



ANALYSIS ON MODELLING IN TWO PHASE BLOOD FLOW THROUGH ARTERIOLES IN CASE OF MAJOR β THALASSEMIA (COOLEY ANEMIA)

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Abstract: A person with major β Thalassemia has affected too few red blood cells. During the course of the disease, haemoglobin production decreases. When the amount of haemoglobin is low, the amount of oxygen supplied to different tissues is less. Affected individuals also suffer from a deficiency of red blood cells, known as anemia, which can lead to weakness, fatigue and further complications.

We implemented Herschel Bulkley's Non-Newtonian model has been transform in to bio-fluid mechanical form setup. Governing equations has been formulated in tensorial form. We collected pathological data of haemoglobin with respect to blood pressure of patients with Cooley's anemia. In Cooley's anemia, the role of haematocrit and blood pressure drop are clearly determined by graphically. The methods of resolution are both analytical and numerical.

Index Terms - Two phase blood flow, Herschel Bulkley's Non-Newtonian Model, Haematocrit, Blood Pressure.

I. INTRODUCTION

Major β Thalassemia is a hereditary blood disease. Due to this, haemoglobin starts to decrease in the body. Dr. Cooley was first reported in Detroit in 1925 as a major β Thalassemia, hence the name Cooley's anemia. It is common in Latin Americans, Middle Easterners and Asians. The presence of haemoglobin β -globins chain in the nasal cavity allows haemoglobin to carry oxygen to the tissues. In this condition, the red blood cells cannot carry oxygen properly.¹¹

Our proposed model assumes a homogeneous mixture of two blood components: the first is the liquid plasma phase and the second is the red blood cells, a fluid collected in transparent membranes in which it can change shape and layers dose from plasma. Some of them have. Both phases adopt a steady flow pattern with uniform velocity and axial symmetry.^[12, 1]

Constitution of blood:

Blood is a fluid that moves through the veins of the bloodstream. The percentage of blood cells in the whole blood is called haematocrit. The total number of leukocytes and platelets is negligible by only 1%. A haematocrit of 42% to 52% in men and 35% to 47% in women is usually considered normal.

II. Real Model:

2.1 Choice of the frame of reference:

In order to model the bleeding condition mathematically, we need to choose a reference frame, taking into account the complexity and generality of the blood flow problem, choose a generalized three-dimensional rectangular curvilinear co-ordinate system, abbreviated E^3 , which named after 3-dim Euclidean. We define blood flow-related quantities in tensorial terms. Let the co-ordinate axis be OX' , where O is the origin and $i = 1, 2, 3$. The mathematical description of the situation is that blood flow is determined by the blood velocity distribution function $v^k = v^k(x^i, t)$ is affected.

2.2 Choice of parameters:

Blood is a non-Newtonian fluid. The constitutive equation of the fluid is $\tau = \eta e^n$. If $n = 1$, the fluid is Newtonian in nature and $n \neq 1$ then the fluid is non-Newtonian in nature. Where the pressure is denoted by τ , the strain rate is denoted by e , τ is indicated by viscosity and depends on the nature of the liquid with the parameter n .

Now we have calculated the blood flow through these arterioles using the following component equation:

$$\tau' = \eta_m e^n + \tau_p (\tau' \geq \tau_p),$$

where τ_p is denoted by yield stress. When strain rate $e = 0$ ($\tau' \geq \tau_p$) a core region is formed which flow just like a plug.

2.3 Constitution of two-phase blood volume:

Blood is a complex fluid consisting of particulate corpuscles suspended in a non-Newtonian fluid. The particulate solids are red blood cells (RBCs), white blood cells (WBCs) and platelets. 55% Of the plasma and 45% of the blood cells in a whole blood and approximately 98% of RBCs in 45% of blood cells and there are a few parts (approximately 2%) of the other cells. Which are ignorable, so one phase of the blood's plasma and 2nd phase of blood is RBCs.

