also. The governing equation of micro-polar fluid for two dimensional flow between two symmetric rollers as follows.

$$(u_v + k_v)\frac{\partial^2 u_x}{\partial y^2} + k_v\frac{\partial V_x}{\partial y} - \frac{\partial p}{\partial_x} = 0 \qquad (4)$$

$$\gamma_v\frac{\partial^2 V_x}{\partial y^2} - k_v\frac{\partial u_x}{\partial y} - 2k_v v_x = 0 \qquad (5)$$

The equation (4) and (5) are solved under the following boundary conditions;

$$u_x = 0 \qquad at\ y = h$$

$$\frac{\partial u_x}{\partial y} = 0 \qquad at\ y = 0 \qquad (6)$$

$$v_x = 0 \qquad at\ y = \pm h$$

We introduce the following non-dimensional quantities;

$$u_x = U_0 u \qquad , y = h_o y^*$$

$$v_x = \frac{U_0 v}{h_0} \qquad , x = \sqrt{2Rh\,x_0}$$

$$h^* = \frac{h}{h_0} = 1 + x^{*2}$$

$$n_2 = \frac{\mu_v h_0^2}{\gamma_v}, \qquad n_3 = \frac{k_v h_0^2}{\gamma_v}$$

$$p_n = \frac{U_0(\mu_v + K_v)P}{L}, \qquad Q^{\Psi} = \frac{Q}{2h_0 U_0}$$

The governing equation take the form after using ;

$$\frac{\partial^2 u}{\partial y*^2} + \frac{n3}{n2+n3}\frac{\partial v}{\partial y*} - \frac{\partial p*}{\partial x*} = 0 \qquad (8)$$

$$\frac{\partial^2 v}{\partial y^{*2}} - n_3 \frac{\partial u}{\partial y*} - 2n_3 v = 0 \qquad (9)$$

From the fig (1) it is clear that the relationships between h, R and x can be easily be obtained in the form

$$h = h_0 + R - \sqrt{R^2 + X^2} \qquad (10)$$

An usual approximation to the equation (10) may be obtained by expanding the square root term in the binomial series and dropping all but first two terms, we get,

$$h = h_0\left(1 + \frac{x^2}{2Rh_0}\right) \qquad (11)$$

$$h^* = 1 + x^{*2} \qquad (12)$$

The boundary condition (6) reduces to

$$u = 0 \qquad at\ y = h'$$

$$\frac{\partial u}{\partial y^*} = 0 \qquad at\ y^* = 0$$

$$v = 0 \text{ at } y^* = \pm h^* \qquad (13)$$

Solution of the problem

Solving the equation (8) and (9) under the boundary condition (13) we get

$$u = 1 + \frac{\partial p^*}{\partial y^*}\left[\frac{(n_2+n_3)}{(2n_2+n_3)}\left(y*^2 - h^{*2}\right) - h^* \frac{(n_2+n_3)}{(2n_2+n_3)^2}\,\alpha\left(\frac{cosh\alpha y^* - cosh\alpha h^*}{sinh\alpha h}\right)\right] \qquad (14)$$

$$V = \frac{n_3}{\alpha^2}\frac{\partial p^*}{\partial y^*}\left[\frac{h^* \sin h\alpha y^*}{sinh\alpha h^*} - y^*\right] \qquad (15)$$

Where

$$\alpha^2 = \frac{n_3(2n_2+n_3)}{n_2+n_3} \qquad (16)$$

The volume flow rate is give by

$$Q = 2\int_0^{h^*} u\,dy \qquad (17)$$

$$Q = 2h^* - \frac{2h^*(n_2+n_3)}{(2n_2+n_3)}\frac{\partial p^*}{\partial y^*}\left[\frac{2}{3}h^2 + \frac{1}{n_2+n_3}\left(1-\alpha^2 h^*\frac{\cosh\alpha h^*}{\sinh\alpha h^*}\right]\right. \qquad (18)$$

Since the rollers are large, and the angle between then is very small m is very small as well as h con not exceed 2. Writing the expansions of coshαh^x and sinhαh^x in ascending power of x,

We get

$$\frac{\partial p^*}{\partial x^*} = \frac{3}{h^2} + \frac{\alpha^2}{2} - \frac{3Q^x}{2h^2} - \frac{Q^x\alpha^2}{4h^x} \qquad (19)$$

