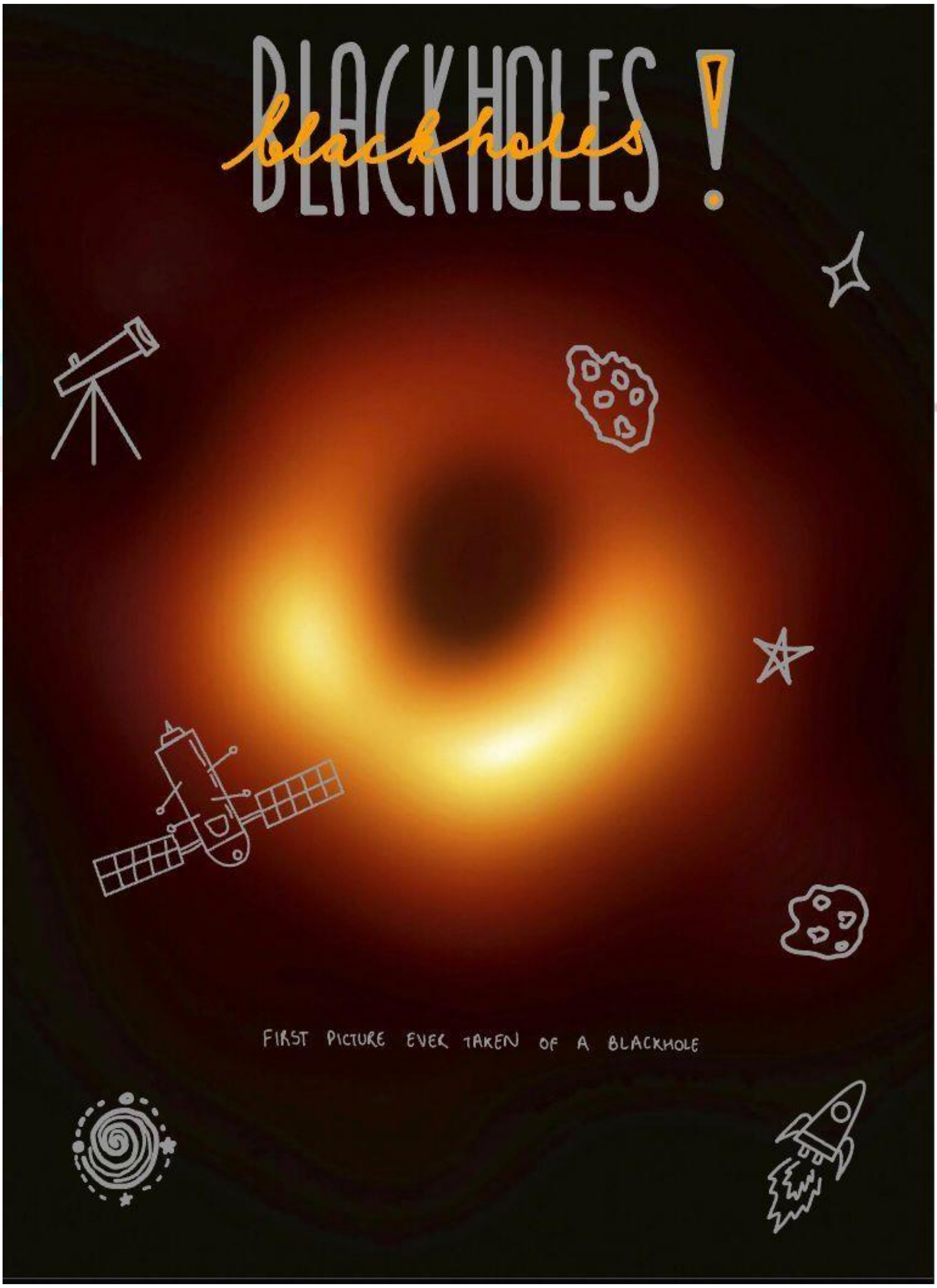
IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

BLACK HOLES

YASHVARDHAN MARDA



INTRODUCTION

After Einstein revealed his great general theory of relativity, physicist realised that it allowed for the possibility of catastrophic gravitational collapse in places of extreme density like the dead core of a massive star where space and timecould be dragged in words to create a hole in the universe. This hole could be entered but from beyond which nothing could escape. Once formed there was nothing in theory or imagination that could bring material consumed back to the outside universe.

This deadly tear in space is what is known as a BLACK HOLE.

HOME EXPERIMENT TO UNDERSTAND THE WORKING OF A BLACK HOLE

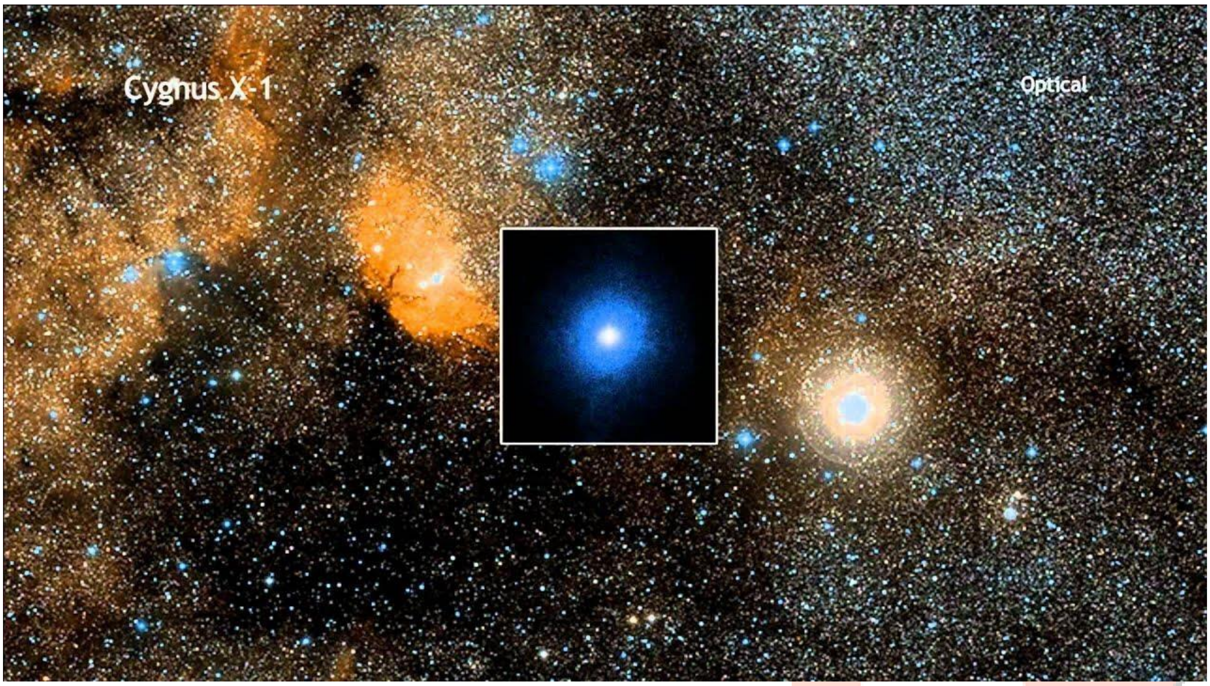
- Set up a trampoline in your lawn. Take around 10 shot put balls and carefully place it at the centre of thetrampoline.
- We can imagine these 10 shot put balls as one black hole whose mass is let's say 10 times the mass of our Sun.
- We can observe that the fabric of the trampoline, that is the fabric of spacetime, has been bent so much by theblack hole that it has warped the spacetime around itself and created infinite gravity.
- That is why it is said that if something falls into a black hole it is almost impossible to get it out, beating gravity,unless it travels at the speed of light which is not pragmatic.

