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HYPERSOFT GENERALIZED β- CLOSED SETS AND HYPERSOFT α-OPEN SETS IN HYPERSOFT TOPOLOGICAL SPACES

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Abstract: In this paper the concept of hypersoft α-open, hypersoft preopen, hypersoft semi open, β-open, semi-generalized closed set (hypersoft sg-closed), generalized-semi closed set(hypersoft gs-closed), α-generalized closed set(hypersoft αg-closed), hypersoft regular generalized closed set(hypersoft rg-closed) and hypersoft generalized β-closed set(hypersoft gβ-closed) in hypersoft topological space is discussed. Since of their properties are discussed in detail.

Keywords: hypersoft α-open, β-open, hypersoft preopen, hypersoft semi open, hypersoft sg-closed, hypersoft gs-closed, hypersoft αg-closed, hypersoft rg-closed, hypersoft gβ-closed

1. Introduction

Researcher deals the complexity of uncertain data in many fields such as economics, engineering, environment science, sociology, medical science etc. In 1999, Molodtsov initiated the novel concept of soft set as a new mathematical tool for dealing with uncertainties. Molodstov[6] (1999) proposed a completely new approach for modelling vagueness and uncertainty in soft set theory. Abbas, Murtaza and Smarandache[1] were introduced the basic operations on hypersoft sets and hypersoft point. . Sagvan Y. Musa, Baravan A [10] was developed the concept of Hypersoft topological spaces. Smarandache [12] expanded the notion of a soft set to a hypersoft set by substituting the function with a multi-argument function described in the cartesian product with a different set of parameters. This concept is more adaptable than the soft set and more useful when it comes to making decisions. Saeed, Ahsan and Rahman [7] introduced a Novel approach to mappings on hypersoft classes with application. Saeed, Ahsan and Siddique[8] presented a study of the fundamentals of hypersoft set theory. Shabir and Naz [9] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Saeed, M., Rahman, A., Ahsan[9] was introduced An Inclusive Study on Fundamentals of Hypersoft Set. In Theory and Application of Hypersoft Set. Later Zorlutuna et al[13] , Aygunoglu and Aygun[3], and Hussain et al. continued to study the properties of soft topological space. They got many important results in soft topological spaces. Arockiarani I and Arokialancy A[2] defined soft β-open sets and continued to study weak forms of soft open sets in soft topological space.

In this paper we have introduced hypersoft α-open, hypersoft preopen, hypersoft semi open, β-open, semi-generalized closed set (hypersoft sg-closed), generalized-semi closed set(hypersoft gs-closed), α-generalized closed set(hypersoft αg-closed), hypersoft regular generalized closed set(hypersoft rg-closed) and hypersoft generalized β-closed set(hypersoft gβ-closed) sets and studied their properties.

2. Preliminaries

Definition 2.1 [1]. Let X be an initial universe and let E be a set of parameters. Let $P(X)$ denote the power set of X and let A be a nonempty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A).

Definition 2.2 [6]. A soft set (F, A) over X is called a null soft set, denoted by Φ; if $e \in A$, $F(e) = 0$.

Definition 2.3 [6]. A soft set (F, A) over X is called an absolute soft set, denoted by $\tilde{A}$; if $e \in A$, $F(e) = X$.

Definition 2.4 [6]. The union of two soft sets of (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup \tilde{B}$ and, for all $e \in C$, $H(e) = \{\{ \{\{ \{ F(e)$, if $e \in A - B$, $G(e)$, if $e \in B - A$, $F(e) \cup \tilde{G}(e)$, if $e \in A \cap B$.$\tilde{}$ (1) We write $(F, A) \cup (G, B) = (H, C)\tilde{}$.

Definition 2.5 [6]. The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X, denoted $(F, A) \cap (G, B)\tilde{}$, is defined as $C = A \cap \tilde{B}$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.6[5]. A soft set (A, E) of a soft topological space (X, τ, E) is called soft α-open set if $(A, E) \tilde{\subseteq}$ int(cl(int(A, E))). The complement of soft α-open set is called soft α-closed set.

