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Nonlinear Ion-Acoustic(IA) Waves In Electron-Positron-Ion Plasma With Arbitrary Degeneracy Electron And Positron

C. Bhowmik
Department of Mathematics
Saheed Nurul Islam Mahavidyalaya, North 24 Parganas-743286, West Bengal

Abstract

The non-linear properties of ion-acoustic (IA) waves in an electron-positron-ion plasma(e-p-i) plasma with arbitrary degeneracy of electrons and positrons. For this purpose hydrodynamic model is used and a variable coefficient Korteweg-de-Vries (KdV) equation is derived by using the standard reductive perturbation technique. The parameter $p = \frac{n_{po}}{n_{eo}}$, which is the equilibrium density ratio of the positron to electron, plays a vital role in the forming of both bright and dark solitons. It is also found that two parameters, defining the ratio of the ion to electron temperature (σ_i) and the parameter(σ_p) describing the ratio of the positron to electron temperature are shown to play crucial roles in the formation of bright solitons. The present results may be relevant to intense laser produced plasma, high density astrophysical plasmas(i.e. white dwarfs, neutron stars) as well as large density electronics devices.

Keywords: Ion-acoustic(IA) waves, Degenerate plasmas, Solitary waves, KdV equation

INTRODUCTION

Degenerate plasmas have gain much interests in strong laser produced plasmas [1], high density astrophysical plasmas such as in white draft or neutron stars [2] or large density electronic devices [3] and laboratory experiments (plasmas of semiconductors and metals[4]).The wave propagation in a degenerate plasma can be studied using hydrodynamic models. But in hydrodynamic models, the momentum equation for electron is made consistent with the equation of state of a degenerate electron Fermi gas [5, 6].

In the present paper, we investigate the non-linear properties of ion-acoustic(IA) waves in an electron-positron-ion plasma(e-p-i) plasma with arbitrary degeneracy of electrons and positrons. The equation of state for electron and positron follow from a local Fermi-Dirac distribution function of an ideal Fermi gas [7, 8]. By using the standard reductive perturbation technique, we have derived Korteweg-de Vries (KdV) equation. The effect of p, i. e. the ratio of the positron to electron number density, on the profiles of the amplitudes and widths of the solitary structures are examined numerically. We have also shown that two parameters, defining the ratio of the ion to electron temperature(σ_i) and the parameter(σ_p) describing the ratio of the positron to electron temperature are shown to play crucial roles in the formation of bright solitons.

BASIC EQUATIONS AND DERIVATION OF THE KdV

A. Basic equations

We consider an electron-positron-ion plasma(e-p-i) plasma with arbitrary degeneracy of electrons and positrons. The basic equations, describing the dynamics of IAWs in an unmagnetized plasmas with arbitrary degeneracy of electrons and positrons, are as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \qquad (1)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x} - \frac{1}{m_i n_i}\frac{\partial p_i}{\partial x}, \qquad (2)$$

$$0 = e\frac{\partial \phi}{\partial x} - \frac{1}{n_e}\frac{\partial p_e}{\partial x}, \qquad (3)$$

$$0 = -e\frac{\partial \phi}{\partial x} - \frac{1}{n_p}\frac{\partial p_p}{\partial x}, \qquad (4)$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_e - n_p - n_i), \qquad (5)$$

where $p_i = n_i k_B T_i$ and φ is the electrostatic potential, n_j represents particle densities (j = e for electron, j = i for ion and j = p for positron), mi is the ion mass. The equation of state 1 of electron and positron are obtained from the local Fermi-Dirac distribution function [7, 8] of an ideal Fermi gas as follows,

$$p_j = k_B T_j n_j \frac{Li_{5/2}[-\exp(\mu_j/k_B T_j)]}{Li_{3/2}[-\exp(\mu_j/k_B T_j)]}, \quad j = e, p \qquad (6)$$

where k_B is the Boltzmann constant, T_j is the temperature (j = e for electron and p for positron) and μ_j is the chemical potential given by

$$n_j = n_{j0}\frac{Li_{3/2}[-\exp(\mu_j/k_B T_j)]}{Li_{3/2}[-\exp(\mu_{j0}/k_B T_j)]}, \quad j = e, p \qquad (7)$$

where $Li_{\nu}(-z)$ is the polylogarithm function with index ν, which for $\nu > 0$ dened [9] by

$$Li_{\nu}(-z) = -\frac{1}{\Gamma(\nu)}\int_0^{\infty} \frac{s^{\nu-1}}{1 + \exp(s)/z} dz, \quad \nu > 0 \qquad (8)$$