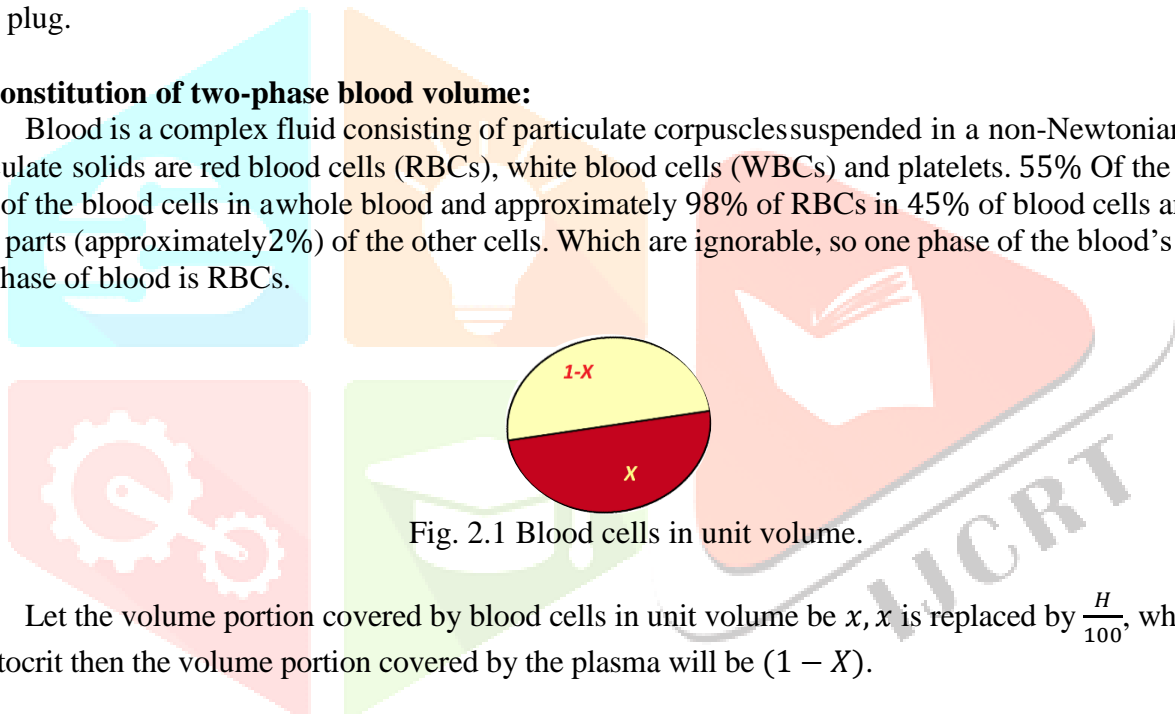


Fig. 2.1 Blood cells in unit volume.

Let the volume portion covered by blood cells in unit volume be x , x is replaced by $\frac{H}{100}$, where H is the hematocrit then the volume portion covered by the plasma will be $(1 - X)$.

III. Mathematical formulation:

Blood is a fluid and non-Newtonian; blood is not an ideal fluid for working with the equations of motion. Another important principle in hydropower is the conservation of energy. The equation of motion is based on this principle. According to this principle, the total energy of any fluid system is conserved in the absence of external forces.

$$\frac{dp}{dt} + P - F_v = 0 \quad (\text{External force}).$$

Blood can be considered as a two-dimensional homogeneous mixture. We get the basic equation of continuity, which is a mathematical expression of the conservation principle.

3.1 Equation of continuity:

If mass ratio of cells to plasma is r then

$$r = \frac{X\rho_c}{(1-X)\rho_p}, \quad (3.1)$$

where ρ_c and ρ_p are densities of red blood cells and plasma respectively. This mass ratio is usually not constant. However, in the present case the mass ratio is assumed to be constant [Upadhyay, 2000]. The mass ratio converts vessels to vessels. However, in specific tissues (i.e. arterioles, arteries, capillary) mass, the rate of increase is relatively constant. All components of the blood, red blood cells and plasma, move normally. We consider both the blood components separately.

$$(3.2) \quad \frac{\partial(X\rho_c)}{\partial t} + (X\rho_c v^i)_{,j} = 0$$

$$(3.3) \quad \text{and} \quad \frac{\partial[(1-X)\rho_p]}{\partial t} + [(1-X)\rho_p v^i]_{,j} = 0,$$

Where v^i = common velocity of two-phase blood cells and plasma, $(X\rho_c v^i)_{,j}$ is covariant derivative of $(X\rho_c v^i)_{,j}$ with respect to X^i and $[(1-X)\rho_p v^i]_{,j}$ is derivative of $[(1-X)\rho_p v^i]_{,j}$ with respect to X^i .

