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SPLIT PERFECT ODD VERTICES DOMINATING SET OF AN INTERVAL GRAPH USING AN ALGORITHM

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1.1. ABSTRACT:

Among the various applications of the theory of perfect domination [1], the most often discussed is communication network. There has been persistent in the algorithmic aspects of interval graphs in past decades spurred much by their numerous applications of interval graphs corresponding to interval families.

The research of the domination in graphs has been an evergreen of the graph theory. Its basic concept is the dominating set and the domination number. The theory of domination in graphs was introduced by O. Ore [4] and C. Berge [2]. A survey on results and applications of dominating sets was presented by E.J. Cockayane and S.T. Hedetniemi [3].

The concept of split and non split domination in graphs and also in Maheswari, B et all [5] . The concept of restrained domination was introduced by J.A. Telle and A. Proskurowski [6], albeit indirectly, as a vertex partitioning problem. One application of domination is that of prisoners and guards.

In this paper, we introduce an algorithm to find a split odd vertices perfect dominating set of an interval graph.

KEY WORDS : Interval graph, Dominating set, Split dominating set, Perfect dominating set, Perfect odd vertices dominating set.

1.2. PRELIMINARIES

A graph $G=(V,E)$ is an interval graph if the vertex set V can be put into one-to-one correspondence with a set of intervals I on the real line R such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I have non-empty intersection. That is if $i = [a_i, b_i]$ and $j = [a_j, b_j]$, then i and j intersect means either $a_j < b_i$ or $a_i < b_j$. The set I is called an interval representation of G and G is referred to as the intersection graph of I. Also we say that the intervals contain both its end points and that no two intervals share a common end point. The intervals and vertices of an interval graph are one and the same thing. The graph G is connected and the list of sorted end points is given and the intervals in I are indexed by increasing right end points that is $b_1 < b_2 < b_3 < \ldots\ldots < b_n$.

Let $I = \{I_1, I_2, I_3, I_4, \ldots\ldots\ldots I_n\}$ be any interval family, where each I_i is an interval on the real line and $I_i = [a_i, b_i]$, for $i = 1,2,3,4,\ldots\ldots\ldots n$. Here a_i is called the left end point labeling and b_i is the right end point labeling of I_i. Without loss of generality, we assume that all the end points of the intervals in I are distinct numbers between 1 and $2n$. Two intervals i and j are said to be intersect each other if they have non - empty intersection.

Let $G=(V,E)$ be a graph. A set $D \subseteq V(G)$ is a dominating set of G if every vertex in V/D is adjacent to some vertex in D. A dominating set D of the graph $G(V,E)$ is a split dominating set if the induced sub graph $< V - D >$ is disconnected. The split domination number $\gamma_S(G)$ of the graph G is the minimum cardinality of the split dominating set.

A dominating set D of a graph is said to be a perfect dominating set if for each vertex v not in D, v is adjacent to exactly one vertex of D.

Odd vertices perfect dominating set is nothing but a perfect dominating set consists all odd vertices.

A perfect odd vertices dominating set PODS of G is a disconnected perfect odd vertices dominating set if the induced sub graph $< V - PODS >$ is disconnected. i.e., A perfect odd vertices dominating set PODS of a graph $G(V,E)$ is a split perfect odd vertices dominating set if the induced sub graph $< V - PODS >$ is disconnected.

1.3. EXPLANATION OF THE ALGORITHM

For finding the perfect odd vertices domination through an algorithm [7], we consider a connected interval graph. Let $G = (V,E)$ be a connected interval graph. Intervals are labeled according to increasing order of their endpoints. This labeling is referred to as IG ordering. In this connected interval graph the vertices are ordered by IG ordering. First of all we treat none of a vertex of $V(G)$ is a member of perfect odd vertices dominating set PODS. Then insert vertices one by one by testing their consistency. If a vertex v is dominated by at least two vertices, then leave it, otherwise take the starting numbered adjacent odd vertex step by step by leaving even vertices from $N[v]$ as a member of PODS with perfect domination, if it is not adjacent to the next member of $N[v]$ or v is not the last vertex, clearly given in the algorithm of section 1.4.