, $\Gamma(\nu)$ is the gamma function and

$$Li_{\nu}(-z) = \left(z\frac{\partial}{\partial z}\right) Li_{\nu+1}(-z), \nu + 1 > 0 \qquad (9)$$

The equilibrium chemical potential μ_{j0} is related to the equilibrium density n_{j0} through

$$-\frac{n_{j0}}{Li_{3/2}[-\exp(\mu_{j0}/k_B T_j)]}\left(\frac{m_j}{2\pi k_B T_j}\right)^{3/2} = 2\left(\frac{m_j}{2\pi\hbar}\right)^{3}, \quad (10)$$

Using Eq.(7), Eqs.(3) and(4) imply

$$0 = e\frac{\partial\phi}{\partial x} - \frac{k_B T_e}{n_{e0}}\frac{Li_{3/2}[-\exp(\mu_{e0}/k_B T_e)]}{Li_{1/2}[-\exp(\mu_e/k_B T_e)]}\frac{\partial n_e}{\partial x}, \quad (11)$$

$$0 = -e\frac{\partial\phi}{\partial x} - \frac{k_B T_p}{n_{p0}}\frac{Li_{3/2}[-\exp(\mu_{p0}/k_B T_p)]}{Li_{1/2}[-\exp(\mu_p/k_B T_p)]}\frac{\partial n_p}{\partial x}, \quad (12)$$

Using the following suitable normalizations

$$x \to \frac{\omega_{pi} x}{c_s}, \quad t \to \omega_{pi} t, \quad n_\alpha \to \frac{n_\alpha}{n_{\alpha 0}}, \quad v_i \to \frac{v_i}{c_s}, \quad \phi \to \frac{e\phi}{m_i c_s^2}, \quad p_\alpha \to \frac{p_\alpha}{n_{\alpha 0} k_B T_\alpha}, \quad \alpha = e, p, i$$

$$\mu_j \to \frac{\mu_j}{k_B T_j}, \quad \mu_{j0} \to \frac{\mu_{j0}}{k_B T_j}, \quad j = e, p \quad (13)$$

The system reduces to the following set of normalized equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \quad (14)$$

$$\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} = -\frac{\partial\phi}{\partial x} - \frac{\nu_e \sigma_i}{n_i}\frac{\partial n_i}{\partial x}, \quad (15)$$

$$0 = \frac{\partial\phi}{\partial x} - \nu_e\frac{Li_{3/2}[-\exp(\mu_{e0})]}{Li_{1/2}[-\exp(\mu_e)]}\frac{\partial n_e}{\partial x}, \quad (16)$$

$$0 = -\frac{\partial\phi}{\partial x} - \nu_e \sigma_p\frac{Li_{3/2}[-\exp(\mu_{p0})]}{Li_{1/2}[-\exp(\mu_p)]}\frac{\partial n_p}{\partial x}, \quad (17)$$

$$\frac{\partial^2\phi}{\partial x^2} = \alpha_e n_e - \alpha_p n_p - n_i, \quad (18)$$

$$n_j = \frac{Li_{3/2}[-\exp(\mu_j)]}{Li_{3/2}[-\exp(\mu_{j0})]}, \quad (19)$$

$$p_j = \frac{Li_{5/2}[-\exp(\mu_j)]}{Li_{3/2}[-\exp(\mu_{j0})]}, \quad (20)$$

where $c_s = \sqrt{k_B T_e/\nu_e m_i}$ is the ion-acoustic (IA) speed and $\omega_{p_i} = \sqrt{4\pi n_{i0} e^2/m_i}$ is the particle plasma frequency, $\nu_e = \frac{Li_{1/2}[-\exp(\mu_{e0})]}{Li_{3/2}[-\exp(\mu_{e0})]}$, $\sigma_i = \frac{T_i}{T_e}$, $\sigma_p = \frac{T_p}{T_e}$, $\alpha_e = \frac{1}{1-p}$, $\alpha_p = \frac{p}{1-p}$, $p = \frac{n_{p0}}{n_{e0}}$, $\mu_{e0} = -\sigma_p \mu_{p0} - 1/\nu_e$.