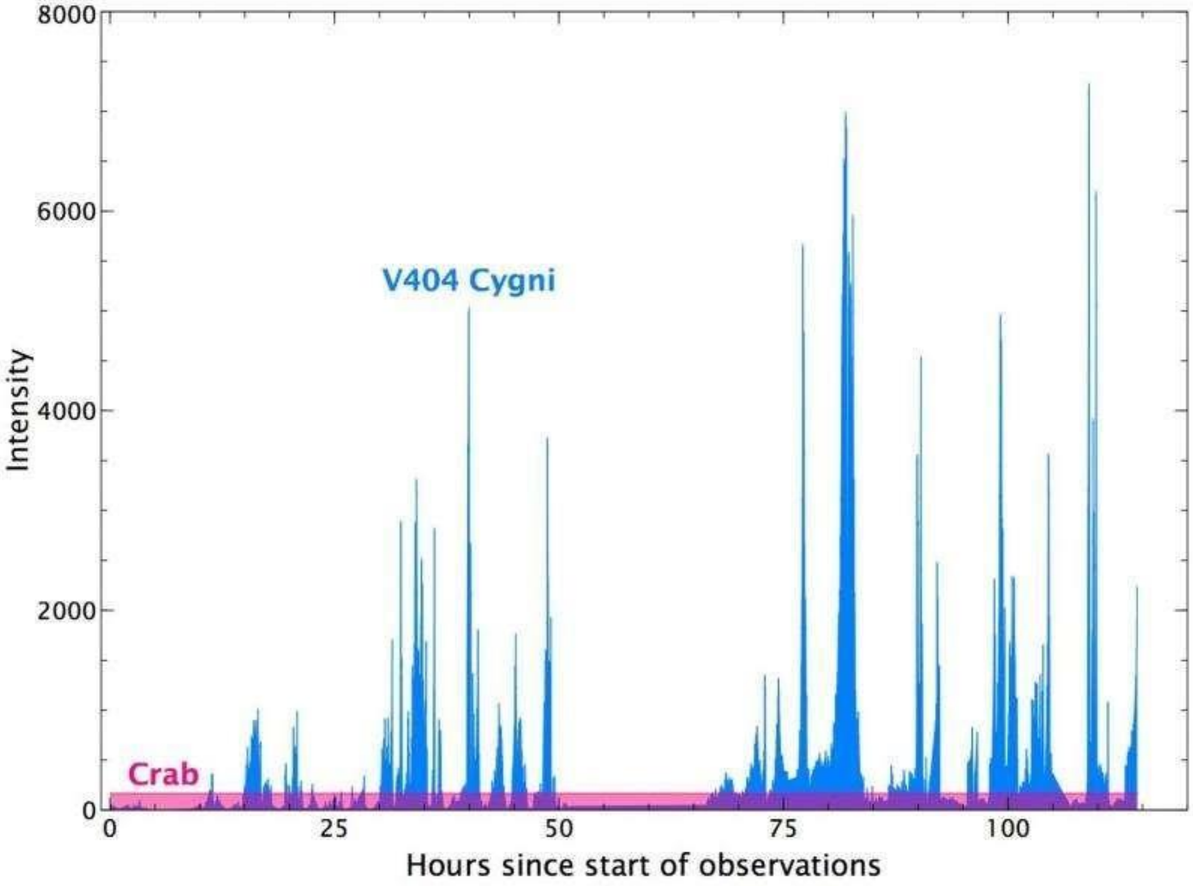
HISTORY OF BLACK HOLES

FIRST BLACK HOLE TO BE DETECTED – CYGNUS X-1

For many decades, black holes were only known to be theoretically correct until 1964. Astronomers at the US Naval Research Lab first detected Cygnus X-1, a galactic X-ray source, located in the constellation Cygnus within the milky way. Now, the problem was that the X-ray emissions were blocked by the Earth's atmosphere, so observation of celestial x-ray sources was not possible directly from the Earth. We needed to lift some instruments at a particular altitude where the X-rays could penetrate. For this reason, the U-huru X-ray satellite was launched that examined over 200 X-ray sources. It also noted that Cygnus X-1 emitted X-rays. When the astronomers looked towards the closest celestial body, HDE 226868, they observed that it revolved around a particular path, an orbit, indicating that it was a part of a binary system. However, they could not find the other companion star in that binary system. All the calculations made complete

sense because: - Firstly, it is very difficult to observe a black hole because it does not allow any light to escape such that itreaches our eyes. And secondly, a black hole emits X-rays which were detected.





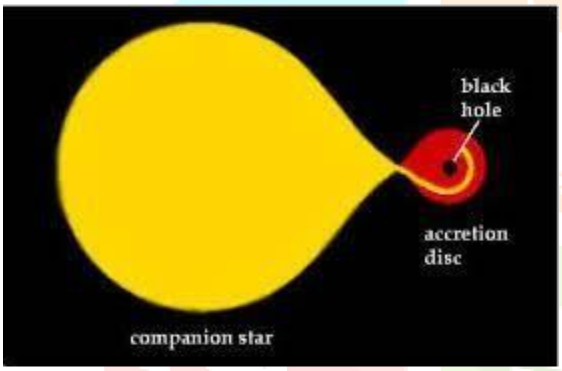
This is a graph that describes the variation in the X-ray luminosity of the Cygnus X-1 black hole as compared to the CrabNebula, one of the brightest sources in the high energy sky. The variations in the luminosity of the X-rays emitted is mainly because of the material pilling up in the accretion disk of the black hole, until eventually reaching a tipping pointthat dramatically changes the black hole's feeding routine for a short period.

Now I know the question that arises in your minds is that if a black hole does not allow light, an electromagneticradiation, to escape. Then how do the X-Rays, which are also a form of electromagnetic radiation escape?

The X-rays originate from the hot gas orbiting around the black hole in a disk called the accretion disk.

ILLUSTARTION TO EXPLAIN HOW HOT GASES FORM THE ACCRETION DISK

Let us suppose a star is orbiting a black hole very closely. The changes we observe in the star's dynamics prove to be veryuseful for us. The black hole's gravity is so strong that it pulls off the outer gases of the star and forms a disk around itselfcalled the accretion disk. As the gas orbits, magnetic forces make the gas loose energy and its angular momentum, which causes the gas to spiral slowly in towards the black hole. The orbital energy of the gas is transformed into thermal energy,heating the gas up to millions of degrees, resulting in the emission of X-Rays. Minimum X-rays are emitted when the gas is closer to the back hole as the effect of gravity becomes stronger and stronger not allowing any kind of electromagnetic radiation to escape. And maximum X-Rays are emitted when the gas is orbiting the black hole at an appreciable distance, where the effect of gravity is comparatively weaker.



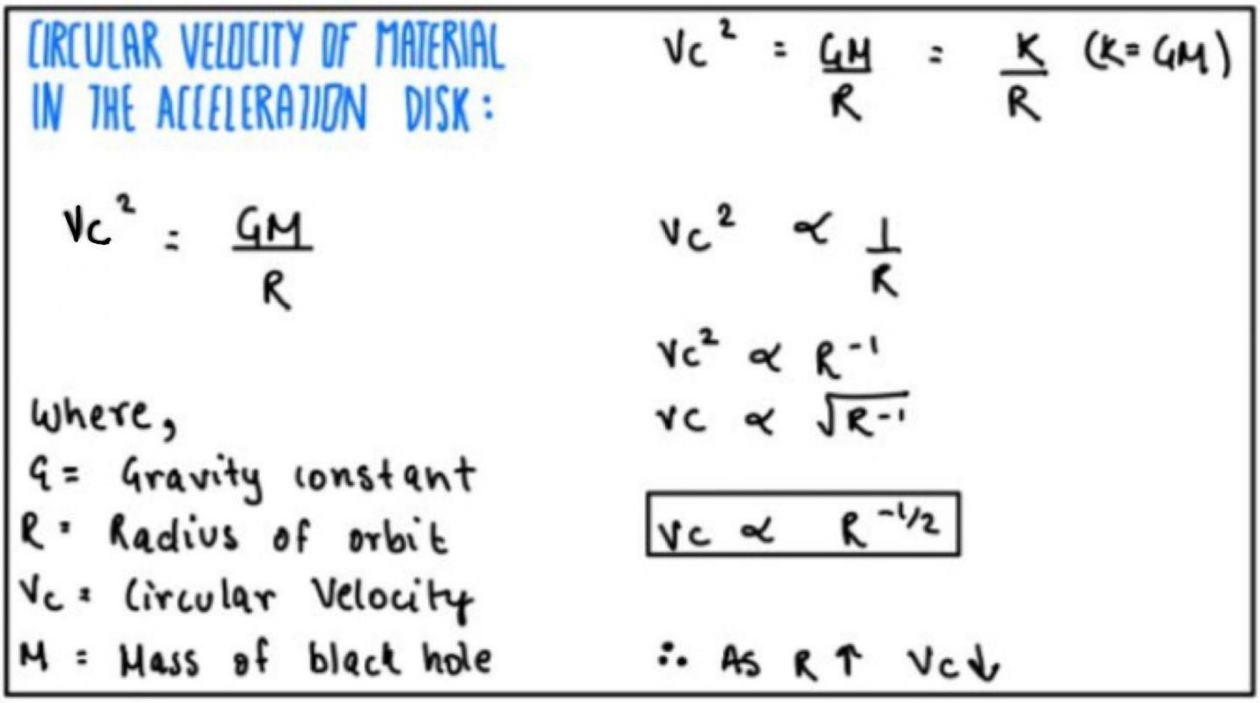
FIRST BLACK HOLE TO BE OBSERVED – M87

Most of the scientists all around the world, had accepted the fact the black hole's do exist after the remarkable discovery of Cygnus X-1. Yet, that hint of spice was missing which could only be obtained after visual evidence of a black hole. The image of the black hole in Messier 87, located in the Virgo constellation, was captured by the Even Horizon telescope – a global network of synchronized radio observatories located in diverse regions of the Earth. When combined,this array acts as a telescope the size of Earth.

ACCRETION DISKS

We have studied in brief about accretion disk in the above paragraphs. Let us try and extend our knowledge about them. An accretion disk is not strictly contingent to a black hole per se. However, we shall be referring to the accretion disk of ablack hole only.

A black hole forms an accretion disk by gaining matter (hot gases, plasma etc) from a giant star for e.g., present in a binary system with the black hole, such that the matter has enough rotational or angular momentum that it cannot fall inward towards the black hole along a straight line. Matter orbiting in the gravitational field of a black hole loses angular momentum and energy as it slowly spirals inwards towards the black hole. The inner part of the accretion disk rotates faster than the outer part. Gas (mostly plasma) present in the accretion disk moves a with a faster velocity when it is closeto the black hole, while the gas moves with a comparatively slower velocity when it is comparatively far away from the black hole.



CIRCULAR VELOCITY OF MATERIAL IN THE ACCELERATION DISK:

$$V_c^2 = \frac{GM}{R}$$

Where,
G = Gravity constant
R = Radius of orbit
Vc = Circular Velocity
M = Mass of black hole

$$V_c^2 = \frac{GM}{R} = \frac{K}{R} \quad (K = GM)$$

$$V_c^2 \propto \frac{1}{R}$$

$$V_c^2 \propto R^{-1}$$

$$V_c \propto \sqrt{R^{-1}}$$

$$\boxed{V_c \propto R^{-1/2}}$$

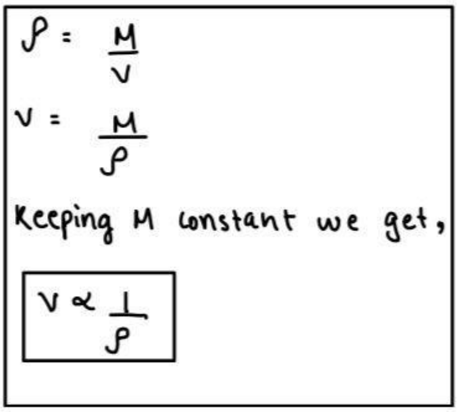
$\therefore$ As R ↑ Vc ↓

From these calculations, we can see that the velocity of the gas in the accretion disk varies with the radius. Such a difference in the outer and inner velocities of the gas in the disk create a fluid frictional force called Viscous force orviscosity.

Let us further understand about the dynamics of the matter in the accretion disk. Taking an analogy, consider a situationin which you open the sink’s tap, letting the water come out of the tap and going into the drain. Keep the tap open.
Similarly matter present along the inner edges of the accretion disk gets sucked by the black hole, however the accretion disk never collapses because the black hole, due to its gravitational attraction, keeps adding more and more matter into itsaccretion disk, until it does not have any nearby gases to feed on.