Let us associate a new term $M_i(v)$, for a vertex $v \in V$, for all $i = 0,1,2,.....,k(k = |N(v)|)$ to each adjacent vertices of v in order to IG ordering of intervals in the following way:

$$M_i(v) = \max\left\{N[v] - \bigcup_{j=0}^{i-1} M_i(v)\right\}, \text{ with } M_0(v) = \max\{N(v)\}$$

In connection with the starting numbered adjacent vertex of v, we call this $M_i(v)$ as the p - th numbered adjacent vertex of v. Let $u, v \in V$, if for some i $(i = 0,1,2,.....,|N(v)|)$, $|N(v)| - i = p$ such that $u = M_i(v)$, then u is called the p- th numbered adjacent vertex of v.

1.4. ALGORITHM FOR PERFECT DOMINATING SET OF AN INTERVAL GRAPH

Input: An interval graph $G = (V, E)$ with IG ordering vertex set $V = \{1, 2, ..., n\}$. **Output:** Perfect odd vertices dominating set PODS.

Step 1: Set $f(j) = 0, \forall j = 1,2,....., n;$

Step 2: Set $i = 1, D = \phi;$

Step 2.1: Compute $W_i(f) = \sum_{v \in N[i]} f(v);$

Step 2.2: If $W_i(f) = 0$, then

Set $f(M_0(i)) = 1$, $f(M_1(i)) = 1$ and $\mathrm{f}(\mathrm{M}_2(i)) = 1$

take PODS = $\{M_2(i)\};$

If it is odd vertex

Step 2.3: else if $W_i(f) = 1$, i is not the last vertex, then

Step 2.3.1: if $f(M_0(i)) = 0$ and $M_0(i)$ is adjacent to $M_1(i);$

PODS remains unchanged;

end if;

Step 2.3.2: otherwise if $f(M_0(i)) = 0$ and $M_0(i)$ is not adjacent to $M_1(i)$;

Set $f(M_0(i)) = 1$, $f(M_1(i)) = 1$ and $\mathrm{f}(\mathrm{M}_2(i)) = 1$;

take PODS = PODS $\cup \{M_1(i)\};$

and $M_2(i)$ is the last neighborhood of that vertex in the p-th numbered table

Here $M_1(i)$ was odd number, if it was the even leave it and take the above neighborhood.

end if;

Step 2.4: else if $W_i(f) = 1$, i is the last vertex, then

PODS remains unchanged;

end if;

Step 2.5: else if $W_i(f)=2$, i is not the last vertex, then

PODS remains unchanged;

end if;

Step 2.6: else if $W_i(f)\geq 3$, then

PODS remains unchanged;

end if;

Step 2.7: Calculate $i=i+1$ and go to Step 2.1 and continue until the last vertex;

end PODS.

1.5. MAIN THEOREMS

1.5.1. Theorem:

Let $I=\{i_1,i_2,.....,i_n\}$ be an n interval family and G is an interval graph corresponding to I. If i and j are any two intervals in I such that $i\in$ PODS, where PODS is a perfect odd vertices dominating set, $j\neq 1$, if there is at least one interval to the left of j that contains j and there is no interval $k\neq i$ to the right of j that intersects j and k intersects exactly one vertex to the left , then split perfect odd vertices domination occurs in G and the split perfect odd vertices dominating set $<V - PODS>$ is disconnected as $|PODS|=3$

Proof:

Let $I=\{i_1,i_2,.....,i_n\}$ be the given n interval family and G is an interval graph corresponding to I. First we will find the perfect odd vertices dominating set corresponding to G using the algorithm as explained above. If i and j are any two intervals in I such that $i\in$ PODS, where PODS is a perfect odd vertices dominating set, $j\neq 1$ and j is contained in i and there is no interval $k\neq i$ to the right of j that intersects j. Then it is obvious that j is not adjacent to k in $<V - PODS>$, so that there will not be any connection in $<V - PODS>$. Since, there is no interval to the right of j that intersects j, there will not be any connection in $<V - PODS>$ to its right. Thus, we get the split perfect odd vertices domination in G. If the interval j does not contains in i and intersects to the left of j then we get a contradiction for the split domination and the vertex k intersects more than one vertex to its left then it leads to the contradiction of split perfect odd domination and which leads to the split perfect odd vertices dominating set using the above algorithm.