B. Dispersion Relation

In order to obtain linear dispersion relation, we linearize the system of Eqs. (14)-(20) by considering the first order perturbations (with a subscripted 1) relative to the equilibrium as follows:

$$n_i = 1 + n_{i1}, \quad n_e = 1 + n_{e1}, \quad n_p = 1 + n_{p1}$$

$$v_i = v_{i1}, \quad \phi = \phi_1, \quad \mu_e = \mu_{e0} + \mu_{p1}, \quad \mu_e = \mu_{p0} + \mu_{p1}$$

Assuming the perturbed quantities are proportional to $\exp[i(kx - \omega t)]$, the dispersion relation yield

$$\omega^2 = \frac{1 + \nu_e \sigma_i \left(\alpha_e + \frac{\alpha_p \nu_p}{\nu_e \sigma_p} + k^2\right)}{\frac{\alpha_e}{k^2} + \frac{\alpha_p \nu_p}{\nu_e \sigma_p k^2} + 1}, \tag{21}$$

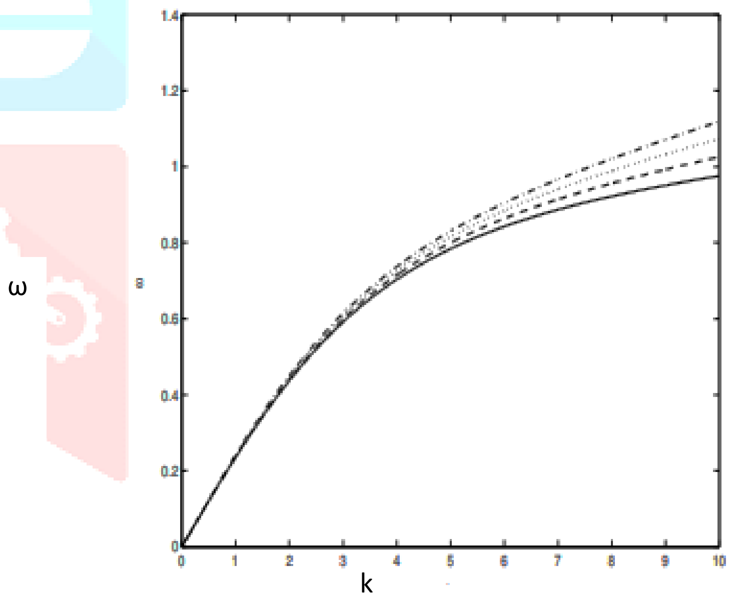

Figure 1: Plot of the dispersion relation relating the frequency ω to the wave number k for different values of σ_i = 0.001 (solid line), 0.002 (dashed line), 0.003 (dotted line) and 0.003 (dashed-dotted line).