COMPONENTS OF A BLACK HOLE:

1. Schwarzschild radius – Let us suppose we get the power to shrink a particular celestial body, say a planet of mass
   M. As we compress and shrink the planet, its mass, which is the internal matter stored, remains the same.However, its density increases drastically. Hence the volume decreases.

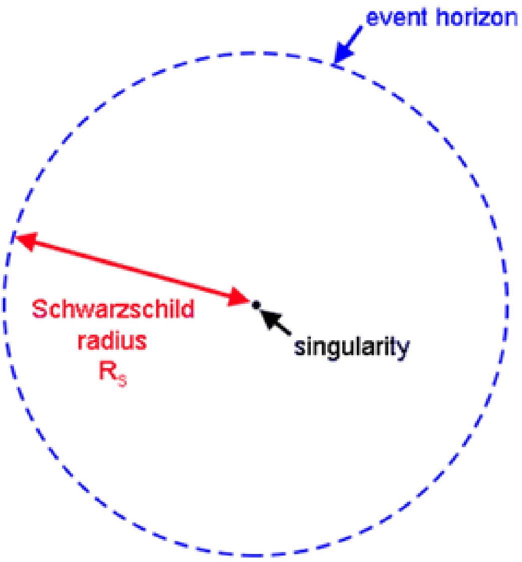


$$\rho = \frac{M}{V}$$

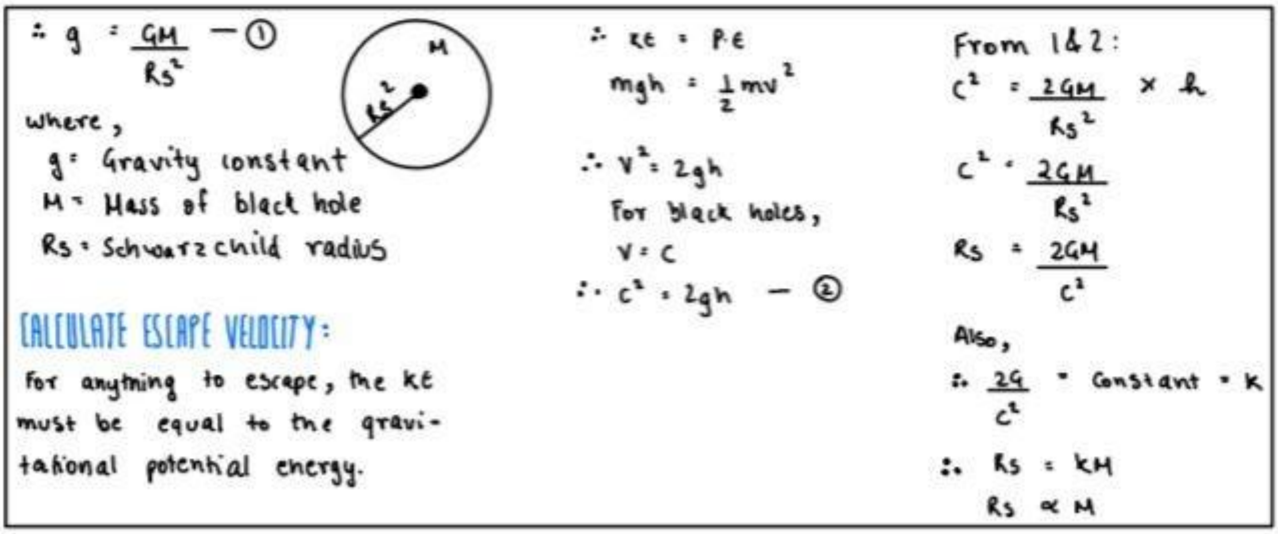
$$V = \frac{M}{\rho}$$

Keeping M constant we get,

$$\boxed{V \propto \frac{1}{\rho}}$$



We keep shrinking the planet until it reaches a critical size, which is known as the Schwarzschild radius. It is the radius of a sphere such that, if all the mass of an object were to be compressed within that sphere, the escapevelocity from the sphere would be equal to the speed of light. Hence, we establish the reason as to why it is not possible to escape a black hole, because pragmatically it is not possible to reach till the speed of light.



$\therefore g = \frac{GM}{Rs^2}$ — ①

where,

g = Gravity constant

M = Mass of black hole

Rs = Schwarzchild radius

CALCULATE ESCAPE VELOCITY:

For anything to escape, the KE must be equal to the gravitational potential energy.

$\therefore KE = P.E$

$mgh = \frac{1}{2}mv^2$

$\therefore v^2 = 2gh$

For black holes,

$v = c$

$\therefore c^2 = 2gh$ — ②

From 1 & 2:

$c^2 = \frac{2GM}{Rs^2} \times h$

$c^2 = \frac{2GM}{Rs^2}$

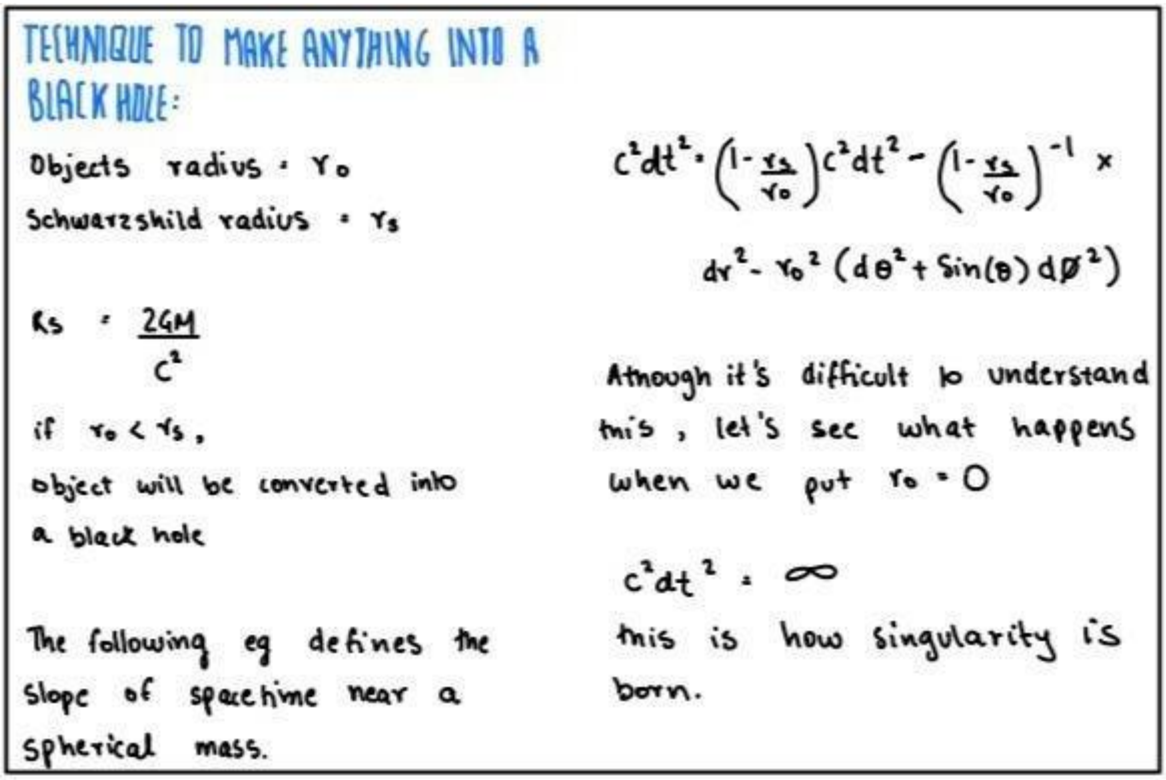
$Rs = \frac{2GM}{c^2}$

Also,

$\therefore \frac{2G}{c^2} = \text{Constant} = k$

$\therefore Rs = kM$

$Rs \propto M$



TECHNIQUE TO MAKE ANYTHING INTO A BLACK HOLE:

Objects radius = r_o

Schwarzshild radius = r_s

$R_s = \frac{2GM}{c^2}$

if $r_o < r_s$, object will be converted into a black hole

The following eq defines the slope of spacetime near a spherical mass.

$$c^2dt^2 = \left(1-\frac{r_s}{r_o}\right)c^2dt^2 - \left(1-\frac{r_s}{r_o}\right)^{-1} \times dr^2 - r_o^2\,(d\theta^2 + \sin(\theta)\,d\phi^2)$$

Atnough it's difficult to understand this, let's see what happens when we put $r_o = 0$

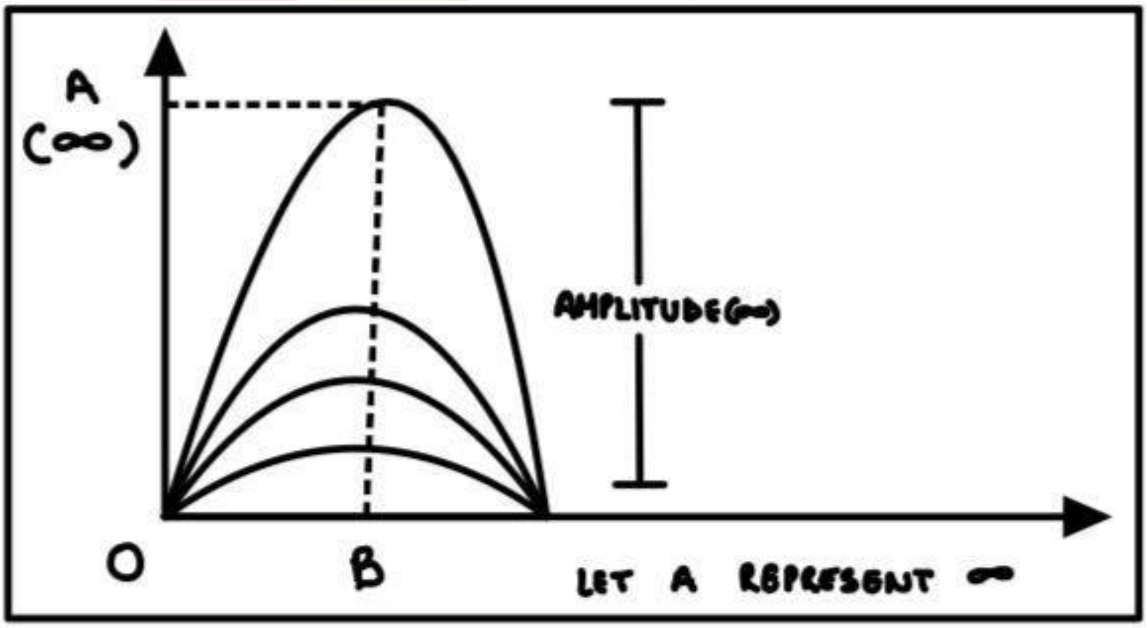
$c^2dt^2 = \infty$

this is how singularity is born.