If we define the uniform density of the blood ρ_m as follow

$$(3.4) \quad \frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p}.$$

Combined both equation (3.2) and (3.3) we get

$$(3.5) \quad \frac{\partial\rho_m}{\partial t} + (\rho_m v^i)_{,j} = 0,$$

where $\rho_m = X\rho_c + (1-X)\rho_p$.

3.2 Equation of motion for blood flow:

The hydro dynamical pressure P between the two phases of blood can be supposed to be uniform because both the phases are always in equilibrium state in blood. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum. We get the equation of motion for two phase of blood cells as follows:

$$(3.6) \quad X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c v^i) v^i_{,j} = -X P_{,j} g^{ij} + X\eta_c (g^{jk} v^i_{,k})_{,j}.$$

The equation of motion for plasma will be as follows-

$$(3.7) \quad (1-X)\rho_p \frac{\partial v^i}{\partial t} + ((1-X)\rho_p v^i) v^i_{,j} = -(1-X) P_{,j} g^{ij} + (1-X)\eta_p (g^{jk} v^i_{,k})_{,j}.$$

Now adding equation (3.6) and (3.7) and using relation (3.4), the equation of motion for blood flow with the both phases will be as follows:

$$(3.8) \quad \rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^i) v^i_{,j} = -P_{,j} g^{ij} + \eta_m (g^{jk} v^i_{,k})_{,j}$$

where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood flow decreases, the viscosity of blood increases. The velocity of blood decreases successively because of the fact arterioles, venules and veins these vessels are relatively low. Secondly these vessels are relatively narrow down more rapidly. In this situation, the blood cells line up on the axis to build up rouleaux. Hence a yield stress is produced. Though this yield stress is very small, even then viscosity of blood is increased.

Applying Herschel Bulkley law on the two-phase blood flow through arterioles.

The constitutive equation is as follows:

$$\tau' = \eta_m e^n + \tau_p \quad (\tau' \geq \tau_p) \quad \text{and} \quad e = 0 \quad (\tau' < \tau_p),$$

where τ_p is yield stress. When strain rate $e = 0$ ($\tau' < \tau_p$) A core region is formed which flow just like a plug. Let the radius of the plug be r_p , the stress acting on the surface of plug will be τ_p . Equation the forces on the plug, we get

$$\Rightarrow \quad P\pi r_p^2 = \tau_p 2\pi r_p$$

$$\Rightarrow \quad r_p = 2 \frac{\tau_p}{P}. \quad (3.9)$$

The constitutive equation for test part of blood vessel is

$$\tau' = \eta_m e^n + \tau_p$$

$$\text{Or} \quad \tau' - \tau_p = \eta_m e^n = \tau_e,$$

where τ_e = effective stress.

Whose generalized form will be as follows –

$$\tau^{ij} = -P g^{ij} + \tau e^{ij},$$

where, $\tau e^{ij} = \eta_m (e^{ij})^n$ while $e^{ij} = g^{jk} v^i_{,k}$, where the symbols have their usual meanings.

Now we describe the basic equations for Herschel Bulkley blood flow as follow-

$$\text{Equation of continuity-} \quad \frac{1}{\sqrt{g}} (\sqrt{g} v^i)_{,i} = 0$$

Equation of motion-
$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_n v_i) v^i_{,j} = \tau^{ij}_{,j}$$

Where all the symbols have their usual meaning.

IV. Analysis:

The blood vessels supposed to be in this case are cylindrical; the above governing equations have to be transformed into cylindrical co-ordinate. As we know earlier:

$$X^1 = r, X^2 = \Theta, X^3 = z$$

Matrix of metric tensor in cylindrical co-ordinates is as follows:

$$g = |g_{ij}| = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = r^2$$

While matrix of conjugate matrix tensor is as follow-

$$[g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Whereas the Christoffel symbol of 2nd kind are as follow-

$$\left\{ \begin{matrix} 1 \\ 2 \end{matrix} \begin{matrix} 1 \\ 2 \end{matrix} \right\} = -r, \left\{ \begin{matrix} 2 \\ 2 \end{matrix} \begin{matrix} 2 \\ 1 \end{matrix} \right\} = \left\{ \begin{matrix} 2 \\ 1 \end{matrix} \begin{matrix} 2 \\ 2 \end{matrix} \right\} = \frac{1}{r}$$

Remaining others is zero.