Using equation (7), we get

$$\frac{\partial p^n}{\partial x^m} = \frac{3}{(1+x^*)^2} + \frac{\alpha^2}{2} - \frac{3Q^*}{2(1+x^*)^3} - \frac{Q^*\alpha^2}{(1+x^2)} \qquad (20)$$

The quantity Q^* is yet to be determined in a suitable form. This can be done by using the condition at $x^p = \lambda^p$, where the fluid looks contact with rollers, where $\frac{\partial p^x}{\partial x} = 0$ which gives the form;

$$\frac{3}{(1+\lambda^2)^2} + \frac{\alpha^2}{2} - Q^x\left[\frac{3}{2(1+\lambda^2)^3} + \frac{\alpha^2}{4(1+\lambda^2)}\right] = 0 \qquad (21)$$

Or

$$Q^* = \frac{\frac{3}{(1+\lambda^{*2})^3} + \frac{\alpha^2}{2}}{\frac{3}{2(1+\lambda^2)^3} + \frac{\alpha^2}{4(1+\lambda^2)}} \qquad (22)$$

Integrating equation (20) using the condition $p^x(\lambda) = 0$,we get on expression for the pressure at any point of the contact region, in the form

$$P^* = A_{11}Q^* + A_{12} \qquad (23)$$

Where

$$A_{11} = \left[\frac{3}{8}\left(\frac{\lambda^*}{(1+\lambda^*)^3} - \frac{x^*}{(1+x^{*2})^3}\right) + \frac{9}{16}\left(\frac{\lambda^*}{(1+\lambda^*)} - \frac{x^*}{(1+x^*)} + \tan^{-1}\lambda - \tan^{-1}x\right) + \frac{\alpha^2}{4}\tan^{-1}\lambda - \tan^{-1}x\,3\right]$$

(24)

$$A_{12} = \left[\frac{3}{2}\left(\frac{x^*}{(1+x^{*2})} - \frac{\lambda}{(1+\lambda^{*2})} + \tan^{-1}x - \tan^{-1}\lambda\right) + \frac{\alpha^2}{4}(x^* - \lambda^*)\right] \qquad (25)$$

The load carrying capacity in given by

$$W = \int_0^{\lambda^*} p^*\,d\lambda^* \qquad (26)$$

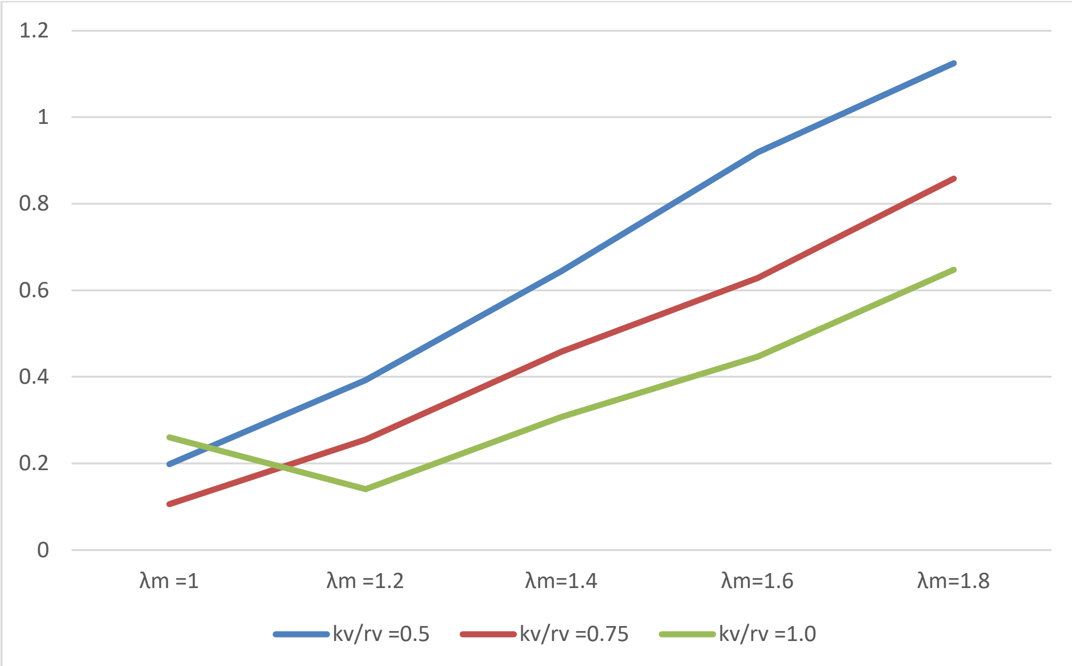
$$W = \left[\frac{9\lambda^{*4}+12\lambda^{*2}-1}{16(1+\lambda^{*2})^2} + \frac{\alpha^2}{8}\log(1+\lambda^{*2}) \right] Q^{*} - \left[\frac{\lambda^{*2}}{2(1+\lambda^{*2})} + \frac{\alpha^2}{u}\lambda^{*2} \right] \quad (27)$$