Definition 2.7[5]. A soft set (A, E) is called soft preopen set [9] (resp., soft semiopen [5]) in a soft topological space X if $(A, E) \tilde{\subseteq}$ int(cl(A, E)) (resp., $(A, E) \tilde{\subseteq}$ cl(int(A, E))).

Definition 2.8. [12] A pair (F, $A_1 \times A_2 \times \dots \times A_n$) is called a hypersoft set over U, where F is a mapping given by F : $A_1 \times A_2 \times \dots \times A_n \to P(U)$. Simply, we write the symbol E for $E_1 \times E_2 \times \dots \times E_n$, and for the subsets of E : the symbols A for $A_1 \times A_2 \times \dots \times A_n$, and B for $B_1 \times B_2 \times \dots \times B_n$. Clearly, each element in A, B and E is an n-tuple element. We can represent a hypersoft set (F,A) as an ordered pair, (F,A) = {(α, F(α)) :α ∈ A}.

Definition 2.9. [12] For two hypersoft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a hypersoft subset of (G,B) if

(1) A ⊆ B, and

(2) F(α) ⊆ G(α) for all α ∈ A. We write (F,A) ⊆ (G,B).

Definition 2.10. [12] Two hypersoft sets (F,A) and (G,B) over a common universe U are said to be hypersoft equal if (F,A) is a hypersoft subset of (G,B) and (G,B) is a hypersoft subset of (F,A).

Definition 2.11. [12] The complement of a hypersoft set (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, A)$ where $F^c : A \to P(U)$ is a mapping given by $F^c(\alpha) = U \setminus F(\alpha)$ for all α ∈ A .

Definition 2.12. [12] A hypersoft set (F,A) over U is said to be a relative null hypersoft set, denoted by (Φ,A), if for all α ∈ A, F(α) = ϕ.

Definition 2.13. [12] A hypersoft set (F,A) over U is said to be a relative whole hypersoft set, denoted by (X,A), if for all α ∈ A, F(α) = U.

Definition 2.14. **[12]** Difference of two hypersoft sets (F,A) and (G,B) over a common universe U, is a hypersoft set (H, C), where C = A ∩ B and for all α ∈ C, H(α) = F(α)\G(α). We write (F,A) \ (G,B) = (H, C).

Definition 2.15. [12] Union of two hypersoft sets (F,A) and (G,B) over a common universe U, is a hypersoft set (H, C), where C = A ∩ B and for all α ∈ C, H(α) = F(α) ∪ G(α). We write (F,A) ∪ (G,B) = (H, C).

Definition 2.16. [12] Intersection of two hypersoft sets (F,A) and (G,B) over a common universe U, is a hypersoft set (H, C), where C = A∩B and for all α ∈ C, H(α) = F(α)∩G(α). We write (F,A) ∩ (G,B) = (H, C).

Definition 2.17[12]. Let τ_H be the collection of hypersoft sets over U, then τ_H is said to be a hypersoft topology on U if

(1) (Φ,E), (X,E) belong to τ_H ,

(2) the intersection of any two hypersoft sets in τ_H belongs to τ_H ,

(3) the union of any number of hypersoft sets in τ_H belongs to τ_H .

Then (U, τ_H ,E) is called a hypersoft topological space over U.

Definition 2.18[] :A hypersoft set (H_S, E) is called a hypersoft generalized closed (hypersoft g-closed) in a hypersoft topological space (X,τ_H, E) if *cl* (H_S,E) (U,E) whenever (H_S,E) ⊂ (U,E) and (U,E) is hypersoft open in X

Definition 2.19[]: A hypersoft set (H_S,E) is called a hypersoft generalized open (hypersoft g-open) in a hypersoft topological space(X,τ_H, E) if the relative complement (H_S, E)' is hypersoft g-closed in X.

3.HYPERSOFT α-OPEN SETS

Definition 3.1

A hypersoft set (F_H, E) of a hypersoft topological space is said to be hypersoft **pre-open** if $(F_H, E) \subseteq int(cl(F_H, E))$ and hypersoft **pre-closed** if $cl(int(F_H, E)) \subseteq (F_H, E)$.