In this procedure we also find the perfect odd vertices dominating set of an interval graph towards the algorithm as given in section 2.4 with an illustration.

1.5.2. Illustration:

Consider the following interval family I as follows,

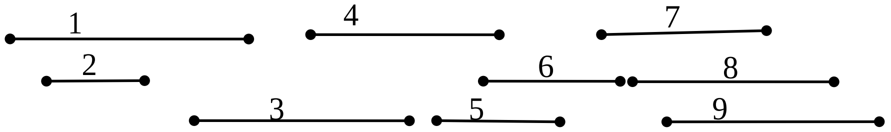

Fig.1: Interval family *I*

The corresponding interval graph G is as follows,

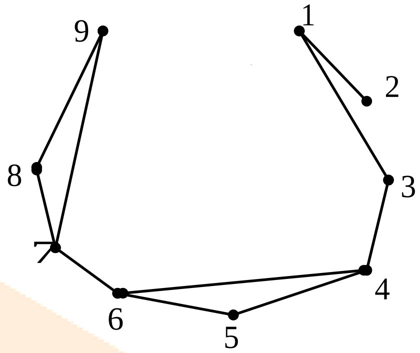

Fig.2: Interval graph *G*

From the above interval graph G, the neighborhoods of each vertex are as follows, nbd [1] = {1,2,3} nbd [6] = {4,5,6,7}

nbd [2] = {1,2} nbd [7] = {6,7,8,9}

nbd [3] = {1,3,4} nbd [8] = {7,8,9}

nbd [4] = {3,4,5,6} nbd [9] = {7,8,9}

nbd [5] = {4,5,6}

To find the perfect odd vertices dominating set, we have to compute all the p - th numbered adjacent vertices as follows,

$M_i(v) \backslash v$	1	2	3	4	5	6	7	8	9
$M_0(v)$	3		4	6	6	7	9	9	9
$M_1(v)$	2	2	3	5	5	6	8	8	8
$M_2(v)$	1	1	1	4	4	5	7	7	7
$M_3(v)$	-	-	-	3	-	4	6	-	-
$M_4(v)$	-	-	-	-	-	-	-	-	-
$M_5(v)$	-	-	-	-	-	-	-	-	-

First set $f(j)=0, \forall j \in V$. In step 2, set $i=1$, PODS = ϕ, that is initially PODS is empty. Step 2 repeats for n times. Here $n=9$, the number of vertices in the interval graph G. We follow the iterations of the illustration through the table.

Iteration (1)

For the first iteration $i = 1$

$N[1] = \{1, 2, 3\}$

$w_1(f) = f(N[1])$

$w_1(f) = f(1) + f(2) + f(3) = 0$

The first condition of if-end if is satisfied. Since $w_1(f) = 0$, we find $M_0(1) = 3, M_1(1) = 2$ and $M_2(1)=1$

Then set $f(3) = 1, f(2) = 1$ and f(1)=1 Also set

$$\text{PODS} = \phi \cup \{1\} \Rightarrow \text{PODS} = \{1\}$$

Iteration (2)

For the second iteration $i = 2$

N[2] = {1,2}

$W_2(f) = f(N[2])$

$W_2(f) = f(1)+f(2)=1+1=2$

So, in this iteration PODS could not be calculated. Hence PODS remains same and i is being increased to 3.