Result and Conclusion – The result for load capacity have been examined for different values of λ^x, μ_v and K_r. The result describes the variation of load capacity w for various value of the parameters. It has been observed that the load capacity decreases with the increase in the value of coupling constant μ_v and K_v. load capacity increases with increase of axial distance and the micro-polar fluid under consideration paves to the better lubricant for two symmetric rollers.

Table 1

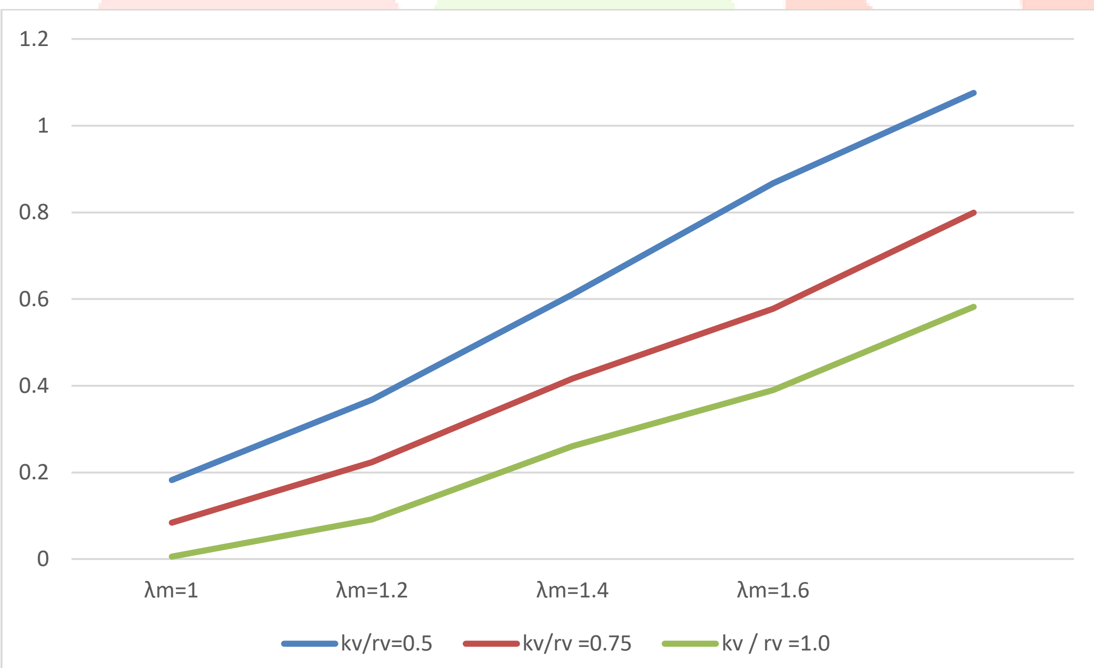
$h_0 = 1$	μ_{v/r_v}	λ^m	$\frac{K_v}{r_v} = 0.5$	$\frac{K_v}{r_v}$=0.75	$\frac{K^v}{r_v} = 1.0$
		1	.1982383	.1056497	0.2602788
		1.2	.3923692	.2549572	.1404296
	.5	1.4	.6446732	.4585927	.3077807
		1.6	.9186253	.6283082	.4462950
		1.8	1.1243310	.8582263	.6471389
		1.0	.1820794	.0839372	.0055607
		1.2	.3679719	.2234524	.0912710
	.75	1.4	.6110949	.4168080	.2608085
		1.6	.8672348	.5776980	.3897832
		1.8	1.0754610	.7995951	.5821184
		1.0	.1714836	0.687232	.0001872
		1.2	.3520790	.2015148	.0779338
	1.0	1.4	.5893610	.4929370	.2265341
		1.6	.8411256	.5427183	.4117947
		1.8	1.0440740	.7556896	.5344372