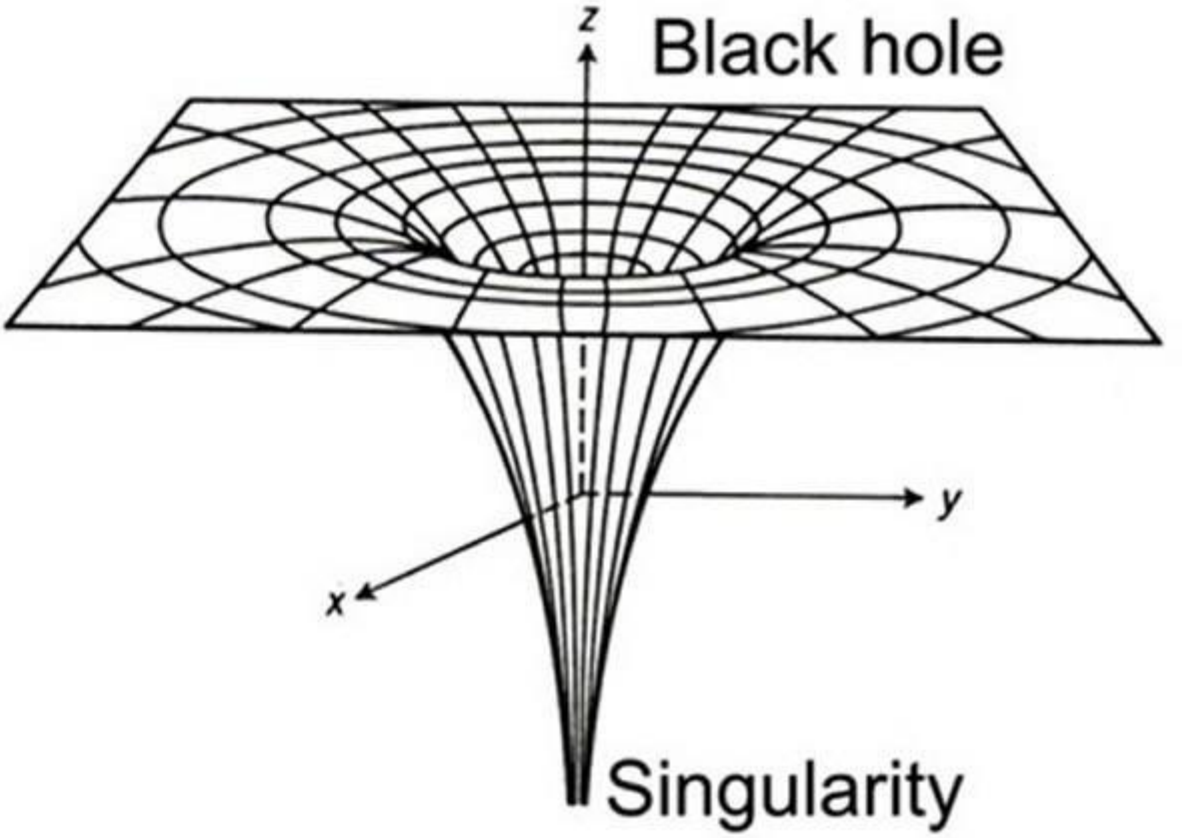
2. Singularity: A singularity has puzzled physicists in the understanding of the cosmos and is by far the most curiousand fascinating component of a black hole. This is because all the fundamental known laws of physics break down at the singularity, which is located at the centre of the black hole. It is a place where matter is compressed down to an infinitely tiny point. As fascinating as this concept might seem, in the real world there is no singularity.

   Let us understand this by taking an analogy.

   Consider a guitar string that vibrates when we play it. A simple model predicts that the vibrations increase exponentially with time, and very soon the amplitude of the wave reaches infinity. But does this even really occurin real life? No, it does not. The guitar string breaks much before.



This indicates that the model has reached its limit. Therefore, to avoid infinity in the equations, we need a muchbetter theory than that of a singularity.

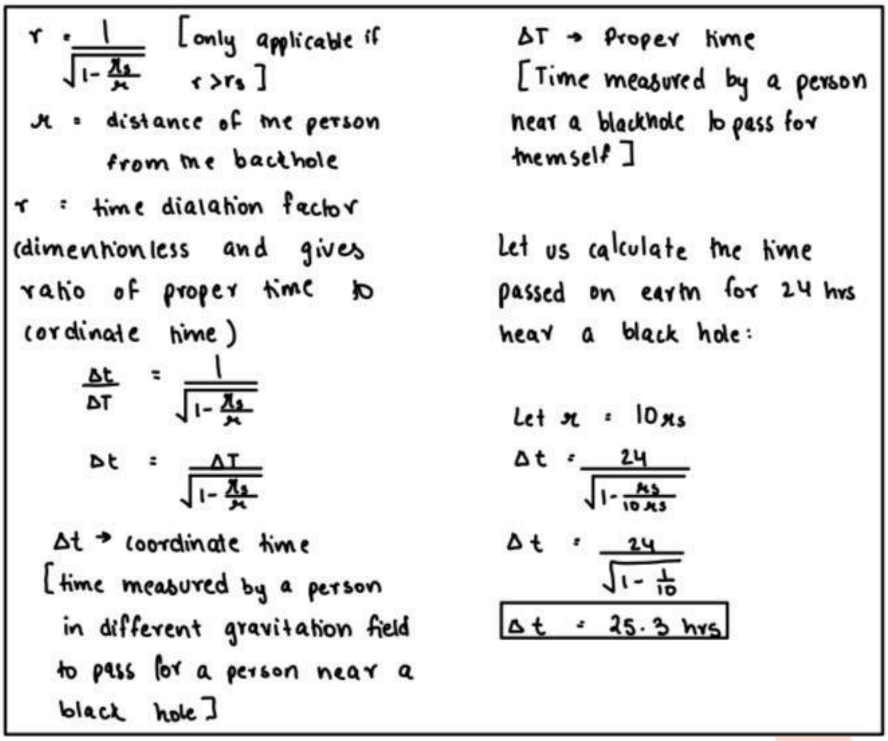


EVENT HORIZON: - Event horizon of a black hole is a boundary that marks the limits of humanity. It is knownas the “point of no return” as beyond the event horizon the escape velocity is equal to the speed of light. Most of the black holes rotate on its axis. The event horizon of such black holes is non spherical in nature. On the other hand, for those rare nonrotating black holes, the event horizon is largely spherical in nature. A misconception about the event horizon is that it is a barrier that restricts any matter from going in. Rather, it is just a mathematically defined demarcated boundary that prevents matter or radiation from exiting it after that boundaryis crossed. We cannot see the event horizon because it does not emit any light or radiation.

Although it is essentially almost impossible to orbit a black hole because of the extreme warping of spacetime, letus say we have extremely strong engines and strong bones. As we orbit the black hole, we experience a gigantic force downwards towards the black hole. However, let us say we have an anti-gravity device installed in our rocket systems that protects us. As we descend, the black hole bends more and more light down into it. Now if weimagine this scenario, during our descent when we look outside of our spaceship, we will see the entire universe bend up around us. We see these visuals only because of our frame of reference. The universe, as seen through another person’s eyes from Earth, would remain the same. The further we go the more light bends away from us and the window of our universe keeps shrinking and shrinking. Now, how do we know that we have reached the event horizon? We can simply detect the event horizon by watching the window of our universe above us, and when it becomes a point that means we are at the event horizon, the point of no return.

TIME DILATION

We know that speed is distance divided by time, and the speed of light is a constant whose value is equal to 300000000m/s for all reference frames. Due to the curvature of spacetime around a black hole, light must travel amuch greater distance near a black hole. The distance that light travels is not merely the distance travelled by light in a Euclidean plane. If we take speed of light as constant and increase the distance, then to keep the ratio constant time must decrease. We can also calculate the exact time dilation around a black hole using the following equation.



$$\gamma = \frac{1}{\sqrt{1-\frac{r_s}{r}}}$$ [only applicable if $r > r_s$]

r = distance of the person from the backhole

γ : time dialation factor (dimentionless and gives ratio of proper time to cordinate time)

$$\frac{\Delta t}{\Delta T} = \frac{1}{\sqrt{1-\frac{r_s}{r}}}$$

$$\Delta t = \frac{\Delta T}{\sqrt{1-\frac{r_s}{r}}}$$

Δt → coordinate time [time measured by a person in different gravitation field to pass for a person near a black hole]

ΔT → Proper time [Time measured by a person near a blackhole to pass for themself]

Let us calculate the time passed on earth for 24 hrs near a black hole:

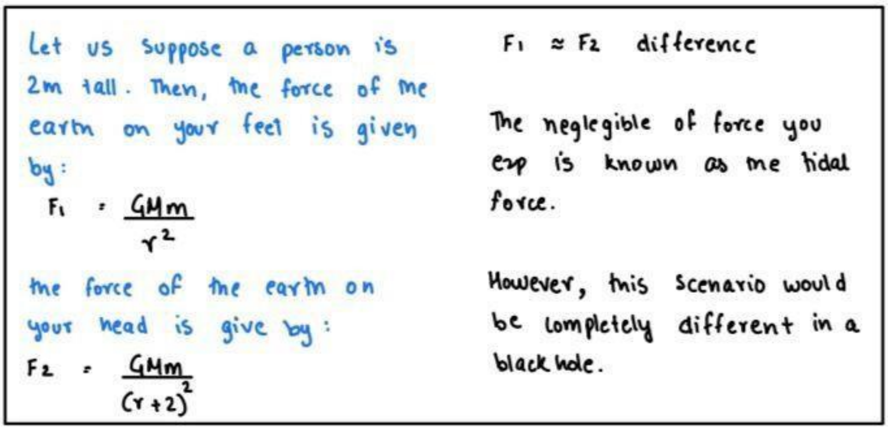
Let $r = 10r_s$

$$\Delta t = \frac{24}{\sqrt{1-\frac{r_s}{10r_s}}}$$

$$\Delta t = \frac{24}{\sqrt{1-\frac{1}{10}}}$$

$$\boxed{\Delta t = 25.3 \text{ hrs}}$$

DEATH BY A BLACK HOLE



Let us suppose a person is 2m tall. Then, the force of the earth on your feet is given by:

$$F_1 = \frac{GMm}{r^2}$$

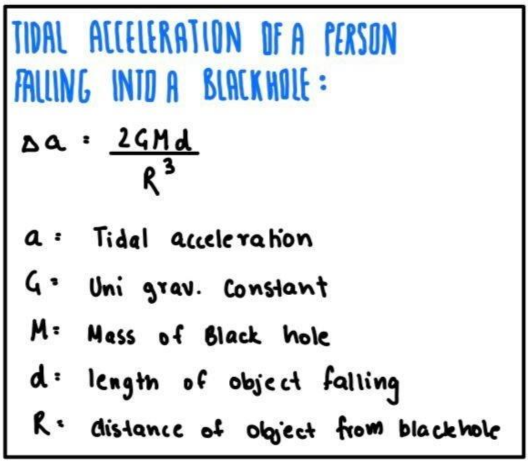
the force of the earth on your head is give by:

$$F_2 = \frac{GMm}{(r+2)^2}$$

$F_1 \approx F_2$ difference

The neglegible of force you exp is known as the tidal force.

However, this scenario would be completely different in a black hole.



TIDAL ACCELERATION OF A PERSON FALLING INTO A BLACKHOLE:

$$\Delta a = \frac{2GMd}{R^3}$$

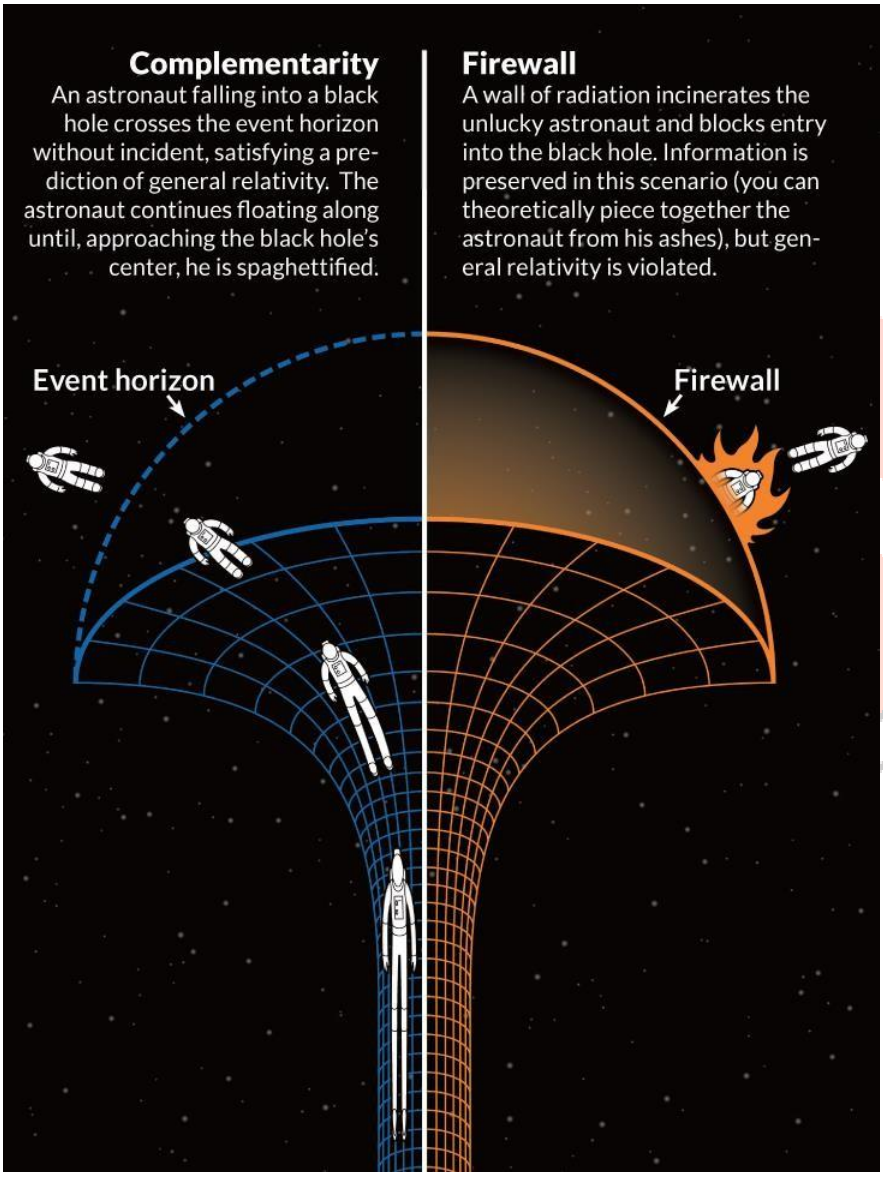
a: Tidal acceleration

G: Uni grav. Constant

M: Mass of Black hole

d: length of object falling

R: distance of object from blackhole



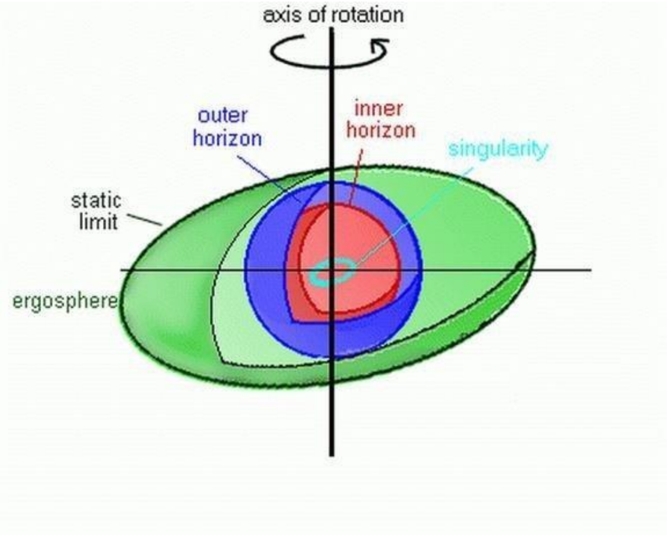
Complementarity

An astronaut falling into a black hole crosses the event horizon without incident, satisfying a prediction of general relativity. The astronaut continues floating along until, approaching the black hole's center, he is spaghettified.

Firewall

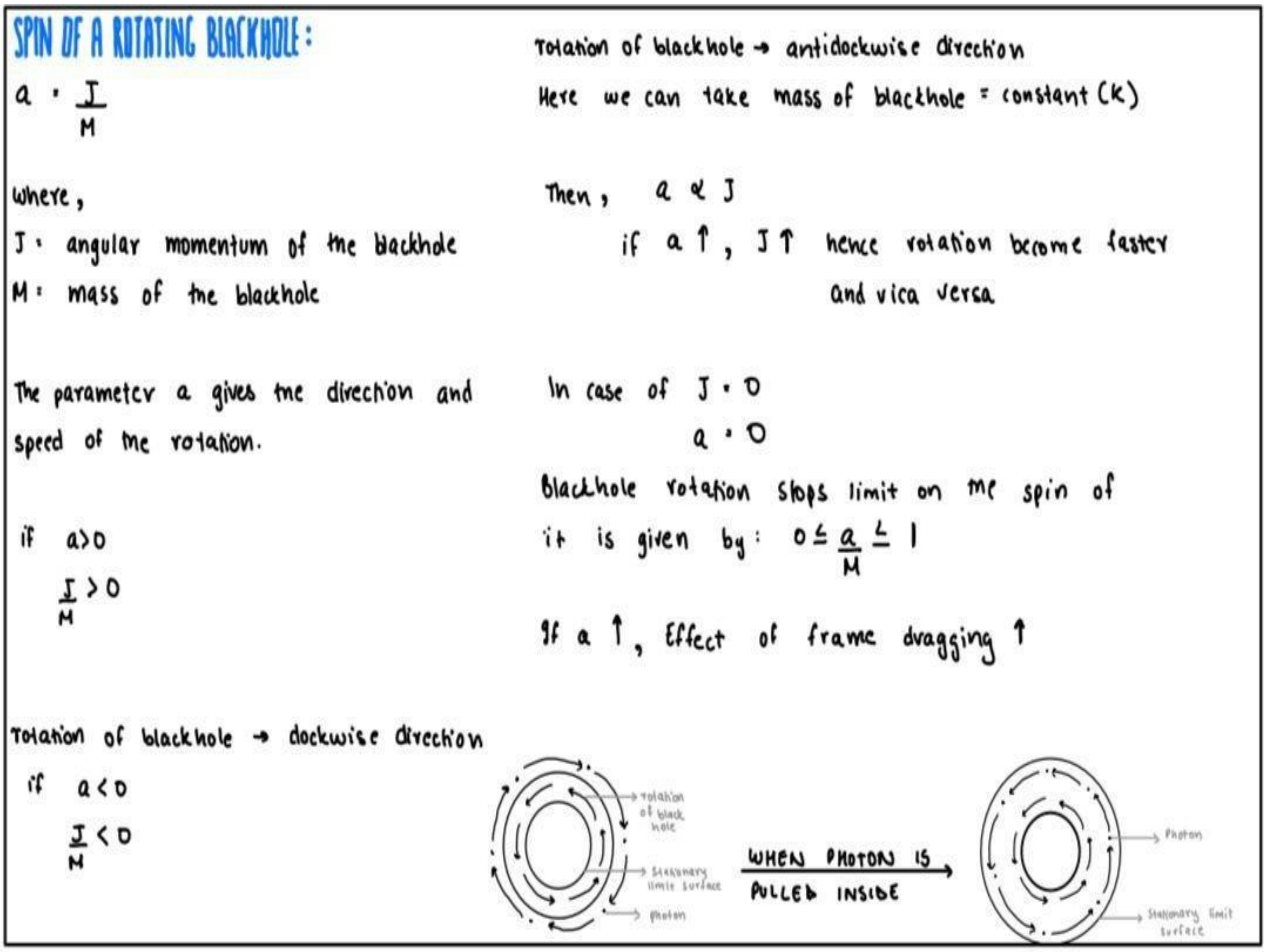
A wall of radiation incinerates the unlucky astronaut and blocks entry into the black hole. Information is preserved in this scenario (you can theoretically piece together the astronaut from his ashes), but general relativity is violated.