Relation between contra variant physical components of the blood flow will be as follows.

$$\begin{aligned} \sqrt{g_{11}} v^1 &= v_1 \Rightarrow v_r \Rightarrow v^1 \\ \sqrt{g_{22}} v^2 &= v_2 \Rightarrow v_\theta \Rightarrow v^2 \\ \sqrt{g_{33}} v^3 &= v_3 \Rightarrow v_z \Rightarrow v^3 \end{aligned}$$

Again, the physical components of $p_{,j} g^{ij}$ is $-\sqrt{g_{11}} p_{,j} g^{ij}$.

Now, equation (3.9) & (3.10) are transformed into cylindrical form so as to solve them as power law model to find

Equation of continuity- $\frac{\partial v}{\partial z} = 0$

Equation of motion-

r – Component- $-\frac{\partial p}{\partial r} = 0$

θ – Component- $0 = 0$

z – Component- $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \left[r \left(\frac{\partial v_z}{\partial r} \right)^n \right]$.

Here, this fact has been taken in view that the blood flow is axially Symmetric in arteries concerned, i.e.

$$V_\theta = 0 \text{ and } V_r = 0$$

Here, this fact has been taken in view that the blood flow is axially symmetric in arterioles concerned, i.e. $v_p = 0$ and v_r, v_z and $p = p(z)$ and

$$0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \left[r \left(\frac{\partial v_z}{\partial r} \right)^n \right]$$

(4.1)

Since, pressure gradient $-\frac{\partial p}{\partial z} = P$

$$r \left(\frac{\partial v}{\partial z} \right)^n = -\frac{Pr^2}{2\eta_m} + A.$$

Applying boundary condition at $r = 0, v = v_0$ then we get $A = 0$

$$\Rightarrow -\frac{dv}{dr} = \left(\frac{Pr}{2\eta_m} \right)^{1/n}, \text{ here replace } r \rightarrow r_p$$

$$-\frac{dv}{dr} = \left(\frac{\frac{1}{2}Pr - \frac{1}{2}Pr_p}{\eta_m} \right)^{\frac{1}{n}},$$

$$\Rightarrow \frac{dv}{dr} = -\left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} (r - r_p)^{\frac{1}{n}}. \quad (4.2)$$

On integration above equation (4.2) under the no slip boundary condition $v = 0$ at $r = R$ so we get:

$$v = \left(\frac{P}{2\eta_m}\right)^{1/n} \frac{n}{n+1} \left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1} \right]. \quad (4.3)$$

This is the formula for velocity of blood flow in arterioles. Putting $r = r_p$ to get the velocity v_p of plug flow as follows:

$$v_p = \frac{n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1}, \quad (4.4)$$

where the value of r_p is taken from (3.7).

V. Patient case history-

Patient name- XYZ

Age- 27 Year

Sex- Male

Table 5.1 Diagnosis: Major β Thalassemia (Cooley's anemia)

Date	Haemoglobin (HB)	Haematocrit	Blood Pressure	Arteries Pressure Drop in Pascal sec. $\Delta P = p_f - p_i$	$\Delta P(m\ pa)$
31-05-23	6.9	20.9	109/70	-4844.033	0.004844
03-06-23	7.6	23.2	124/80	-5510.64267	0.005511
05-06-23	7.6	23.5	102/35	-4532.948	0.004533
15-06-23	8.1	24.8	98/61	-4355.1853334	0.0043552

The two-phase blood flow in arterioles is

$$Q = \int_0^{r_p} 2\pi r v_p dr + \int_{r_p}^R 2\pi r v dr$$

$$= \int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} dr + \int_{r_p}^R 2\pi r \left(\frac{P}{2\eta_m}\right)^{1/n} \frac{n}{n+1} \left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1} \right] dr$$