Iteration (3)

For the third iteration $i = 3$

N [3]={1,3,4}

$w_3(f) = f(N[3])$

$w_3(f) = f(1) + f(3) + f(4) = 1 + 1 + 0 = 2$

In this iteration PODS remains unchanged. The iteration number i is being increased to 4.

Iteration (4)

For the fourth iteration $i = 4$

N [4] ={3,4,5,6}

$w_4(f) = f(N[4])$

$w_4(f) = f(3) + f(4) + f(5) + f(6) = 1 + 0 + 0 + 0 = 1$

In this iteration also PODS remains unchanged. The iteration number i is being increased to 5.

Iteration (5)

For the fifth iteration $i = 5$

N [5] ={4,5,6}

$w_5(f) = f(N[5])$

$w_5(f) = f(4) + f(5) + f(6) = 0 + 0 + 0 = 0$

The first condition of if-end if is satisfied. Since $w_5(f) = 0$, we find $M_0(5)=6$, $M_1(5)=5$and$M_2(5)=4$.

Then set . Also f(4)=1, f(5)=1 and f(6)=1also set

$$\text{PODS} = \text{PODS} \cup \{5\} \Rightarrow \text{PODS} = \{1,5\}$$

The iteration number i is being increased to 6.

Iteration (6)

For the sixth iteration $i = 6$

N [6]={4,5,6,7}

$$w_6(f) = f(N[6])$$
$$w_6(f) = f(4) + f(5) + f(6) + f(7) = 1 + 1 + 1 + 0 = 3$$

In this iteration also PODS remains unchanged. The iteration number i is being increased to 7

Iteration (7)

For the seventh iteration $i = 7$

N [7]={6,7,8,9}

$$w_7(f) = f(N[7])$$
$$w_7(f) = f(6) + f(7) + f(8) + f(9) = 1 + 0 + 0 + 0 = 1$$

In this iteration PODS remains unchanged. The iteration number i is being increased to 8.

Iteration (8)

For the eighth iteration $i = 8$

N [8]={7,8,9}

$$w_8(f) = f(N[8])$$
$$w_8(f) = f(7) + f(8) + f(9) = 0 + 0 + 0 = 0$$

Here set f(7)=1,f(8)=1 and f(9)=1 and take the next odd vertex 9 into PODS

The iteration number i is being increased to 9.

Iteration (9)

For the ninth iteration $i = 9$

N [9]={7,8,9}

$$w_9(f) = f(N[9])$$
$$w_9(f) = f(7) + f(8) + f(9) = 1 + 1 + 1 = 3$$

In this iteration PODS could not be calculated. Hence PODS remains unchanged.

$$\therefore \text{PODS} = \{1,5,9\}$$

$$\Rightarrow |PODS| = \text{The cardinality of PODS} = 3.$$

Thus we get the non-spilt perfect odd vertices dominating set $<V - PODS>$ as follows,

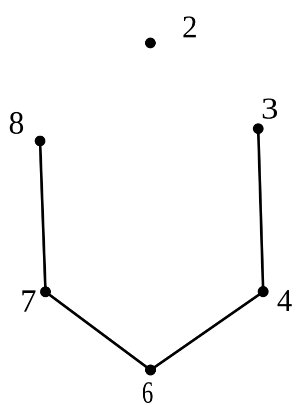

Fig.3: Vertex induced sub graph <*V* - *PODS*> - Disconnected graph from *G*

Hence the theorem is proved.

1.5.3. Theorem:

Let G be an interval graph corresponding to an n interval family $I = \{i_1, i_2, ..., i_n\}$. If i , j and k are any three intervals in I such that $i \in PODS$, $i = 1$, PODS is a perfect odd vertices dominating set and i contains j also there is no more interval to the right of j that intersects j , then split perfect odd vertices dominating set occurs in G as <V-PODS>=2.