SPIN OF A BLACK HOLE



A spin of a black hole is characterized by its rotation on its axis. Thus, spinning black holes are in fact rotating black holes. A rotating black hole is further described by its mass and angular momentum. Such a black hole also forces the spacetime around it to rotate with it. This phenomenon is known as frame dragging in which the frames of reference of allthe observers rotate with the black hole. All the matter dragged inside a rotating black hole must rotate in the direction of the rotation of the black hole inside a particular stationary limit surface. Even photons, particles of light, which travel at the cosmic speed limit rotate in the direction of the rotation of the black hole. Let us consider a situation in which a photon rotates in the opposite direction of the rotation of the black hole outside the stationary limit surface. As the rotating black hole's gravity pull's the photon inside its stationary limit surface, the photon changes its initial direction of rotation and then rotates in the direction of the rotation of the black hole.

The region between the stationary limit surface and the event horizon is known as the ergosphere of a black hole.



SPIN OF A ROTATING BLACKHOLE:

$a = \frac{J}{M}$

where,

J : angular momentum of the blackhole

M : mass of the blackhole

The parameter a gives the direction and speed of the rotation.

if $a > 0$

$\frac{J}{M} > 0$

rotation of blackhole → clockwise direction

if $a < 0$

$\frac{J}{M} < 0$

rotation of blackhole → anticlockwise direction

Here we can take mass of blackhole = constant (K)

Then, $a \propto J$

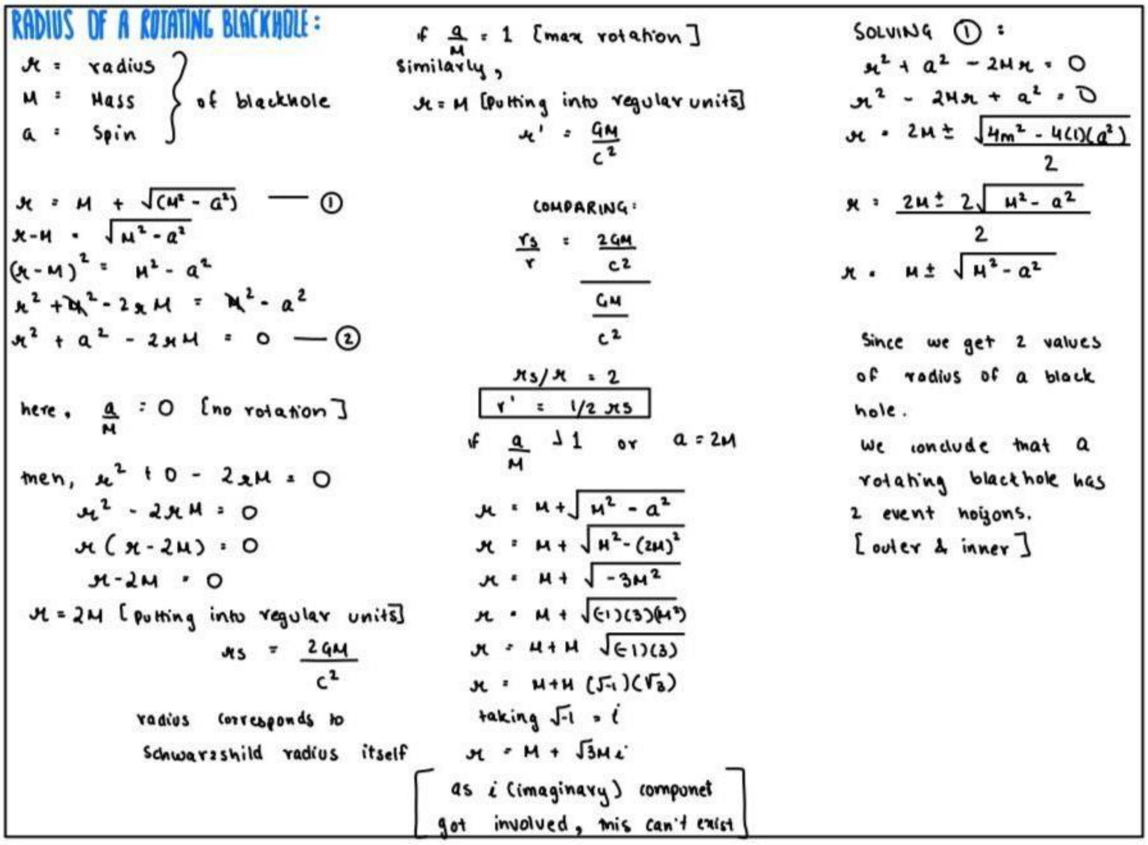
if a ↑, J ↑ hence rotation become faster and vica versa

In case of $J = 0$

$a = 0$

Blackhole rotation stops limit on the spin of it is given by: $0 \le \frac{a}{M} \le 1$

If a ↑, Effect of frame dragging ↑



RADIUS OF A ROTATING BLACKHOLE:

r : radius
M : Mass
a : Spin
} of blackhole

$r = M + \sqrt{(M^2 - a^2)}$ —— ①
$r - M = \sqrt{M^2 - a^2}$
$(r - M)^2 = M^2 - a^2$
$r^2 + \cancel{M^2} - 2rM = \cancel{M^2} - a^2$
$r^2 + a^2 - 2rM = 0$ —— ②

here, $\frac{a}{M} = 0$ [no rotation]

then, $r^2 + 0 - 2rM = 0$
$r^2 - 2rM = 0$
$r(r - 2M) = 0$
$r - 2M = 0$
$r = 2M$ [putting into regular units]
$r_s = \frac{2GM}{c^2}$

radius corresponds to Schwarzshild radius itself

if $\frac{a}{M} = 1$ [max rotation]
Similarly,
$r = M$ [putting into regular units]
$r' = \frac{GM}{c^2}$

COMPARING:
$$\frac{r_s}{r} = \frac{\frac{2GM}{c^2}}{\frac{GM}{c^2}}$$
$r_s / r = 2$
$\boxed{r' = 1/2\, r_s}$

if $\frac{a}{M} > 1$ or $a = 2M$

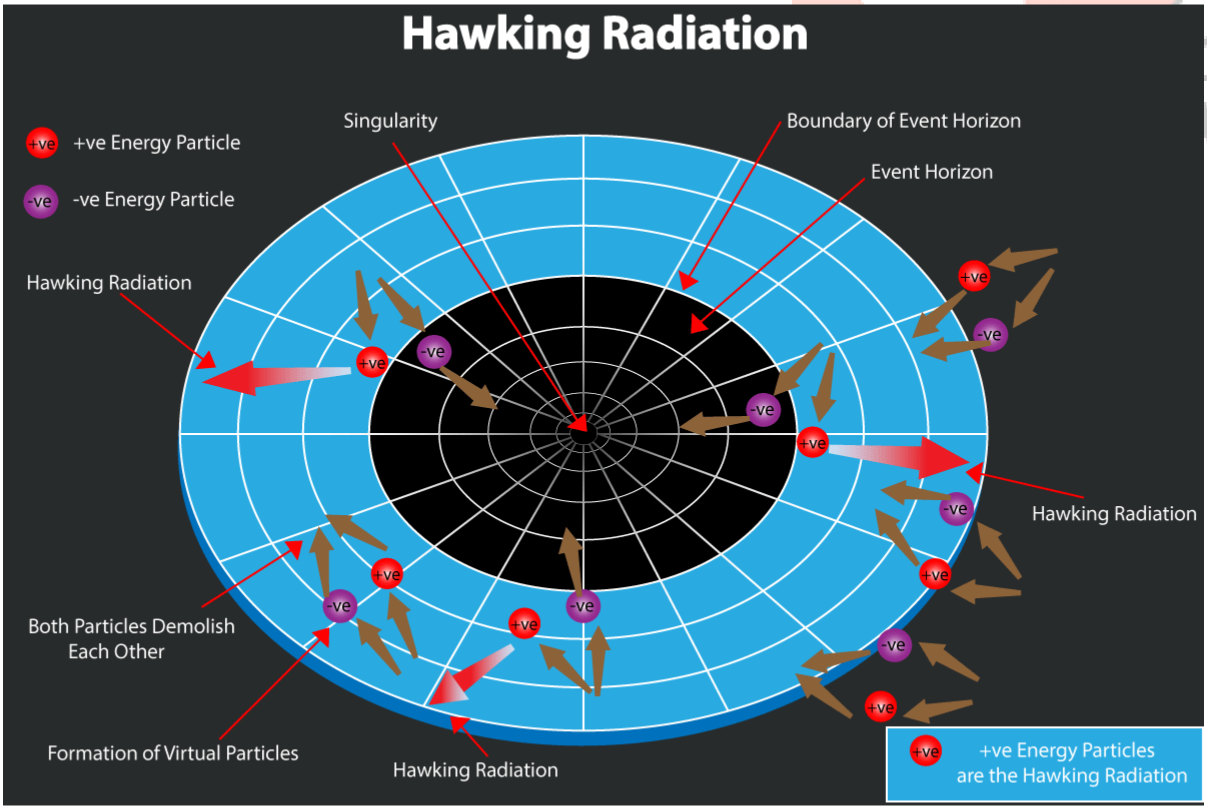
$r = M + \sqrt{M^2 - a^2}$
$r = M + \sqrt{M^2 - (2M)^2}$
$r = M + \sqrt{-3M^2}$
$r = M + \sqrt{(-1)(3)(M^2)}$
$r = M + M\sqrt{(-1)(3)}$
$r = M + M(\sqrt{-1})(\sqrt{3})$
taking $\sqrt{-1} = i$
$r = M + \sqrt{3}Mi$