Using equation (4.2) and (4.4) we get

$$Q = \frac{2\pi n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} \left[\frac{r^2}{2} \right]_0^{r_p}$$

$$+ \frac{2\pi n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \left[\frac{r^2}{2} (R - r_p)^{\frac{1}{n}+1} - \frac{r(R - r_p)^{\frac{1}{n}}}{\frac{1}{n} + 2} + \frac{(R - r_p)^{\frac{1}{n}+3}}{\left(\frac{1}{n} + 1\right) \left(\frac{1}{n} + 3\right)} \right]_{r_p}^R$$

$$Q = \frac{\pi n}{n+1} \left(\frac{P}{2\eta_m}\right)^{1/n} R^{\frac{1}{n}+3} \left[\frac{r_p}{R^2} \left(1 - \frac{r_p^2}{R}\right)^{\frac{1}{n}+1} + \left(1 + \frac{r_p}{R}\right)^{\frac{1}{n}+1} \left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2} - \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2}}{\left(\frac{1}{n}+2\right)} + \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)} \right]. \quad (5.1)$$

Now we have $Q = 16.6667\ ml/s^{[4]}$

$$R = 1, r_p = 1/3$$

$$\eta_p = 0.0192\ mpa. s. \quad [4]$$

$$\eta_c = 0.01179\ mpa. s. \quad [4]$$

$$H = 23.2$$

$$\text{Arterioles pressure drop} = 0.005511\ mpa. s. \quad [4]$$

$$\text{Arterioles length} = 0.05\ mm \quad [4]$$

We know that $\eta_m = \eta_c X + \eta_p(1 - X)$, where $X = \frac{H}{100} = 0.209$

$$\eta_m = 0.01179(0.232) + 0.0192(1 - 0.232),$$

$$\eta_m = 0.00273528 + 0.014746$$

$$\eta_m = 0.01748128\ mpa. s.$$

Using this relation, we get

$$\eta_m = 0.01179 \left(\frac{H}{100} \right) + 0.0192 \left(1 - \frac{H}{100} \right)$$

$$\eta_m = 0.0001179H + 0.0192 - 0.0001192H$$

$$\eta_m = 0.0192 - 0.000741H$$

Now putting the value of r_p and R in equation (5.1) we get

$$Q = \frac{2\pi}{27} \left[\frac{P}{2\eta_m} \cdot \frac{2}{3} \right]^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

Let $A = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$

$$\frac{P}{3\eta_m} = \left(\frac{27 \cdot Q}{2\pi A} \right)^n \Rightarrow P = \left(\frac{27 \cdot Q}{2\pi A} \right)^n \cdot 3\eta_m$$

$$P = -\frac{dp}{dz} \Rightarrow -dp = Pdz$$

And limit of the pressure from z_f to z_i then

$$\int_{P_f}^{P_i} dP = - \int_{z_f}^{z_i} \left(\frac{27Q}{2\pi A} \right)^n \cdot 3\eta_m dz$$

Where $P_f - P_i =$ pressure drop and $z_f - z_i =$ arterioles length

$$P_f - P_i = \left(\frac{27Q}{2\pi A} \right)^n \cdot 3\eta_m (z_f - z_i)$$

$$\frac{27Q}{2\pi A} = \left(\frac{P_f - P_i}{3\eta_m (z_f - z_i)} \right)^{\frac{1}{n}}$$

$$\frac{27Q}{2\pi} = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right] \left(\frac{P_f - P_i}{3\eta_m (z_f - z_i)} \right)^{\frac{1}{n}}$$

$$\frac{27 * 16.6667}{2 * 3.14} = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right] \left(\frac{0.005511}{3 * 0.01748128 * 0.05} \right)^{\frac{1}{n}}$$

$$\frac{450.0009}{6.28} = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right] \left(\frac{0.005511}{0.002622192} \right)^{\frac{1}{n}}$$

$$71.6561942675 = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right] (2.10167676509)^{\frac{1}{n}}$$

On solving, we get

$$n = 0.179671$$

Now again using equation

$$P_f - P_i = \left(\frac{27Q}{2\pi A} \right)^n \cdot 3\eta_m (z_f - z_i)$$

$$\Delta P = (2.10168014178) \cdot (0.00288 - 0.000033345H)$$

$$\Delta P = 0.00605283881 - 0.00007008014178H.$$

Table 5.2: for Haematocrit v/s Blood Pressure drop

Haematocrit (H)	20.9	23.2	23.5	24.8
Blood Pressure Drop	0.00458816385	0.00442697952	0.00440595548	0.00431485128

Figure 5.1: Graphical Presentation of Pathological Data.