Definition 3.2

A hypersoft set (F_H, E) of a hypersoft topological space is said to be hypersoft **α-open** if $(F_H, E) \subseteq int(cl(int(F_H, E)))$ and hypersoft **α-closed** if $cl(int(cl(F_H, E))) \subseteq (F_H, E)$.

Definition 3.3

A hypersoft set (F_H, E) is called hypersoft preopen set in a hypersoft topological space X if, $(F_H, E) \subseteq int(cl(F_H, E))$ and hypersoft pre closed if $(F_H, E) \subseteq cl(int(F_H, E))$

Hypersoft open set
↓
Hypersoft α-open set ↗ Hypersoft semi open set
↘ Hypersoft preopen set

Proposition 3.4

Arbitrary union of hypersoft α-open set is a hypersoft α-open set.

Proof:

Let $\{(F_{Hi}, E) : i \in A\}$ be a collection of hypersoft α-open sets. Then for each $i \in A$

$(F_{Hi}, E) \subseteq int\,(cl\,(int\,(F_{Hi}, E)))$

Now $\cup\,(F_{Hi}, E) \subseteq \cup\, int\,(cl\,(int\,(F_{Hi}, E)))$

$= int(\cup\,(cl\,(int\,(F_{Hi}, E))))$

$\subseteq int\,(cl\,(\cup\,(int\,(F_{Hi}, E))))$

$= int\,(cl\,(int\,(F_{Hi}, E))))$

Hence the arbitrary union of that is $\cup\,(F_{Hi}, E)$ is a hypersoft α-open set.

Theorem3.5

Arbitrary intersection of hypersoft α-closed set is a hypersoft α-closed set.

Proof:

Let $\{(G_{Hi}, E) : i \in I\}$ be a collection of hypersoft α-closed sets.

Then $\cap\,\{(G_{Hi}, E) : i \in I\}$ is a collection of hypersoft α-open sets.

Then from the above theorem, $\{\{(G_H, E)_i\}$ is a hypersoft α-open set.

$((G_H, E)| \quad |i)^C$ is a hypersoft α-open set.

$(G_H, E)_i$ is a hypersoft α-closed set.

Hence the arbitrary intersection of hypersoft α-closed set is a hypersoft α-closed set.

4. HYPERSOFT GENERALIZED β-CLOSED SETS

Definition 4.1

A hypersoft subset (F_H, E) of a hypersoft topological space (X, τ_H, E) is called a hypersoft generalized β-closed set (**hypersoft gβ-closed**) if $\beta cl(F_H, E) \subseteq (U, E)$ whenever $(F_H, E) \subseteq (U, E)$ and (U,E) is hypersoft open in (X, τ_H, E).

Definition 4.2

A hypersoft set (F_H, E) of a hypersoft topological space (X, τ_H, E) is said to be hypersoft **β-open** if $(F_H, E) \subseteq cl(int(cl(F_H, E)))$ and hypersoft **β-closed** if $int(cl(int(F_H, E))) \subseteq (F_H, E)$.

Definition 4.3

A hypersoft subset (F_H, E) of a hypersoft topological space (X, τ_H, E) is called a hypersoft semi-generalized closed set (**hypersoft sg-closed**) if $scl(F_H, E) \subseteq (U, E)$ whenever $(F_H, E) \subseteq (U, E)$ and (U,E) is hypersoft semi open in X.

Definition 4.4

A hypersoft subset (F_H, E) of a hypersoft topological space (X, τ_H, E) is called a hypersoft generalized-semi closed set (**hypersoft gs-closed**) if $scl(F_H, E) \subseteq (U, E)$ whenever $(F_H, E) \subseteq (U, E)$ and (U,E) is hypersoft open in X.

Definition 4.5

A hypersoft subset (F_H, E) of a hypersoft topological space (X, τ_H, E) is called a hypersoft α-generalized closed set (**hypersoft αg-closed**) if $\alpha cl(F_H, E) \subseteq (U, E)$ whenever $(F_H, E) \subseteq (U, E)$ and (U,E) is hypersoft open in X.