$\frac{\mu_v}{r_v} = .5$



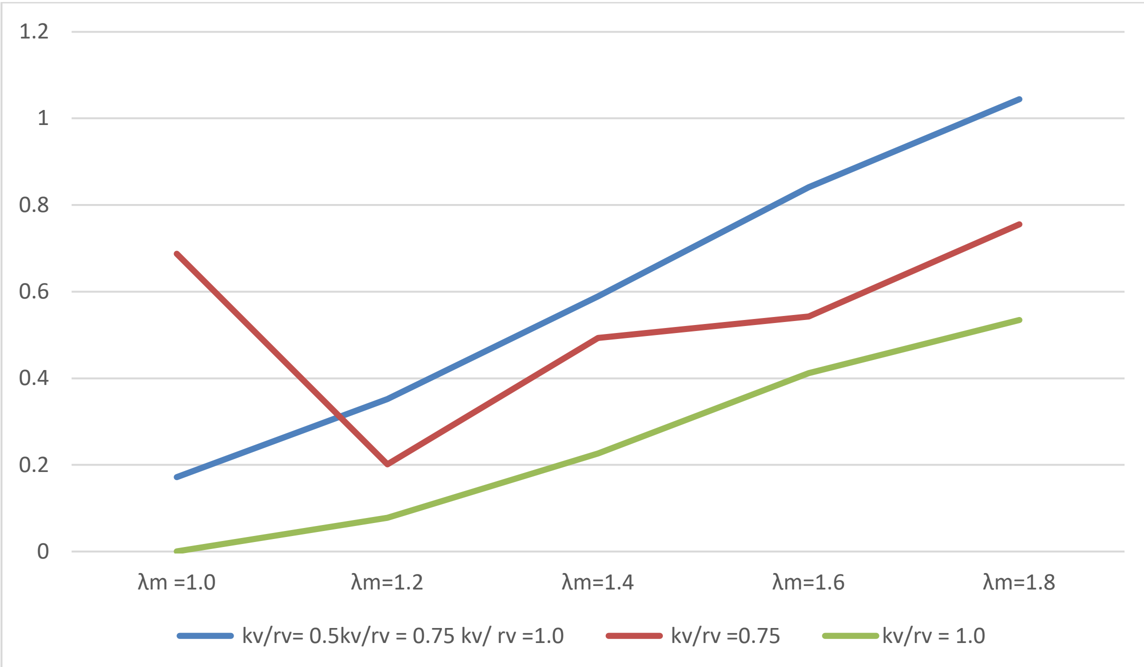
$$\frac{K_v}{r_v} = 0.5 \ , \frac{K_v}{r_v} = 0.75, \frac{K^v}{r_v} = 1.0$$

$$\frac{\mu_v}{r_v} = .75$$



$$\frac{\mathrm{K_v}}{\mathrm{r_v}} = 0.5 \ , \frac{\mathrm{K_v}}{\mathrm{r_v}} = 0.75, \frac{\mathrm{K^v}}{\mathrm{r_v}} = 1.0$$