Proof:

Let $I = \{i_1, i_2, ..., i_n\}$ be the given n interval family and G is an interval graph corresponding to I. First we will find the perfect odd vertices dominating set corresponding to G using the algorithm as explained in section 2.4. Now let i=1 be an interval contains the interval j. Suppose the interval k intersect j to its right then it leads to the contradiction of split perfect domination. So, if j intersects one more interval other than k, it intersects i also as that interval already intersects other vertex in the perfect dominating set. So, we get contradiction. Other than that we get the split perfect odd vertices domination in *G*.

We will find the perfect odd vertices dominating set as follows from an interval family using the algorithm as explained in section 2.4.

1.5.4. Illustration:

Now, consider the following interval family *I*,

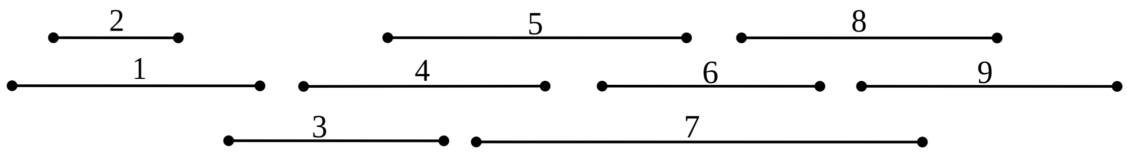

Fig.4: Interval family I

The corresponding interval graph G is as follows,

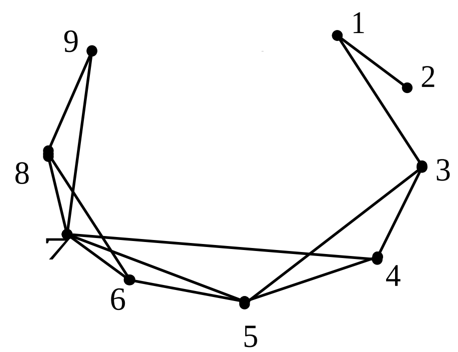

Fig.5: Interval graph *G*

The neighborhoods of each vertex from the interval graph G are as follows,

nbd [1] = {1,2,3}

nbd [2] = {1,2}

nbd [3] = {1,3,4,5}

nbd [4] = {3,4,5,7}

nbd [5] = {3,4,5,6,7}

nbd [6] = {5,6,7,8}

nbd [7] = {4,5,6,7,8,9}

nbd [8] = {6,7,8,9}

nbd [9] = {7,8,9}

To find the restrained dominating set, we have to compute all the p - th numbered adjacent vertices as follows,

$M_i(v)\backslash v$	1	2	3	4	5	6	7	8	9
$M_0(v)$	3	2	5	7	7	8	9	9	9
$M_1(v)$	2	1	4	5	6	7	8	8	8
$M_2(v)$	1	-	3	4	5	6	7	7	7
$M_3(v)$	-	-	1	3	4	5	6	6	-
$M_4(v)$	-	-	-	-	3	-	5	-	-
$M_5(v)$	-	-	-	-	-	-	4	-	-

First set $f(j)=0, \forall j \in V$. In step 2, set $i=1$, PODS = ϕ, that is initially PODS is empty. Step 2 repeats for n times. Here $n=9$, the number of vertices in the interval graph G. As follows the iterations through the table,

Iteration (1)

For the first iteration $i=1$

$N[1]=\{1,2,3\}$

$w_1(f)=f(N[1])$
$w_1(f)=f(1)+f(2)+f(3)=0+0+0=0$

The first condition of if-end if is satisfied. Since $w_1(f)=0,$ we find $M_0(1)=3, M_1(1)=2$ and M_2 (1) =1 Then set $f(3)=1, f(2)=1$, f(1)= 1 Also set

$$\text{PODS} = \phi \cup \{1\} \Rightarrow \text{PODS} = \{1\}$$

Iteration (2)

N[2] = {1,2}

W_2(f) = f(N[2])

W_2(f) = f(1)+f(2)=1+1=2

So, in this iteration PODS could not be calculated. Hence PODS remains same and i is being increased to 3.