Definition 4.6

A hypersoft subset (F_H, E) of a hypersoft topological space (X, τ_H, E) is called a hypersoft regular-generalized closed set (**hypersoft rg-closed**) if $cl(F_H, E) \subseteq (U, E)$ whenever $(F_H, E) \subseteq (U, E)$ and (U,E) is hypersoft regular open in X.

Theorem 4.7

Arbitrary union of hypersoft β-open set is a hypersoft β-open set.

Proof:

Let $\{(F_{Hi}, E) : i \in I\}$ be a collection of hypersoft β-open sets. Then for each $i \in I$

$(F_{Hi}, E) \subseteq cl\,(int\,(cl\,(F_{Hi}, E)))$

Now $\cup (F_{Hi}, E) \subseteq \cup (cl\,(int\,(cl\,(F_{Hi}, E))))$

$= cl\,(\cup\,(int\,(cl\,(F_{Hi}, E))))$

$\subseteq cl\,(int\,(\cup\,(cl\,(F_{Hi}, E))))$

$= cl\,(int\,(cl\,(F_{Hi}, E))))$

Hence the arbitrary union of that is $\cup (F_{Hi}, E)$ is a hypersoft β-open set.

Theorem 4.8

Arbitrary intersection of hypersoft β-closed set is a hypersoft β-closed set.

Proof:

Let $\{(G_{Hi}, E) : i \in I\}$ be a collection of hypersoft β-closed sets.

Then $\cap \{(G_{Hi}, E) : i \in I\}$ is a collection of hypersoft β-open sets.

Then from the above theorem, $\{\{(G_H, E)_i\}$ is a hypersoft β-open set.

$\left((G_H, E)\middle|\ \ \middle|i\right)^C$ is a hypersoft β-open set.

$(G_H, E)_i$ is a hypersoft β-closed set.

Hence the arbitrary intersection of hypersoft β-closed set is a hypersoft β-closed set.

Theorem 4.9

Every hypersoft closed set is hypersoft gβ-closed.

Proof:

Let (A_H,E) be a hypersoft closed set and let (B_H,E) be a hypersoft open set in X such that $(A_H,E) \subseteq (B_H,E)$

$cl(A_H,E) = (A_H,E) \subseteq (B_H,E)$

Thus, $\beta cl(A_H,E) \subseteq cl(A_H,E) \subseteq (B_H,E)$

Hence (A_H,E) is a hypersoft gβ-closed.

Theorem 4.10

Every hypersoft g-closed set is hypersoft gβ-closed.

Proof:

Suppose $(A_H,E) \subseteq (U,E)$ and (U,E) is hypersoft open in X.

By assumption $cl(A_H,E) \subseteq (U,E)$

Since $\beta cl(A_H,E) \subseteq cl(A_H,E) \subseteq (U,E)$

We have, $\beta cl(A_H,E) \subseteq (U,E)$

Hence (A_H,E) is a hypersoft gβ-closed.

Theorem 4.11

Every hypersoft sg-closed set is hypersoft gβ-closed.

Proof:

Consider $(A_H,E) \subseteq (U,E)$ and (U,E) is hypersoft open in X.

Always $\beta cl(A_H,E) \subseteq scl(A_H,E)$

Which implies, $\beta cl(A_H,E) \subseteq (U, E)$

Hence (A_H,E) is a hypersoft gβ-closed.

Theorem 4.12

Every hypersoft gs-closed set is hypersoft gβ-closed.

Proof:

We know that, $\beta cl(A_H,E) \subseteq scl(A_H,E)$ for every (A_H,E) of (X, τ_H, E).

Hence if $(A_H,E) \subseteq (U, E)$ and (U,E) is hypersoft open in X.

$\beta cl(A_H,E) \subseteq (U, E)$. hence (U,E) is hypersoft gβ-closed.

Theorem 4.13

Every hypersoft β-closed set is hypersoft gβ-closed.