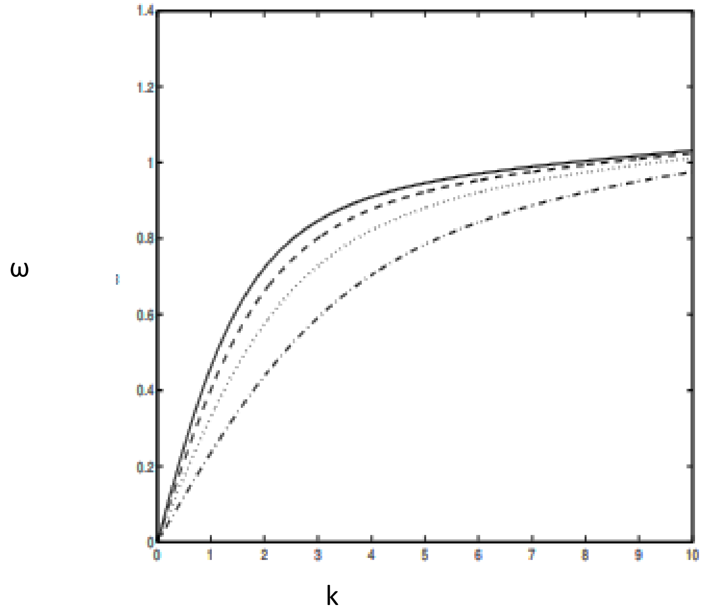

Figure 2: Plot of the dispersion relation relating the frequency ω to the wave number k for different values of p =0.6 (solid line), 0.7 (dashed line), 0.8 (dotted line) and 0.9 (dashed-dotted line).

In Fig. 1 and Fig. 2, we have displayed the dispersion curves of ion-acoustic mode for various values of σ_i and p. It is clear that the wave frequency ω increases when k increases.

C. Derivation and Solitonic solution of the KdV

To investigate the propagation of IA waves in e-p-i plasmas, we employ the standard reductive perturbation technique to obtain the KdV. The independent variables are stretched as$\zeta = \epsilon^{\frac{1}{2}}(x - v_0 t)$,$\tau = \epsilon^{3/2} t$and the dependent variables are expanded as:

$$n_\alpha = 1 + \epsilon n_{\alpha 1} + \epsilon^2 n_{\alpha 2} + ..., \quad \alpha = e, i, p$$

$$v_i = \epsilon v_{i1} + \epsilon^2 v_{i2} + ...,$$

$$\phi = 1 + \epsilon\phi_1 + \epsilon^2\phi_2 + \epsilon^3\phi_3 + ...$$

and

$$\mu_j = \mu_{j0} + \epsilon\mu_{j1} + \epsilon^2\mu_{j2} + ..., \quad j = e, p. \tag{22}$$

where ϵ is a small parameter measuring the weakness of the dispersion and v_0 is the phase velocity of the IAWs.

Substituting the expressions from Eq. (22) in Eqs. (14) - (20)and collecting the terms in different powers of ϵ, we obtain in the lowest order of ϵ as

$$n_{i1} = \frac{1}{v_0^2 - \nu_e\sigma_i}\phi_1, \quad v_{i1} = \frac{v_0}{v_0^2 - \nu_e\sigma_i}\phi_1, \quad \mu_{e1} = \frac{\phi_1}{\nu_e}, \tag{23}$$

$$n_{e1} = \phi_1, \quad n_{p1} = -\frac{\nu_p}{\nu_e\sigma_p}\phi_1, \quad \mu_{p1} = -\frac{\phi_1}{\nu_e\sigma_p} \tag{24}$$

where

$$\nu_p = \frac{Li_{1/2}[-\exp(\mu_{p0})]}{Li_{3/2}[-\exp(\mu_{p0})]}$$

and

$$v_0 = \pm\sqrt{\frac{\nu_e\sigma_i(\nu_e\alpha_e\sigma_p + \alpha_p\nu_p) + \nu_e\sigma_p}{\nu_e\alpha_e\sigma_p + \alpha_p\nu_p}}, \quad (25)$$

which describes phase velocity of the IAWs.

For the next order in ϵ, we obtain

$$\frac{\partial n_{i1}}{\partial \tau} - v_0\frac{\partial n_{i2}}{\partial \xi} + \frac{\partial v_{i2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{i1}v_{i1}) = 0, \quad (26)$$

$$\frac{\partial v_{i1}}{\partial \tau} - v_0\frac{\partial v_{i2}}{\partial \xi} + v_{i1}\frac{\partial v_{v1}}{\partial \xi} = -\frac{\partial \phi_2}{\partial \xi} - \nu_e\sigma_i\left(\frac{\partial n_{i2}}{\partial \xi} - n_{i1}\frac{\partial n_{i1}}{\partial \xi}\right), \quad (27)$$

$$\frac{\partial n_{e2}}{\partial \xi} = \frac{\partial \phi_2}{\partial \xi} + \alpha_1\phi_1\frac{\partial \phi_1}{\partial \xi}, \quad (28)$$