Iteration (3)

For the third iteration $i=3$

N [3] ={1,3,4,5}

$w_3(f)=f(N[3])$
$w_3(f)=f(1)+f(3)+f(4)+f(5)=1+1+0+0=2$

In this iteration PODS remains unchanged. The iteration number i is being increased to 4.

Iteration (4)

For the fourth iteration $i=4$

N [4] ={3,4,5,7}

$w_4(f)=f(N[4])$
$w_4(f)=f(3)+f(4)+f(5)+f(7)=1+0+0+0=1$

In this iteration also PODS remains unchanged. The iteration number i is being increased to 5.

Iteration (5)

For the fifth iteration $i=5$

N [5] ={3,4,5,6,7}

$w_5(f)=f(N[5])$
$w_5(f)=f(3)+f(4)+f(5)+f(6)+f(7)=1+0+0+0+0=1$

In this iteration also PODS remains unchanged. The iteration number i is being increased to 6.

Iteration (6)

For the sixth iteration $i=6$

N [6] = {5,6,7,8}

$w_6(f)=f(N[6])$
$w_6(f)=f(5)+f(6)+f(7)+f(8)=0+0+0+0=0$

The first condition of if-end if is satisfied. Since $W_6(f)=0$ we find M_3 (6)=5, M_2 (6)=6 and M_1 (6) =7 Then set f(5)= 1 f(6)= 1 and f(7)= 1 Also set

$$PODS = \{1,7\}$$

The iteration number i is being increased to 7

Iteration (7)

For the seventh iteration $i=7$

N [7]={4,5,6,7,8,9}

$$w_7(f)=f(N[7])$$
$$w_7(f)=f(4)+f(5)+f(6)+f(7)+f(8)+f(9)=0+1+1+1+0+0=3$$

In this iteration PODS remains unchanged. The iteration number i is being increased to 8.

Iteration (8)

For the eighth iteration $i=8$

N [8]={6,7,8,9}

$$w_8(f)=f(N[8])$$
$$w_8(f)=f(6)+f(7)+f(8)+f(9)=1+1+0+0=2$$

In this iteration PODS remains unchanged. The iteration number i is being increased to 9.

Iteration (9)

For the ninth iteration $i=9$

N [9]={7,8,9}

$$w_9(f)=f(N[9])$$
$$w_9(f)=f(7)+f(8)+f(9)=1+0+0=1$$

In this iteration also PODS could not be calculated. Hence PODS remains unchanged. Since 9 is the last vertex, iteration process will be end.

$$\therefore \text{PODS} = \{1,7\}$$

$$\Rightarrow |PODS| = \text{The cardinality of PODS} = 2.$$

Thus we get the spilt perfect odd vertices dominating set <*V-PODS*> as follows,

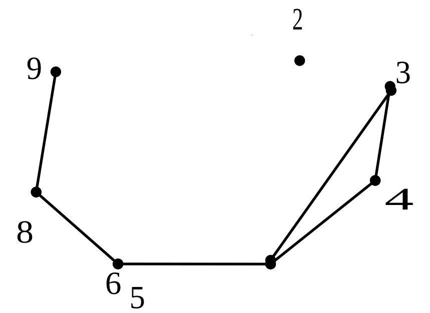

Fig.6: Vertex induced sub graph <*V - PODS*> - Disconnected graph from G

Hence the theorem is proved.

1.5.5. Theorem:

Let us consider an n interval family $I = \{i_1, i_2, \ldots\ldots, i_n\}$ and G be an interval graph of I. If i, j, k are three intervals such that j < i < k and $i \in PODS$, perfect odd vertices dominating set, i intersects j, j does not intersects k and i intersects k. Then split perfect odd vertices domination occurs in G and the split perfect odd vertices dominating set $<V - PODS>$ is disconnected as $|PODS| = 3$.