Proof: The proof is obvious.

Theorem 4.14

Every hypersoft αg-closed set is hypersoft gβ-closed.

Proof:

If $(A_H,E) \subseteq (U,E)$ and (U,E) is hypersoft open set in X then,

$\alpha cl(A_H,E) \subseteq (U,E)$ which yields a relation $\beta cl(A_H,E) \subseteq \alpha cl(A_H,E) \subseteq (U,E)$

Thus (A_H,E) is a hypersoft gβ-closed.

Theorem 4.15

Every hypersoft α-closed set is hypersoft gβ-closed.

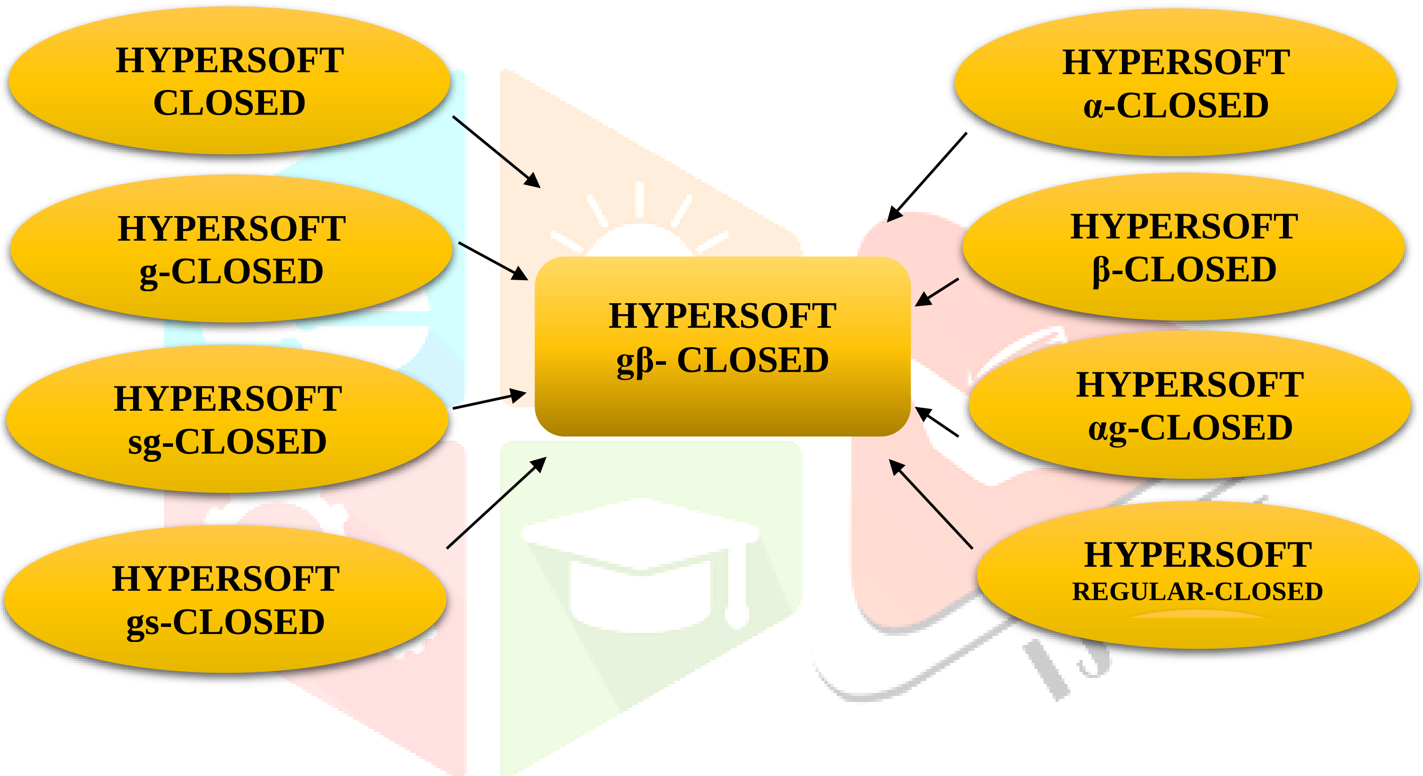
Proof: The proof is obvious and straight forward.