$$\frac{\boldsymbol{\mu_v}}{\boldsymbol{r_v}} = \mathbf{1.0}$$



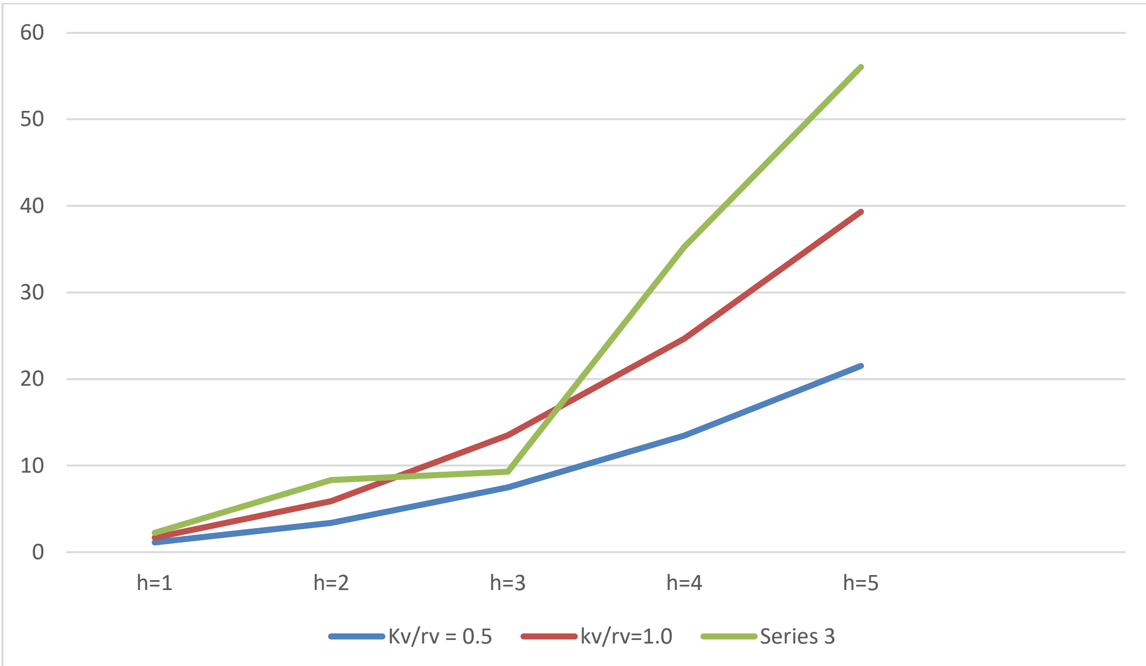
$$\frac{K_v}{r_v} = 0.5 \ , \frac{K_v}{r_v} = 0.75, \frac{K^v}{r_v} = 1.0$$

Table 2

M=.2	μ_v/r_v	H	$\frac{\mathrm{K_v}}{\mathrm{r_v}} = 0.5$	$\frac{K_v}{r_v} = 1.0$	$\frac{K^v}{r_v} = 1.5$
	0.5	1	1.118261	1.686699	2.2275
		2	3.384191	5.892076	8.31217
		3	7.466909	13.512565	9.3000
		4	13.46698	24.673478	35.2429
		5	21.50678	39.335454	56.049144
		1	1.19850	1.851886	2.4554
	1.0	2	3.7334372	6.629148	8.31232
		3	8.3127	15.280411	21.7287
		4	15.083332	27.911757	39.64
		5	24.067890	44.565477	63.0184
		1	2.455463	1.951521	2.608414
	1.5	2	4.090427	7.074890	10.02484

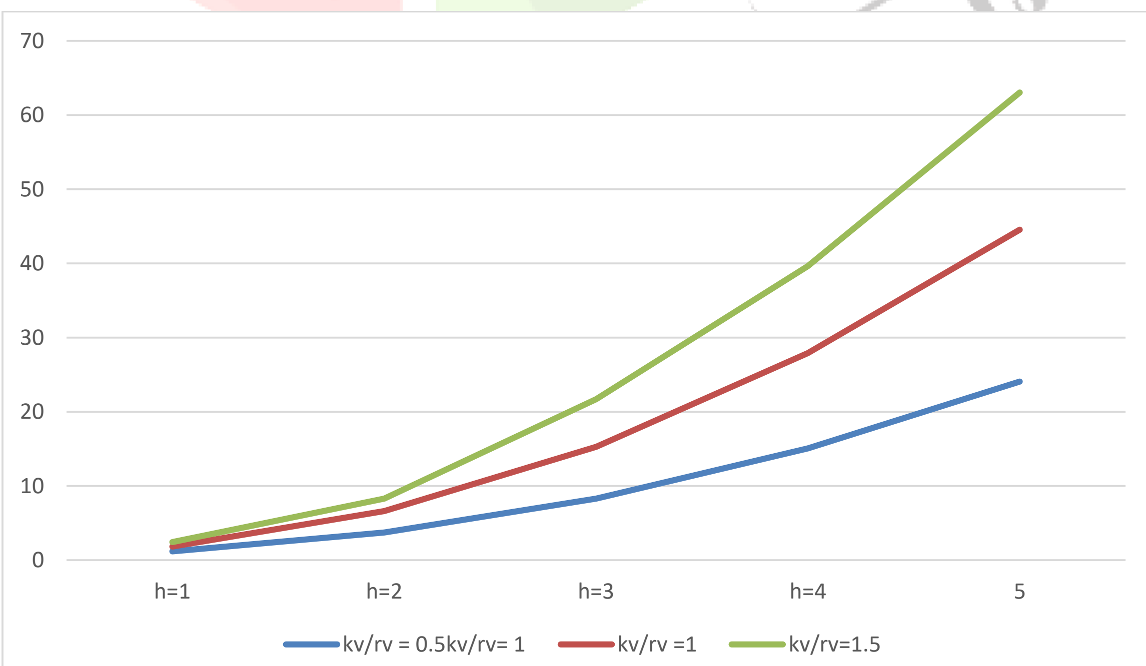
		3	8.738333	16.347170	23.3626
		4	15.872540	29.861333	42.6170
		5	25.330040	47.549607	67.6704