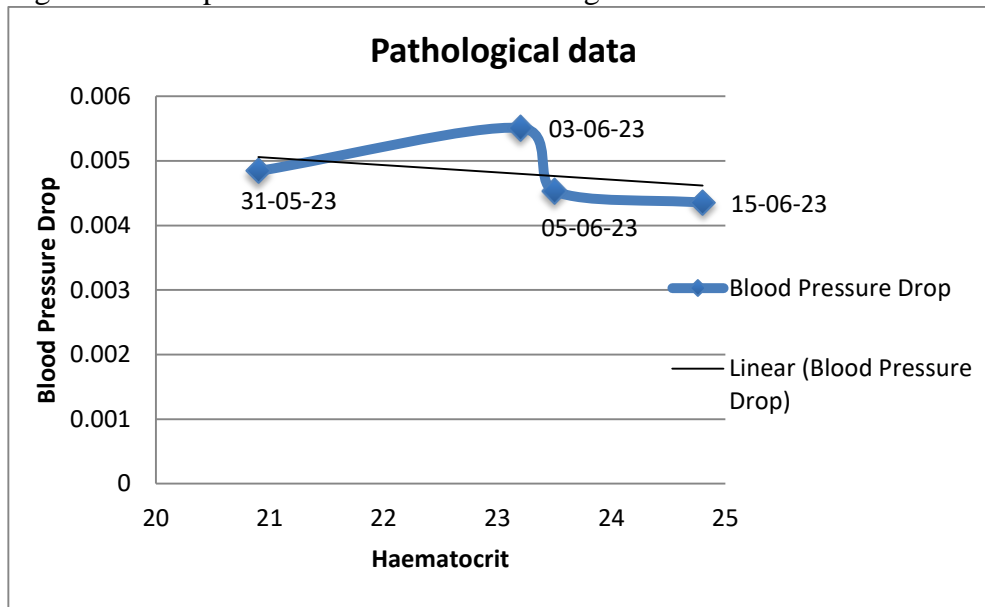
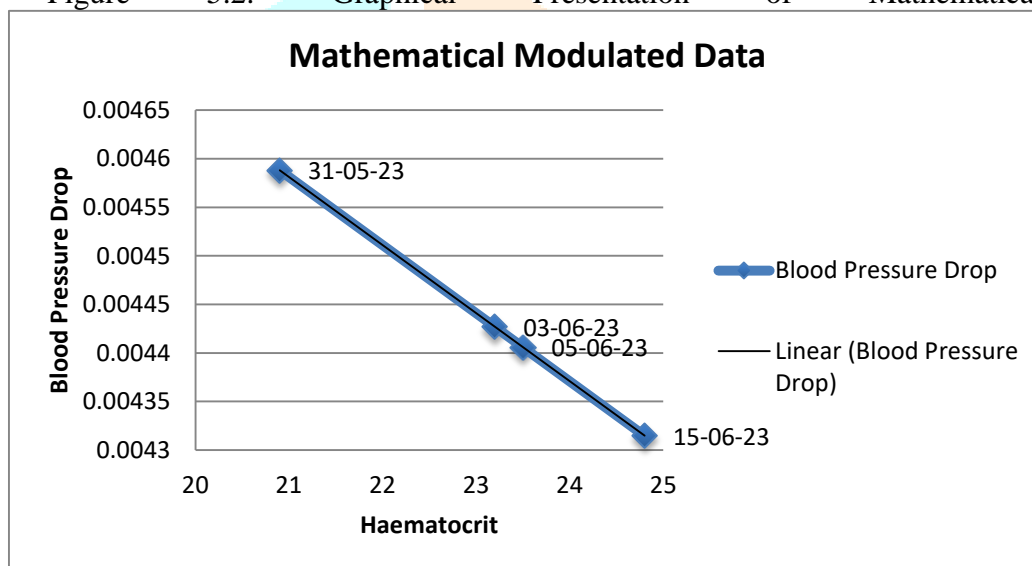


Figure 5.2: Graphical Presentation of Mathematical Modulated Data.

**Conclusion:**

According to this study of haematocrit graph and blood pressure drop in patients with silent. Haematocrit is increasing, blood pressure drop is decreasing and a linear graph is shown.

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