Theorem 4.16

Every hypersoft regular-closed set is hypersoft gβ-closed.

Proof: The proof is obvious and straight forward.

The following diagrammatic representation shows the relation between hypersoft gβ-closed sets with the other defined hypersoft sets.



Theorem 4.17

If (A_H,E) is a hypersoft open and hypersoft gβ-closed then (A_H,E) is a hypersoft β-closed.

Proof:

Suppose (A_H,E) is a hypersoft open and hypersoft gβ-closed, by definition,
$\beta cl(A_H,E) \subseteq (A_H,E)$ always $(A_H,E) \subseteq \beta cl(A_H,E)$
thus $(A_H,E) = \beta cl(A_H,E)$ that (A_H,E) is hypersoft β-closed.

Theorem 4.18

For any hypersoft set (A_H,E) in (X, τ_H, E) the following conditions are equivalent,

(i) (A_H,E) is hypersoft open and hypersoft gs-closed.
(ii) (A_H,E) is hypersoft regular open.

Proof:

(i) implies (ii)

$scl(A_H,E) \subseteq (A_H,E)$ since (A_H,E) is hypersoft open and hypersoft gs-closed.
Thus $scl(A_H,E) = (A_H,E)$ U $int(cl(A_H,E))$
Since (A_H,E) is hypersoft open, it is hypersoft pre-open thus, $(A_H,E) \subseteq int(cl(A_H,E))$
Therefore $int(cl(A_H,E)) \subseteq (A_H,E) \subseteq int(cl(A_H,E))$
Which implies $(A_H,E) = int(cl(A_H,E))$
Thus (A_H,E) is hypersoft regular open.

(ii) implies (i)

Let (A_H,E) is hypersoft regular open then $(A_H,E) = int(cl(A_H,E))$
From this we have, $int(cl(A_H,E)) \subseteq (A_H,E)$
Which implies (A_H,E) is hypersoft semi-closed and hence (A_H,E) is hypersoft sg-closed.
But every hypersoft sg-closed set is hypersoft gs-closed.
Hence (A_H,E) is hypersoft gs-closed.

Corollary 4.19

If (A_H,E) is hypersoft open and hypersoft gs-closed set of (X, τ_H, E) then (A_H,E) is hypersoft semi-closed.

Proof:

Let (A_H,E) be hypersoft open and hypersoft gs-closed set
Which gives $scl(A_H,E) \subseteq (A_H,E)$ but $(A_H,E) \subseteq scl((A_H,E))$
Which implies $(A_H,E) = scl\ (A_H,E)$
Therefore (A_H,E) is hypersoft semi-closed.

Theorem 4.20

For any subset $(A_H,E) \subseteq (X, \tau_H, E)$ then the following conditions are equivalent,

(i) (A_H,E) is hypersoft open and hypersoft gβ-closed.
(ii) (A_H,E) is hypersoft regular open.

Proof:

(i) implies (ii)

By assumption $\beta cl(A_H,E) \subseteq (A_H,E)$, since (A_H,E) is hypersoft open and hypersoft gβ-closed,
it is true that, $int(cl\ (A_H,E) \subseteq (A_H,E)$
As $\beta cl\ (A_H,E)= (A_H,E)$ U $int(cl(A_H,E))$ and (A_H,E) is hypersoft pre-open,
we have $(A_H,E) \subseteq int(cl\ (A_H,E)$
Hence (A_H,E) is hypersoft regular open.

(ii) implies (i)

If (A_H,E) is hypersoft regular open then it is hypersoft open.
As $(A_H,E) = int(cl(A_H,E))$ we have $int(cl(A_H,E)) \subseteq (A_H,E)$
Thus (A_H,E) is hypersoft semi-closed. Every hypersoft semi-closed set is hypersoft β-closed.
By theorem, (A_H,E) is hypersoft gβ-closed.

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