$\frac{\mu_v}{r_v} = 0.5$



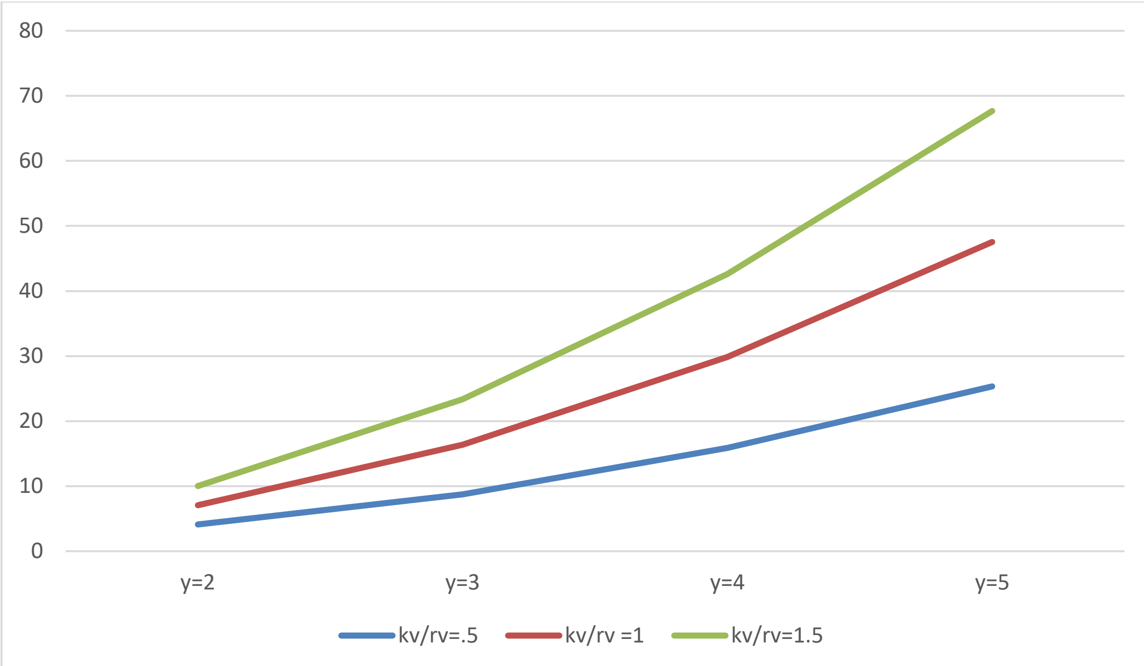
$$\frac{K_v}{r_v} = 0.5\ ,\frac{K_v}{r_v} = 1,\frac{K^v}{r_v} = 1.5$$

$\frac{\mu_v}{r_v} = 1$



$$\frac{K_v}{r_v} = 0.5 \,, \frac{K_v}{r_v} = 1, \frac{K^v}{r_v} = 1.5$$

$\frac{\mu_v}{r_v} = 1.5$



$$\frac{K_v}{r_v} = 0.5 \,, \frac{K_v}{r_v} = 1, \frac{K^v}{r_v} = 1.5$$

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