Proof:

Let $I = \{i_1, i_2, \ldots\ldots, i_n\}$ be an n interval family and G be an interval graph of I. Let i, j, k be three consecutive intervals satisfies the hypothesis. i.e., If i, j, k are three intervals such that j < i < k and $i \in PODS$, i intersects j, j does not intersects k and i intersects k. Now j intersects k for the vertices in the interval graph implies that j and k are adjacent in $<V - PODS>$. So, which leads to the contradiction that there will not be any connection for the vertices j and k in $<V - PODS>$.

Now we will find perfect odd vertices dominating set using the algorithm as given in section 2.4 as follows,

1.5.6. Illustration:

For this consider the interval family I as follows,

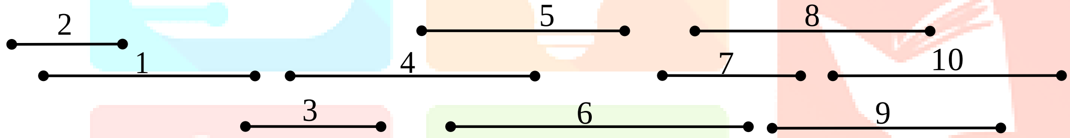

Fig.7: Interval family *I*

The corresponding interval graph G is as follows,

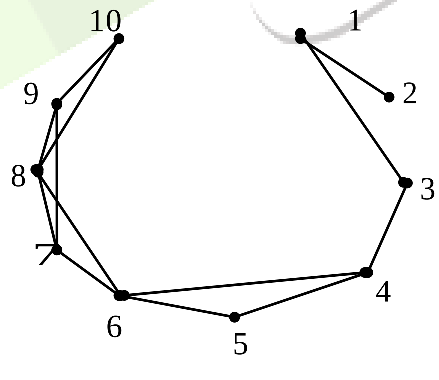

Fig.8: Interval graph *G*

The corresponding neighborhoods of each vertex of the above interval graph G are as follows,

nbd [1] = {1,2,3}   nbd [6] = {4,5,6,7,8}

nbd [2] = {1,2}   nbd [7] = {6,7,8,9}

nbd [3] = {1,3,4}   nbd [8] = {6,7,8,9,10}

nbd [4] = {3,4,5,6}   nbd [9] = {7,8,9,10}

nbd [5] = {4,5,6}   nbd [10] = {8,9,10}

To find the restrained dominating set, we have to compute all the p - th numbered adjacent vertices as follows,

$M_i(v)\backslash v$	1	2	3	4	5	6	7	8	9	10
$M_0(v)$	3	2	4	6	6	8	9	10	10	10
$M_1(v)$	2	1	3	5	5	7	8	9	9	9
$M_2(v)$	1	-	1	4	4	6	7	8	8	8
$M_3(v)$	-	-	-	3	-	5	6	7	7	-
$M_4(v)$	-	-	-	-	-	4	-	6	-	-

First set $f(j)=0, \forall j \in V$. In step 2, set $i=1$, PODS = ϕ, that is initially PODS is empty. Step 2 repeats for n times. Here $n=10$, the number of vertices in the interval graph *G*. As follows iterations,

Iteration (1)

For the first iteration $i=1$

$N[1]=\{1,2,3\}$

$w_1(f)=f(N[1])$

$w_1(f)=f(1)+f(2)+f(3)=0+0+0=0$

The first condition of if-end if is satisfied. Since $w_1(f)=0$, we find $M_0(1)=3, M_1(1)=2$ and also $M_2(1)=1$ Then set $f(3)=1, f(2)=1$ also f(1)=1 Also set

$$\text{PODS} = \phi \cup \{1\} \Rightarrow \text{PODS} = \{1\}$$

Iteration (2)

For the second iteration $i=2$

N[2] = {1,2}

$W_2(f) = f(N[2])$

$W_2(f) = f(1)+f(2)=1+1=2$

So, in this iteration PODS could not be calculated. Hence PODS remains same and i is being increased to 3.