[as i (imaginary) componet got involved, this can't exist]

SOLVING ①:
$r^2 + a^2 - 2Mr = 0$
$r^2 - 2Mr + a^2 = 0$
$$r = \frac{2M \pm \sqrt{4m^2 - 4(1)(a^2)}}{2}$$
$$r = \frac{2M \pm 2\sqrt{M^2 - a^2}}{2}$$
$r = M \pm \sqrt{M^2 - a^2}$

Since we get 2 values of radius of a black hole.
We conclude that a rotating blackhole has 2 event horizons.
[outer & inner]

HAWKING RADIATION

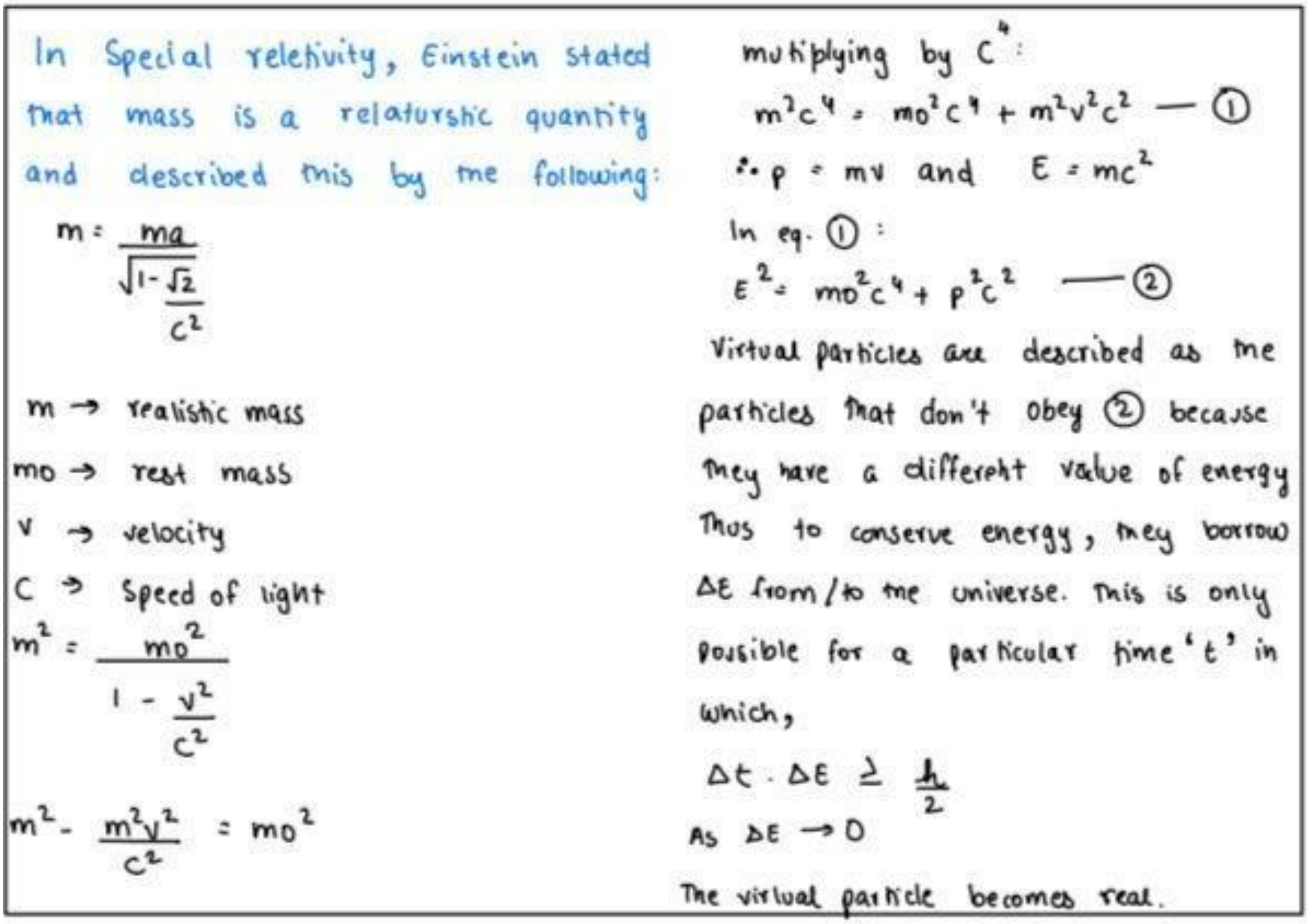


Until now, we have learnt how black holes feed on everything. But can the reverse be possible? Can a black hole ever lose its mass? Can it ever die? Let us look at the answers to these intriguing questions by comprehending hawking radiation.

It is not physically possible for us to observe what happens beyond the event horizon, but from the outside we

can see thatthey are absolutely black. However, this is not completely true as black holes emit radiation which makes it glow for a short period. This radiation emitted by a black hole was given by Stephen Hawking and is known as hawking radiation.
Let us further understand the dynamics of this radiation.

There are two types of particles found in nature: Virtual and Real. While real particles obey the law of conservation energy and momentum, virtual particles do not. They are in fact just a temporary existence of real particles that participatein intermediate effects in between the measurements. Real particles fluctuate in in energy due to quantum uncertainty, andthese fluctuations give rise to virtual particles.



In Special relativity, Einstein stated that mass is a relatursric quantity and described this by the following:

$$m = \frac{ma}{\sqrt{1-\frac{\sqrt{2}}{c^2}}}$$

m → realistic mass
mo → rest mass
v → velocity
C → Speed of light

$$m^2 = \frac{mo^2}{1-\frac{v^2}{c^2}}$$

$$m^2 - \frac{m^2v^2}{c^2} = mo^2$$

mutiplying by C^4:

$$m^2c^4 = mo^2c^4 + m^2v^2c^2 \quad — ①$$

$\therefore p = mv$ and $E = mc^2$

In eq. ①:

$$E^2 = mo^2c^4 + p^2c^2 \quad — ②$$

Virtual particles are described as the particles that don't obey ② because they have a different value of energy Thus to conserve energy, they borrow ΔE from/to the universe. This is only possible for a particular time 't' in which,

$$\Delta t \cdot \Delta E \geq \frac{\hbar}{2}$$

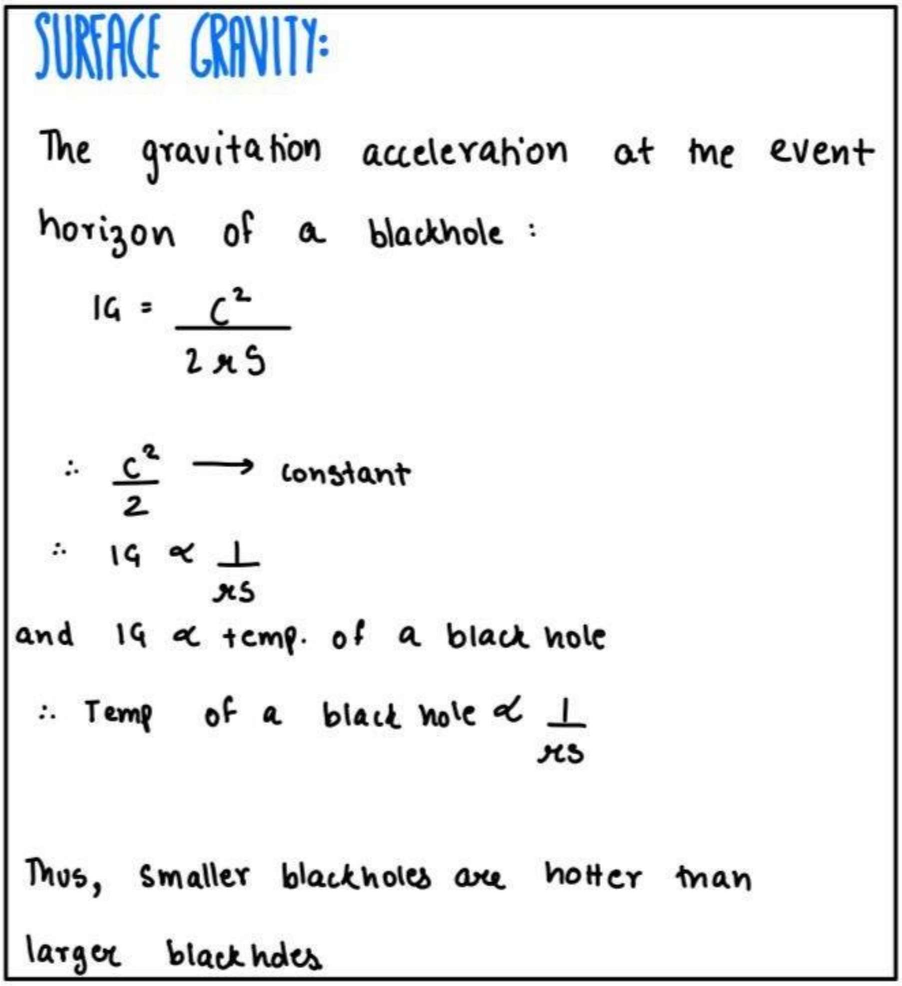
As $\Delta E \rightarrow 0$

The virtual particle becomes real.

Let us imagine a simple quantum field tracing a particular path in space. This field is in a perfect vacuum state before theformation of a black hole in that path. We will observe that as this field interacts with the black hole and due to the blackhole's extreme curvature of spacetime, it disturbs the field that defines the fluctuations of the vacuum. As the trajectory finds its way back into flat space again, we observe that those fluctuations in the quantum field look like real particles.
From our frame of reference, we would be flustered by the visuals of the radiations coming from the event horizon of the black hole. Pairs of virtual matter and antimatter particles keep popping up just outside the event horizon of a black hole due to quantum fluctuations in vacuum as discussed above. By convention one of them has positive energy, and the other(anti-matter particle) has negative energy. Now in normal flat space, these particles would annihilate each other with their positive and negative energies. However, in curved spacetime this scenario completely changes. Due to the conservation of energy, the particle that falls inside a black hole must have negative energy. While the particle that escapes out of the event horizon has positive energy. The negative energy virtual anti matter particle contributes to the loss of the black hole's energy and as a result its mass. This is how a black hole finally evaporates. In a dramatic ending, the black hole loses its mass in a short period of time. This is accompanied by abrupt explosions, which can be detected as gamma rayoutbursts.