$$\frac{\partial n_{p2}}{\partial \xi} = -\frac{\nu_p}{\nu_e\sigma_p}\frac{\partial \phi_2}{\partial \xi} + \frac{\alpha_2\nu_p^2}{\nu_e^2\sigma_p^2}\phi_1\frac{\partial \phi_1}{\partial \xi}, \quad (29)$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \alpha_e n_{e2} - \alpha_p n_{p2} - n_{i2}, \quad (30)$$

where

$$\alpha_1 = \frac{Li_{-1/2}[-\exp(\mu_{e0})]}{\nu_e Li_{1/2}[-\exp(\mu_{e0})]}$$

$$\alpha_2 = \frac{Li_{-1/2}[-\exp(\mu_{p0})]}{\nu_p Li_{1/2}[-\exp(\mu_{p0})]}$$

Eliminating the second order quantities from Eqs. (26) - (30), we obtain the KdV as

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0, \quad (31)$$

where the non linearity coefficient A and the dispersion coefficient B are given by

$$A = \frac{(v_0^2 - \nu_e\sigma_i)^2}{2v_0}\left[\frac{\alpha_p\alpha_2\nu_p^2}{\nu_e^2\sigma_p^2} + \frac{3v_0^2 - \nu_e\sigma_i}{(v_0^2 - \nu_e\sigma_i)^3} - \alpha_e\alpha_1\right]$$

$$B = \frac{(v_0^2 - \nu_e\sigma_i)^2}{2v_0}$$

The effect of arbitrary degeneracy of electrons appears in both the non linear and dispersive coefficient in the KdV equation (31) but the effect of arbitrary degeneracy of positrons appears in the non linear coefficient.

The equation (31) has a solitary wave solution for a moving frame with a speed u_0

$$\phi_1 = \frac{3u_0}{A}sech^2\left[\sqrt{\frac{u_0}{4B}}(\xi - u_0\tau)\right], \quad (32)$$

where u_0 is the speed of the soliton and $\phi_m = \frac{3u_0}{A}$ is the amplitude and $w = \sqrt{\frac{4B_0}{u_0}}$ is the width of the soliton. So, the IA solitary structures are hump-type or dip-type according as A/B> 0 or < 0 or

$$\frac{(p\alpha_2\nu_p^2/\nu_e^2\sigma_p^2 - \alpha_1)(v_0^2 - \nu_e\sigma_i)^3 + (3v_0^2 - \nu_e\sigma_i)(1-p)}{(1-p)(v_0^2 - \nu_e\sigma_i)^3} > or < 0 \qquad (33)$$

Now, if$v_0 > \nu_e\sigma_i$ then $A/B > 0$ for $p < 1, \frac{\alpha_1}{\alpha_2} < \left(\frac{\nu_p}{\nu_{i\sigma_i}}\right)^2$ and $A/B < 0$ for $> 1, \frac{\alpha_1}{\alpha_2} > \left(\frac{\nu_p}{\nu_{i\sigma_i}}\right)^2$. Thus, the condition (33) mainly depends on the parameters α_1, α_2, ν_p, ν_e, $\sigma_p$and p.

D. Numerical results and discussions

In this section, we will study numerically the effects of p, σ_i and σ_p on the profiles of the solitary waves, given by Eq. (31). Eq. (31) describes both hump (or bright)-type and dip (or dark)-type solitary wave. Fig. 3 shows two different types of solitary waves with increasing p. When the value of p increases from 0.88 to 0.99, amplitude of the hump-type solitary wave decreases rapidly and when the value of p crosses the point p = 0.99, the hump-type solution changes into a dip-type solution and the amplitude of the dip-type solitary wave deceases. Thus, the transformation from hump-type to dip-type solitary wave takes place at higher value of p > 1.

Fig. 4 shows that the effects caused by ion and electron temperatures on the hump-type solution given by (31). As the value of $\sigma_i$increases from 0.5 to 0.7, the amplitude of the soliton increases. On the other hand, Fig. 5 shows that amplitude of the hump-type KdV solitary wave increases as $\sigma_p$increases from 0.4 to 0.9.