Iteration (3)

For the third iteration $i=3$

N [3] ={1,3,4}

$w_3(f)=f(N[3])$

$w_3(f)=f(1)+f(3)+f(4)=1+1+0=2$

In this iteration PODS remains unchanged. The iteration number i is being increased to 4.

Iteration (4)

For the fourth iteration $i = 4$

N [4] ={3,4,5,6}

$$w_4(f) = f(N[4])$$
$$w_4(f) = f(3) + f(4) + f(5) + f(6) = 1 + 0 + 0 + 0 = 1$$

In this iteration also PODS remains unchanged. Here i is not the last vertex and $M_0(i)$ is adjacent to $M_1(i)$. The iteration number i is being increased to 5.

Iteration (5)

For the fifth iteration $i = 5$

N [5] ={4,5,6}

$$w_5(f) = f(N[5])$$
$$w_5(f) = f(4) + f(5) + f(6) = 0 + 0 + 0 = 0$$

The first condition of if-end if is satisfied. Since $W_5(f)=0$ we find $M_0(5)=6$, $M_1(5)=5$ and also $M_2(5)=4$ Then set f(4)=1 , f(5)=1 also f(6)=1 Also set

$$\text{PODS} = \{1,5\}$$

Iteration (6)

For the sixth iteration $i = 6$

N [6] = {4,5,6,7,8}

$$w_6(f) = f(N[6])$$
$$w_6(f) = f(4) + f(5) + f(6) + f(7) + f(8) = 1 + 1 + 1 + 0 + 0 = 3$$

In this iteration also PODS remains unchanged. The iteration number i is being increased to 7

Iteration (7)

For the seventh iteration $i = 7$

N [7]={6,7,8,9}

$$w_7(f) = f(N[7])$$
$$w_7(f) = f(6) + f(7) + f(8) + f(9) = 1 + 0 + 0 + 0 = 1$$

In this iteration PODS remains unchanged. The iteration number i is being increased to 8.

Iteration (8)

For the eighth iteration $i = 8$

N [8]={6,7,8,9,10}

$$w_8(f) = f(N[8])$$
$$w_8(f) = f(6) + f(7) + f(8) + f(9) + f(10) = 1 + 0 + 0 + 0 + 0 = 1$$

In this iteration PODS remains unchanged. The iteration number i is being increased to 9.

Iteration (9)

For the ninth iteration $i=9$

N [9]={7,8,9,10}

$w_9(f) = f(N[9])$

$w_9(f) = f(7) + f(8) + f(9) + f(10) = 0+0+0+0 = 0$

The first condition of if-end if is satisfied. Since $W_9(f)=0$ we find $M_3(9)=7$, $M_2(9)=8$, $M_1(9)=9$and also $M_0(9)=10$ Then set f(7)=1 , f(8)=1 also f(9)=1 Also set

$$\text{PODS} = \{1,5,9\}$$

Iteration (10)

For the ninth iteration$_{i=10}$

N [10]={8,9,10}

$w_{10}(f) = f(N[10])$

$w_{10}(f) = f(8) + f(9) + f(10) = 1+1+0 = 2$

PODS remains unchanged.

10 is the last vertex, then

$$\therefore \text{ PODS} = \{1,5,9\}$$

$$\Rightarrow |PODS| = \text{The cardinality of PODS} = 3.$$

Thus we get the spilt perfect odd vertices dominating set $<V - PODS>$ as follows,

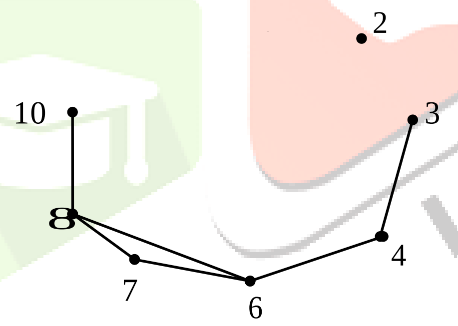

Fig.9: Vertex induced subgraph $<V – PODS>$ - disconnected graph from G

Hence the theorem is proved.

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