TEMPERATURE OF BLACK HOLES

Just outside the black hole, the material that is being pulled inside is accelerated near to the speed of light. The moleculescontained in such a material collide with such force and vigour that it gets heated up to a temperature of hundreds of millions of degrees.



SURFACE GRAVITY:

The gravitation acceleration at the event horizon of a blackhole :

$$IG = \frac{c^2}{2rS}$$

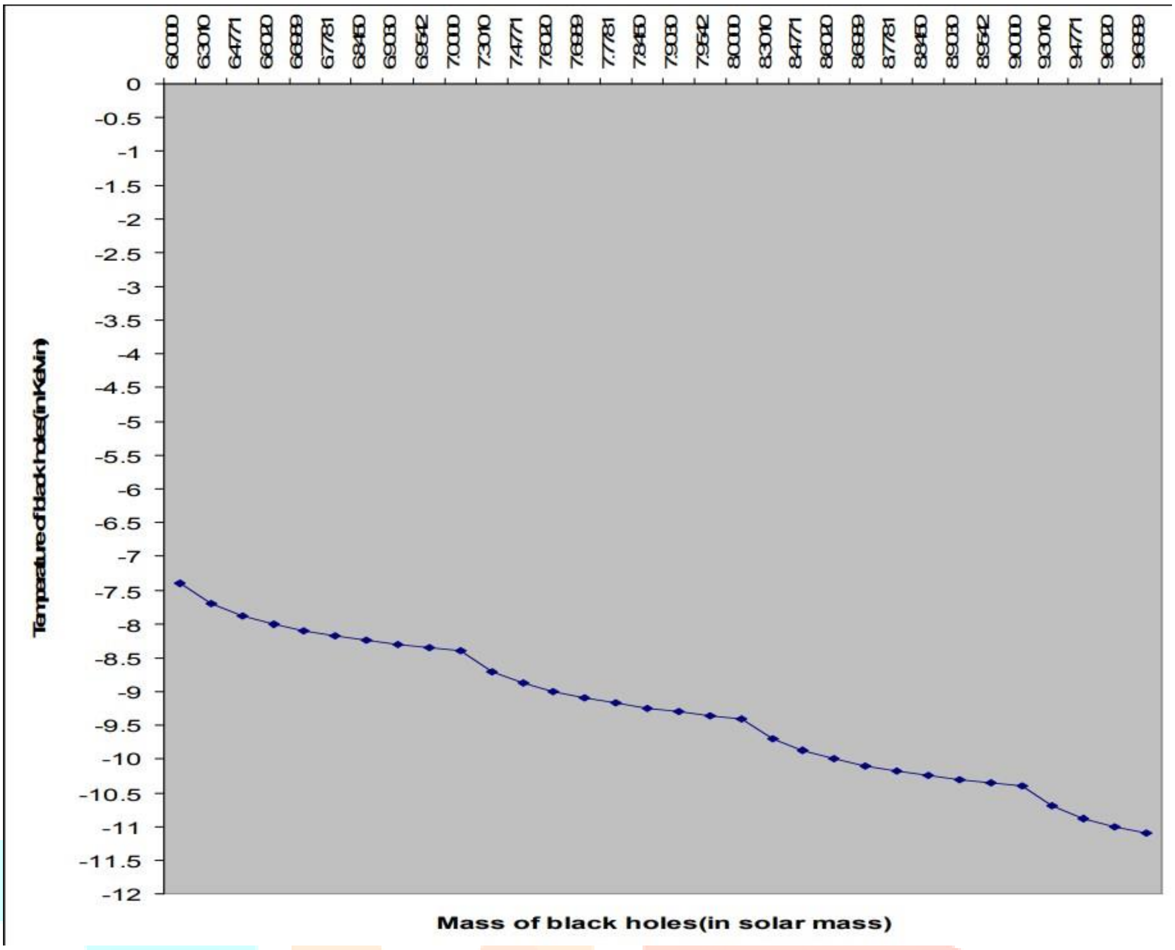
$$\therefore \frac{c^2}{2} \longrightarrow \text{constant}$$

$$\therefore IG \propto \frac{1}{rS}$$

and $IG \propto$ temp. of a black hole

$\therefore$ Temp of a black hole $\propto \frac{1}{rS}$

Thus, smaller blackholes are hotter than larger blackholes



As evident from the above graph also, black holes with a smaller mass are hotter as compared to the black holes with alarger mass.

THE FAMOUS INFORMATION PARADOX

Information is understood as a property of the arrangement of particles. This means, for e.g., we see carbon atoms: whenwe arrange them in a certain way they make coal, but by simply rearranging them they turn into a diamond, similarly it can be turned into lead. Atoms remain the same, but what changes here is the information contained.

Without information everything in the universe would be the same. According to the theory of quantum mechanics,information is indestructible. It might change shape, but it can never be lost.

Another example would be, burning a piece of paper to get ash. If you’re able to carefully collect every single carbon atom in the ash and measure the exact properties of the smoke and heat, you could in theory reconstruct the paper. The information is not lost it is just hard to read. Furthermore, information tells us how things are different from each other, but black holes do the opposite by sucking everything and making it the same. They destroy information. This creates theinformation paradox.

POSSIBLE SOLUTIONS:

1. Information is lost: This is not a feasible solution, as this would mean there is no way to retrieve the informationwhatsoever, which is again not pragmatic.

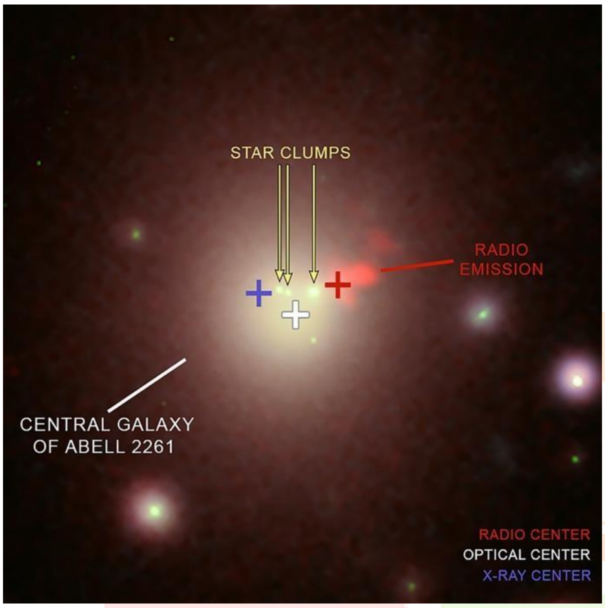
2. Information is hidden: It is erratically possible for the black hole to split into a miniature universe and all the information inside the black hole would be transferred into this new place. We could never interact with it, but it is notlost. The miniature universe model may also be a possible contender to replace the singularity of a black hole.

3. Information is safe: One would think that if a blackhole is at maximum capacity, it cannot take in any more

information. But it turns out that a black hole grows it surface by a tiny pixel for every bit of information we throw into it.Information means more surface area. For example, if we throw a stone into a pond, we won’t be able to see it but we understand something has gone in with the ripples created on the surface of the water and also the increase in the volumeof the water. Even the smallest black hole can store more and more information on its surface and all the data ever produced in human history.

4. The Hawking radiation possibility: If the information is stored on the boundary of a blackhole, the Hawking radiation has a chance of learning about the information. So, when a black hole fades away, the information might get carried awayby the hawking radiation and is not lost.

MISSING BLACK HOLE ABELL 2261



The Nobel prize in physics was awarded to Reinhard Genzel, Andrea Ghez and Roger Penrose. While Roger Penrose had established the fact that black holes are a robust prediction of the General Theory of Relativity, Reinhard Genzel and Andrea Ghez proved that each galaxy has a supermassive black hole at its centre. The bigger the galaxy the more massiveis the supermassive black hole at its centre. For eg – Our milky way galaxy has a supermassive black hole called Sagittarius A at its centre. However, in one such galaxy Abell 2261, the radio emissions detected near the centre of the galaxy suggested supermassive black hole’s activity had taken place there 50 million years ago. Since then, there has been no sign of a black hole at its centre. One possible solution is that the black hole is present, but it has gone silent. Now, what do we mean by this statement? A black hole doesn’t just constantly munch on the space around itself. Eventually they run out of nearby matter and go quiet, lying-in wait until a stary bit of gas passes by. The same could have happened with the black hole present in Abell 2261. Another possible solution could be the merger of the supermassive black hole with another black hole to produce gravitational waves. If the huge amount of gravitational waves generated by such an event were stronger in one direction than the other, then to conserve momentum, the new black hole such formed would be carried away from the centre of the galaxy in the opposite direction. The mystery surrounding Abell 2261 galaxy continues, and scientists hope to resolve this by the new launch of the James Webb Space Telescope. Just like the mysteryof the black hole in Abell 2261, black holes continue to surprise us in every which way possible. While some answers may be known after some time, others may not. To put it simply, it is just one of nature’s most spectacular phenomenon.

CITATIONS:

#What Is a Black Hole?, NASA, 2014

https://www.nasa.gov/audience/forstudents/5-8/features/nasa-knows/what-is-a- black-hole-58.html

#Gravitational Waves from Gravitational Collapse, Kimberly C. B. New, 2003

https://www.researchgate.net/publication/26386593_Gravitational_Waves_from_Gravitational_Collapse

#Schwarzschild Radius, Cosmos, retrieved 2020 https://astronomy.swin.edu.au/cosmos/S/Schwarzschild+Radius

#Black Hole Parameters, Ian Mallett, retrieved 2020 https://space.geometrian.com/calcs/black-hole-params.php

#Information Loss in Black Holes, Stephen Hawking, 2005 https://arxiv.org/pdf/hep-th/0507171.pdf

#Black Holes, Wallace H Tucker, retrieved 2020 https://history.nasa.gov/SP-466/ch17.htm

#Other links: https://physics.stackexchange.com/

https://science.jrank.org/

https://www.physicsforums.com/threads/defining-the-surface-gravity-of-a-black- hole.912044/