Fig. 6 and Fig. 7 show the plot of the solution (31) against ξ and τ for different values of the parameter p = 0.8, p = 1.01. Notice that the soliton appears for p = 0.8 which corresponds to the bright soliton and other is the dark soliton for p = 1.01.

E. Conclusion

The non-linear ion-acoustic (IA) waves in an electron-positron-ion plasma (e-p-i) plasma with arbitrary degeneracy of electrons and positrons have been investigated. Starting from a set of fluid equations for classical ions and Fermi Dirac distribution for electrons and positrons

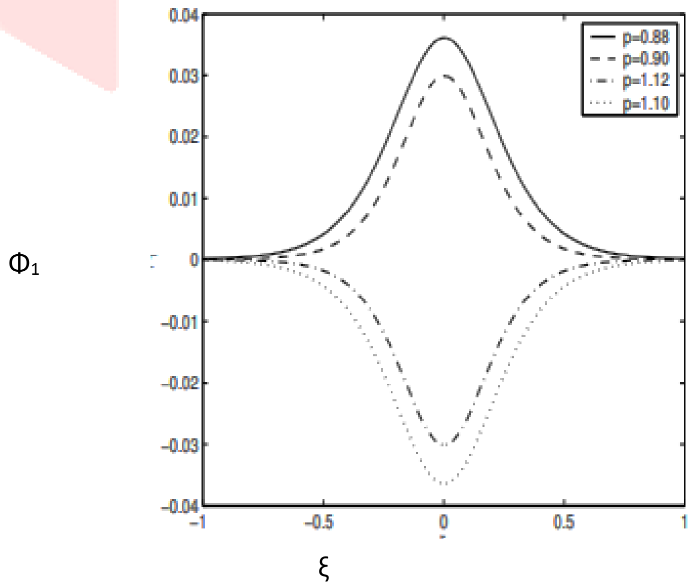

Figure 3: Plot of solitary wave profiles for different values of p.

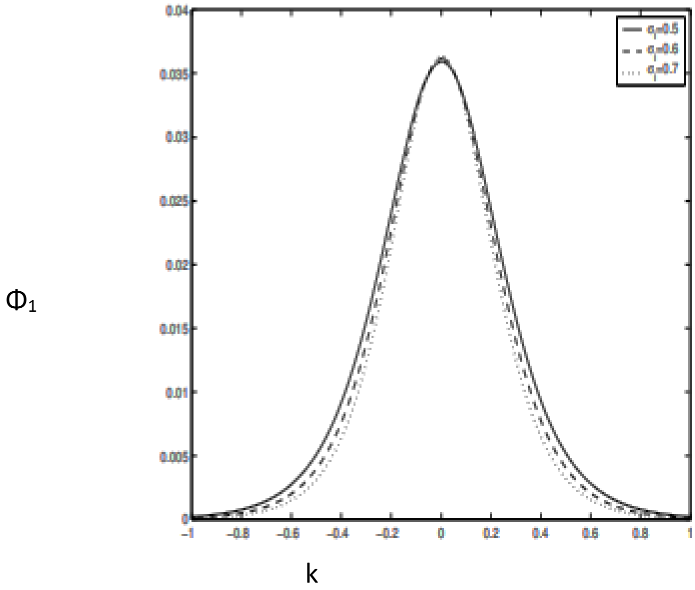

Figure 4: Plot of solitary wave profiles for different values of σ_i.

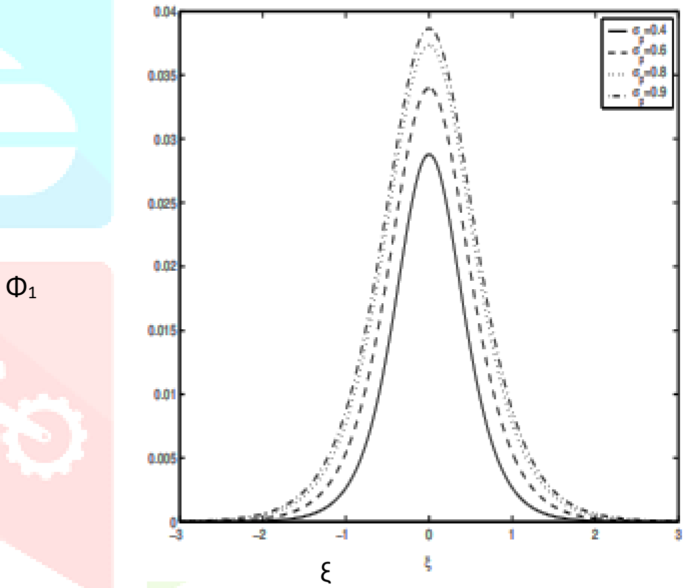

Figure 5: Plot of solitary wave profiles for different values of σ_p.

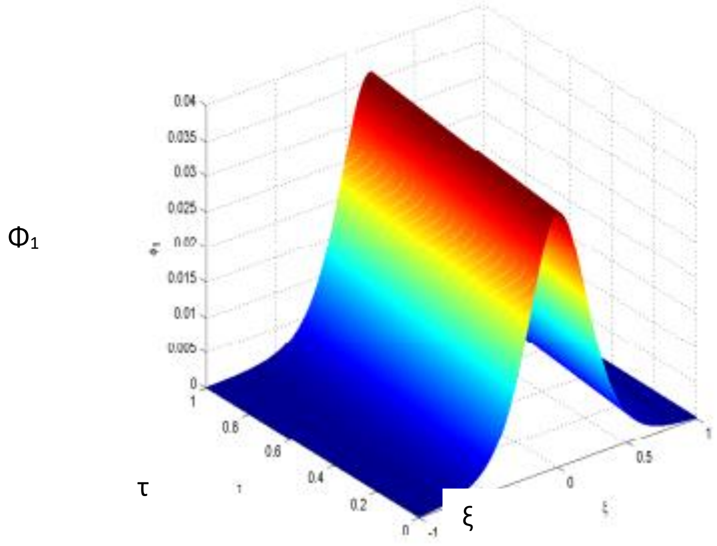

Figure 6: (Color online) The solitary wave solution for Eq.(31) with parameters $\sigma_p = 0.62$, $\sigma_i = 0.58$, $\delta = 10$, $\mu_{p0} = 0.035$: The bright soliton appears at p = 0.8.

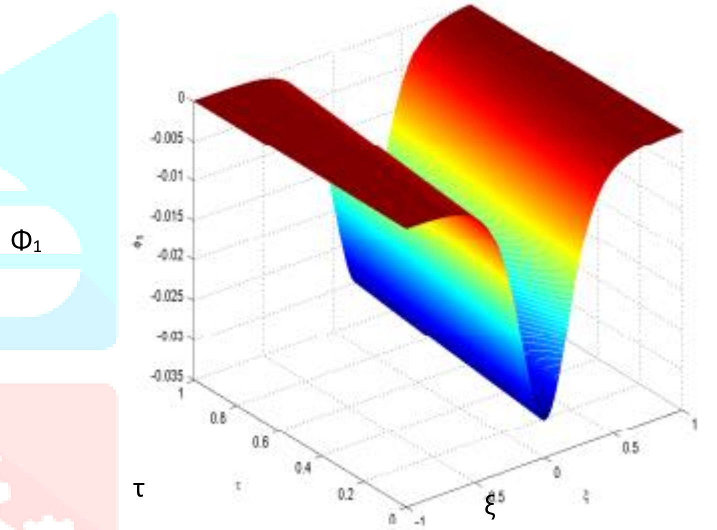

Figure 7: (Color online) The solitary wave solution for Eq.(31) with parameters $\sigma_p = 0.62$, $\sigma_i = 0.58$, $\delta = 10$, $\mu_{p0} = 0.035$: The dark soliton appears at p = 1.01

a linear dispersion relation for IA waves is derived. The non linear theory of IA waves is studied with the help of KdV equation. Different domains in parameter space for the existence of IA solitary waved obtained. The results should be useful for understanding of ion-acoustic solitary waves propagation in electron-positron-ion plasma with arbitrary degeneracy electron and positron.

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